Multicomponent elastic imaging: new insights from the old equations

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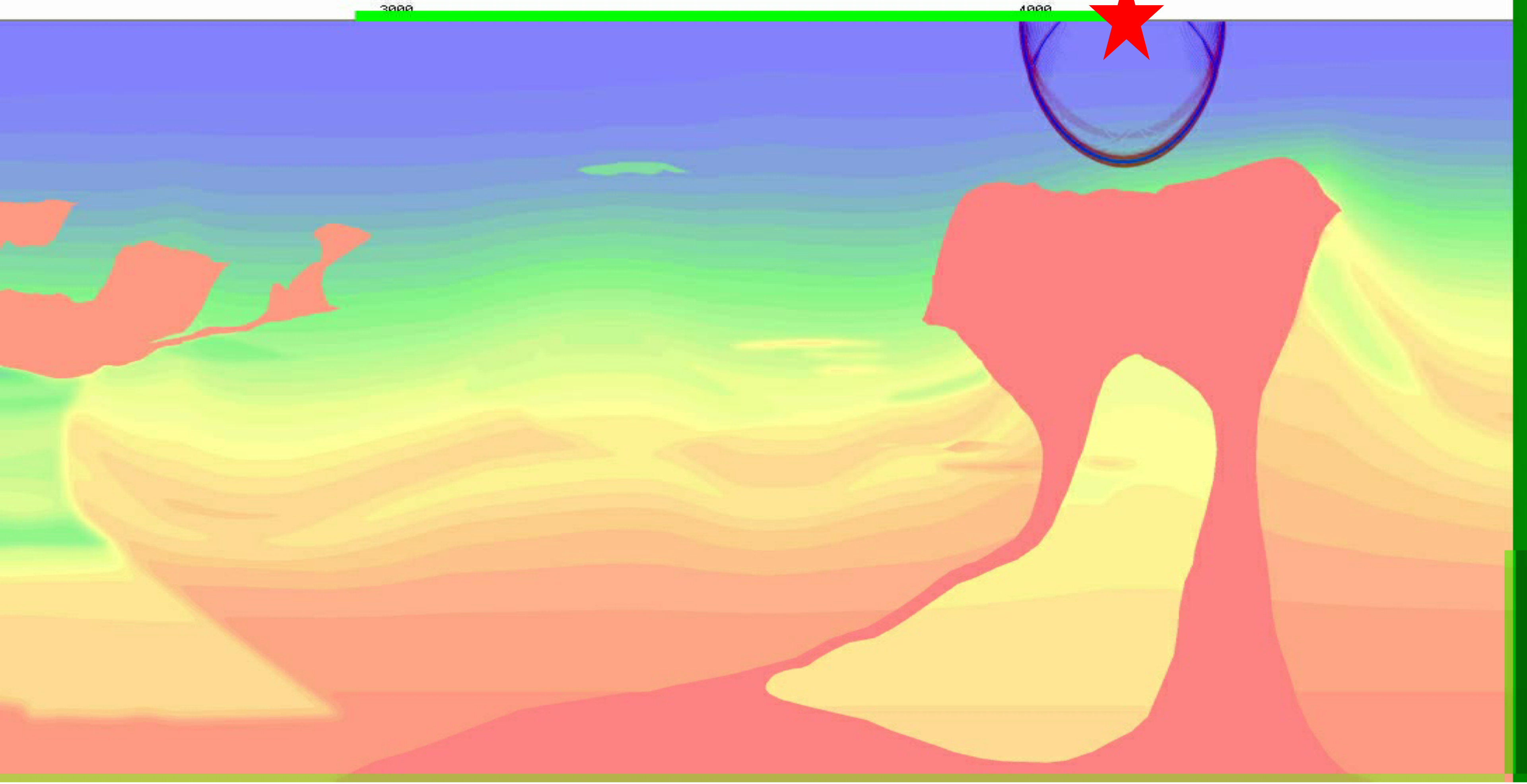
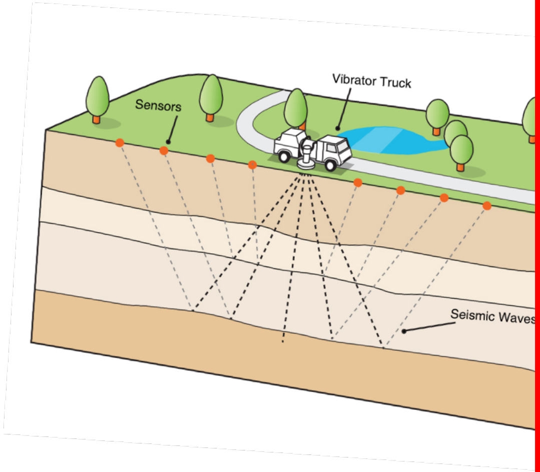
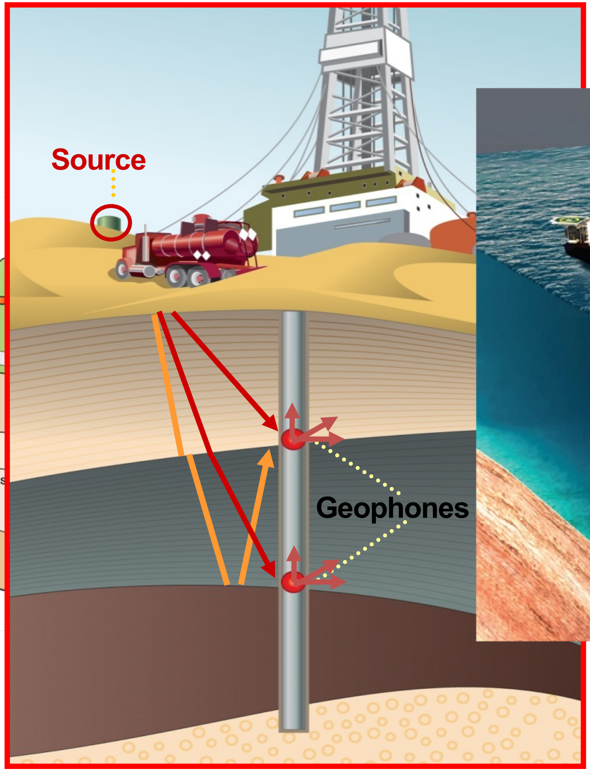
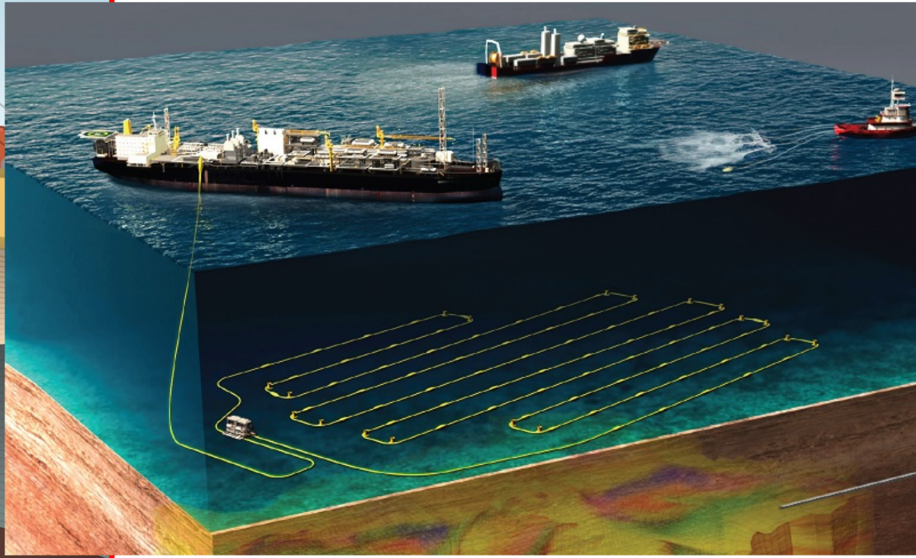


Simulation of a field scale seismic wave acquisition experiment

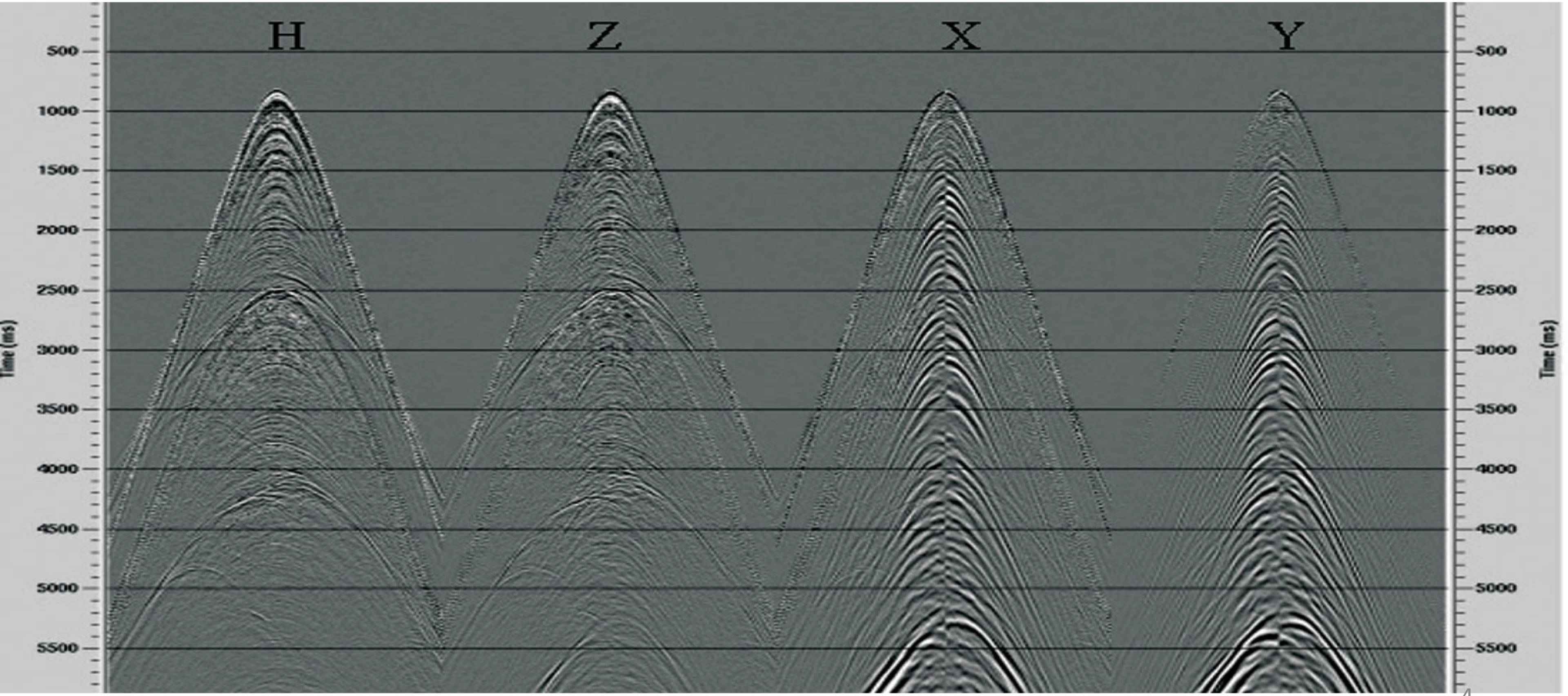
Multicomponent data acquisition







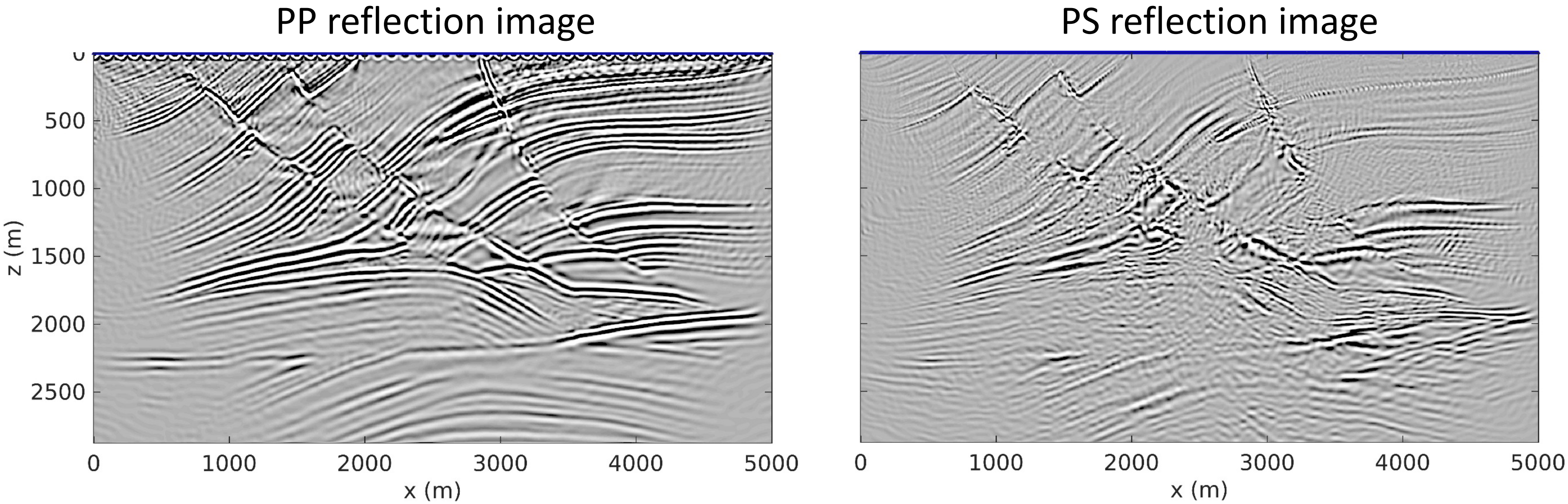
OBN acquisition: 4C data



Elastic imaging is not widely applied

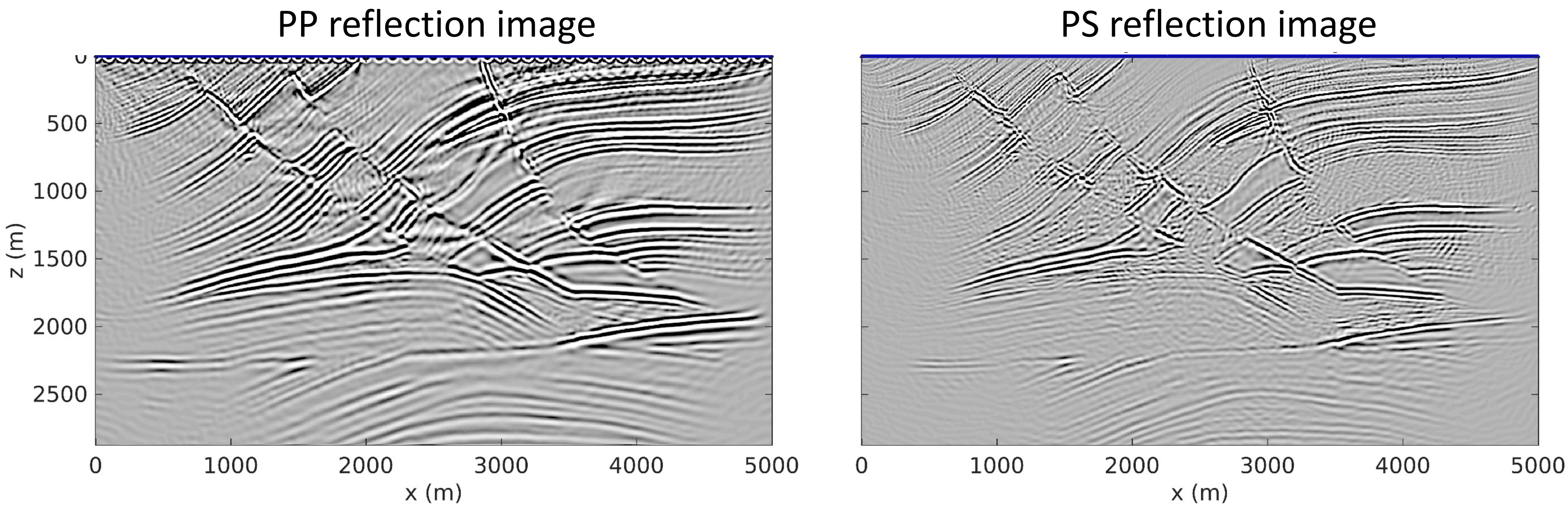
- Large computational cost compared with acoustic imaging (Kelly et al., 1976; Virieux, 1984, 1986)
  - 5 times in runtime and memory in 2D
  - 9 times in runtime and memory in 3D
- Deteriorated image for converted waves (Chang and McMechan, 1987; Yan and Sava, 2008; Cheng et al., 2016)
  - Polarity reversal at normal incidence
  - Complicated, cumbersome, and ad hock

Industry standard imaging algorithm



- Converted wave imaging appears noisier, less coherent, and challenging for joint interpretation
- Images are obtained with 5 times the computation and memory cost of the acoustic images

Proposed imaging algorithm



- ✓ Converted wave imaging shows consistent geological features with higher resolution
- ✓ Imaging cost are reduced by 60% in computation and 80% in memory

Outline

- Elastic wave equations
  - Revisit of the elastic wave equations
  - A new set of separated P- and S-wave equations
- The elastic imaging condition
  - PP and PS images from inverse problem formulation
  - Source-free converted wave imaging condition
- Discussions and conclusions

Seismology 101: elastodynamic system

- Linear, isotropic, elastic medium (Aki and Richards, 1980)

Newton's Law:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \tau_{ij} + f_i$$

Hooke's Law:

$$\tau_{ij} = \lambda \delta_{ij} \partial_k u_k + \mu(\partial_i u_j + \partial_j u_i)$$

u_i particle displacement

τ_{ij} element of the stress tensor

f_i force

ρ, λ, μ density and Lame constants

✧ Need to propagate (and store) 5 fields in 2D, and 9 fields in 3D

✧ Cannot interpret the P- and S-wave directly from the equations

Seismology 101: elastodynamic system

- The second-order system (Aki and Richards, 1980)

$$\rho\ddot{\mathbf{u}} = (\nabla\lambda)(\nabla\cdot\mathbf{u}) + \nabla\mu\cdot[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda+2\mu)\nabla\nabla\cdot\mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u} + \mathbf{f}$$

u_i particle displacement

ρ, λ, μ density and Lamé constants

✧ Need to propagate (and store) 3 fields in 2D, and 3 fields in 3D

✧ Require more strict stability condition

✧ Cannot interpret the P- and S-wave directly from the equations

P- and S-wave separation in homogenous medium

- Assuming constant density and smooth Lame constants

$$\ddot{\mathbf{u}} = \alpha \nabla \nabla \cdot \mathbf{u} - \beta \nabla \times \nabla \times \mathbf{u} + \mathbf{f}$$

$$\alpha = \frac{\lambda + 2\mu}{\rho} = V_p^2, \beta = \frac{\mu}{\rho} = V_s^2$$

$$\ddot{P} - \alpha \nabla^2 P = \nabla \cdot \mathbf{f}$$

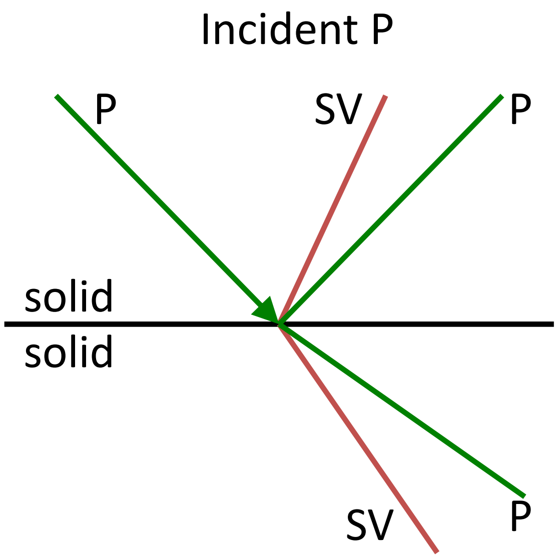
$$\ddot{\mathbf{S}} - \beta \nabla^2 \mathbf{S} = \nabla \times \mathbf{f}$$

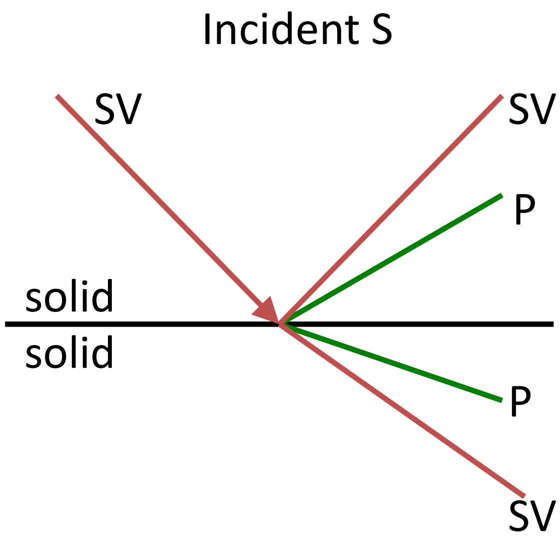
$$P = \nabla \cdot \mathbf{u}, \quad \mathbf{S} = \nabla \times \mathbf{u}$$

- ✧ Fully decoupled P- and S-wave propagations
- ✧ Cannot interpret the mode-conversion directly from the equations

(Aki and Richards, 1980)

Seismology 101: mode conversion



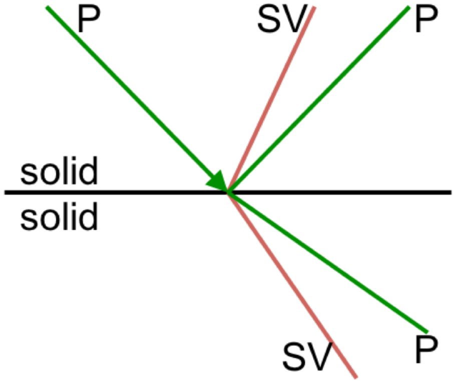


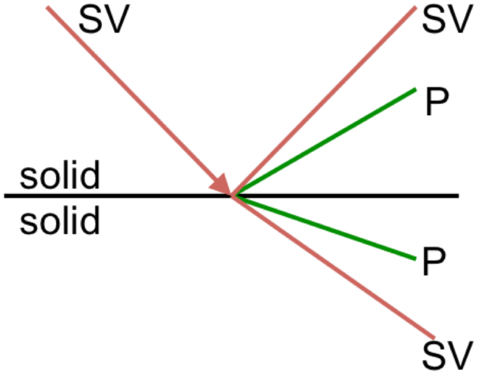
- ✧ Is mode conversion unconditional at solid interfaces?
- ✓ New set of equations: clear mode conversion and its condition

New set of separated P- and S-wave equations

$$
\begin{aligned}
\boxed{\ddot{P} - \alpha \nabla^2 P} = & \nabla \cdot \mathbf{f} && \leftarrow \text{Source term} \\
& + P\nabla^2\alpha + 2\nabla\alpha \cdot \nabla P && \leftarrow \text{P-wave interacts with } V_p \text{ boundary} \\
& - 2P\nabla^2\beta && \leftarrow \text{P-wave interacts with } V_s \text{ boundary} \\
& - 2\nabla\beta \cdot \nabla \times \mathbf{S} && \leftarrow \text{S-wave interacts with } V_s \text{ boundary}
\end{aligned}
$$

P-wave propagation





Li et. al., Geophysics, 2018

New set of separated P- and S-wave equations

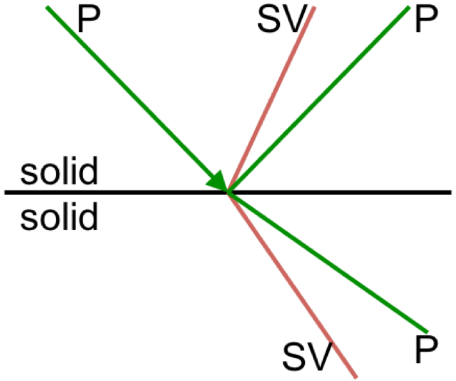
$$
\begin{aligned}
\boxed{\ddot{\mathbf{S}} - \beta \nabla^2 \mathbf{S}} = &\nabla \times \mathbf{f} \\
&+ \nabla \beta \cdot \nabla \mathbf{S} - (\nabla \beta) \times (\nabla \times \mathbf{S}) \\
&+ 2(\nabla \beta) \times (\nabla P)
\end{aligned}
$$

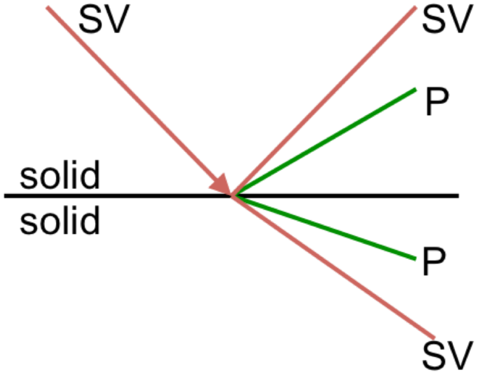
S-wave propagation

← Source term

← S-wave interacts with V_s boundary

← P-wave interacts with V_s boundary





Li et. al., Geophysics, 2018

Insights from the equations

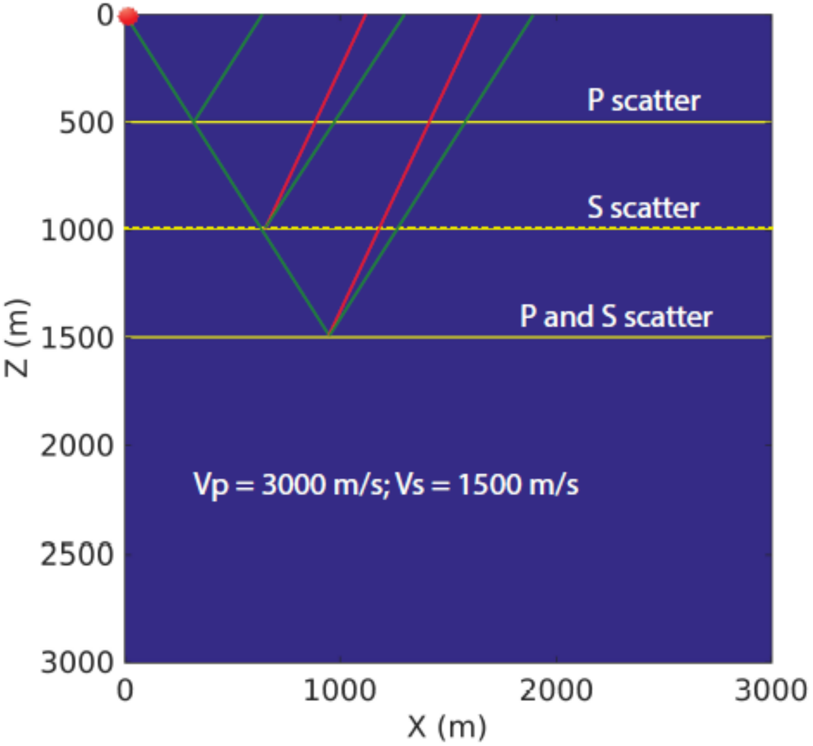
$$\ddot{P} - \alpha\nabla^2 P = P\nabla^2\alpha + 2\nabla\alpha\cdot\nabla P - 2P\nabla^2\beta \boxed{- 2\nabla\beta\cdot\nabla\times\mathbf{S}} + \nabla\cdot\mathbf{f}$$

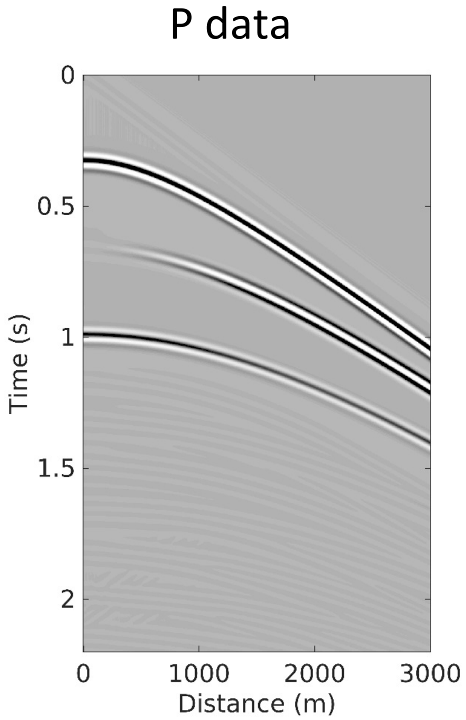
$$\ddot{\mathbf{S}} - \beta\nabla^2\mathbf{S} = \nabla\beta\cdot\nabla\mathbf{S} - (\nabla\beta)\times(\nabla\times\mathbf{S}) + \boxed{2(\nabla\beta)\times(\nabla P)} + \nabla\times\mathbf{f}$$

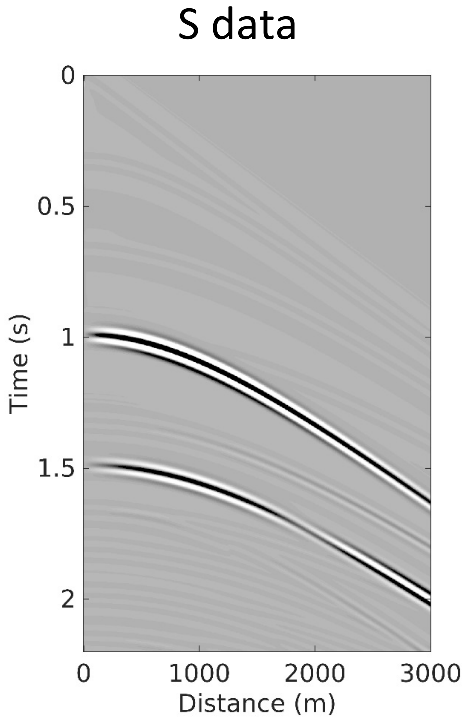
- ✓ New set of equations: coupled but separated for P- and S-propagations in heterogeneous (Lamé) media (constant density)
- ✓ Wave-medium interactions can be directly interpreted
- ✓ Mode-conversion only happens at S-wave discontinuities!
- ✓ Discontinuities only in V_p are transparent to S-wave

Li et. al., Geophysics, 2018

Elastic simulations in heterogeneous media







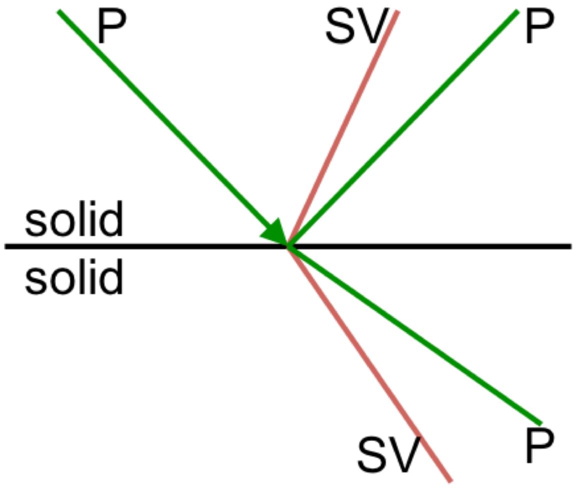
(Removed direct arrival)

Outline

- Elastic wave equations
  - Revisit of the elastic wave equations
  - A new set of separated P- and S-wave equations
- The elastic imaging condition
  - PP and PS images from inverse problem formulation
  - Source-free converted wave imaging condition
- Discussions and conclusions

Imaging condition

image = source wavefield meets scattered wavefield

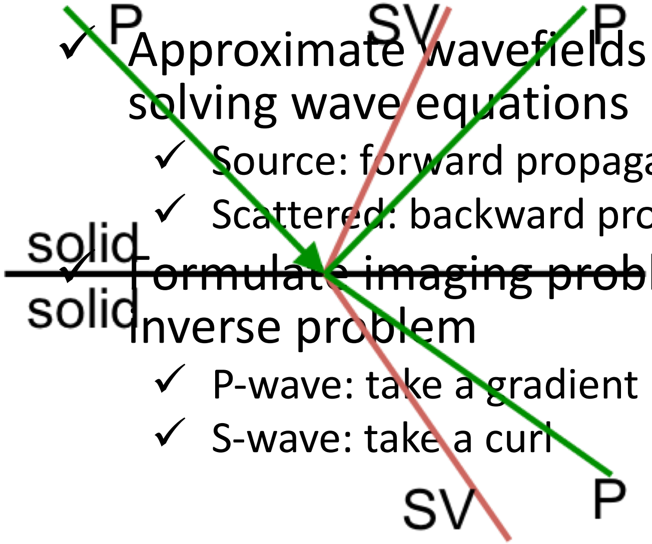


Imaging condition

image = source wavefield meets scattered wavefield

- ✧ Wavefields only recorded on the boundary
  - ✧ Source: source signature
  - ✧ Scattered: receiver recordings
- ✧ How do the wavefields meet?
  - ✧ P-wave: scalar
  - ✧ S-wave: vector

- ✓ Approximate wavefields by solving wave equations
  - ✓ Source: forward propagation
  - ✓ Scattered: backward propagation
- ✓ Formulate imaging problem as an inverse problem
  - ✓ P-wave: take a gradient
  - ✓ S-wave: take a curl



Imaging as an inverse problem

- Match the modeled P-wave data with the recorded P-wave data

$$J_p(\alpha, \beta) = \frac{1}{2}||d_p - d_{p_0}||_2^2$$

- Conventional PP-image

$$\nabla_\alpha J_p = \left(\frac{\partial P}{\partial \alpha}\right)^* \Bigg|_{\alpha=\alpha_0, \beta=\beta_0} (d_p - d_{p_0})$$
$$= 4 \boxed{\left(\nabla^2 P_0\right)}^* \boxed{(\Pi_p)^{-*} \delta d_p}$$

$$I_{pp} = \int_t dt \ \boxed{\text{Forward propagated source P-wavefield}} \times \boxed{\text{Backward propagated “scattered” P-wavefield}}$$

Li et. al., Geophysics, 2018

Imaging as an inverse problem

- Match the modeled S-wave data with the recorded S-wave data

$$J_s(\alpha, \beta) = \frac{1}{2}||\mathbf{d_s} - \mathbf{d_{s_o}}||_2^2$$

- Converted PS-image

$$\begin{aligned}\nabla_\beta J_s &= \left(\frac{\partial \mathbf{S}}{\partial \beta}\right)^* \Bigg|_{\alpha=\alpha_0, \beta=\beta_0} (\mathbf{d_s} - \mathbf{d_{s_o}}) \\ &= -\boxed{2(\nabla P_0)}^* \cdot \boxed{(\nabla \times \Pi_s^{-*} \delta \mathbf{d_s})}\end{aligned}$$

$$I_{ps} = \int_t dt \ \text{grad} \left(\boxed{\text{Forward propagated source P-wavefield}} \right) \times \text{curl} \left(\boxed{\text{Backward propagated “scattered” S-wavefield}} \right)$$

Li et. al., Geophysics, 2018

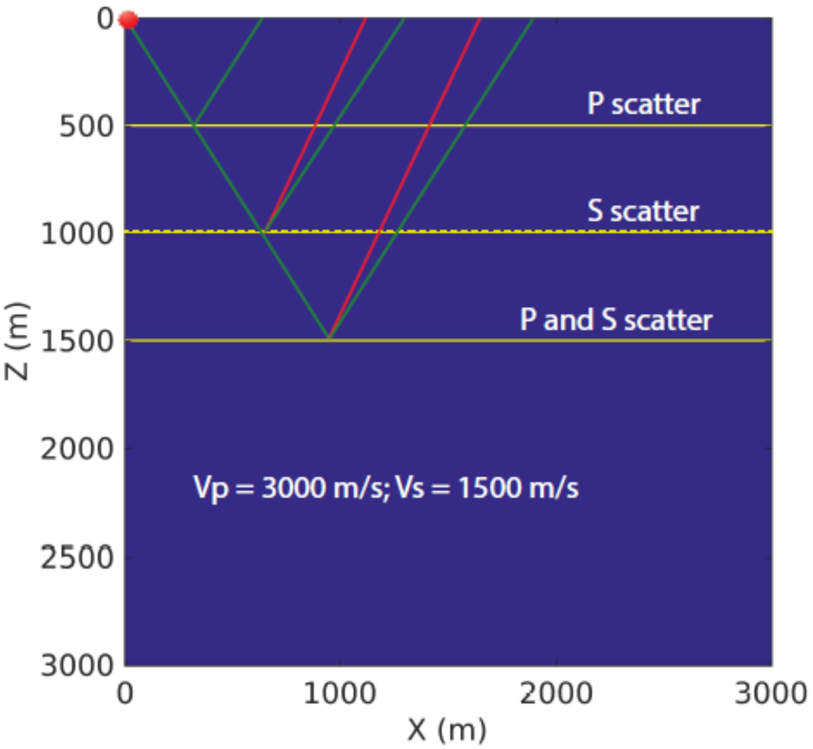
Elastic imaging using acoustic propagators

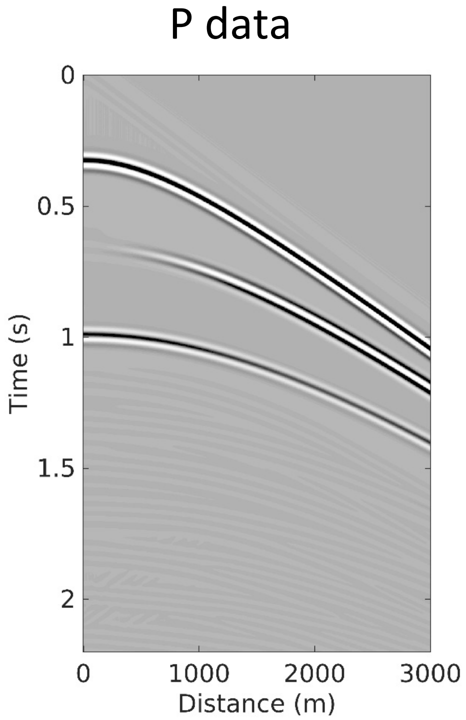
$$\nabla_\alpha J_p = \left(\frac{\partial P}{\partial \alpha}\right)^* \bigg|_{\alpha=\alpha_0,\beta=\beta_0} (d_p - d_{p_0})$$
$$=4\left(\nabla^2 P_0\right)^* (\Pi_p)^{-*}\delta d_p$$

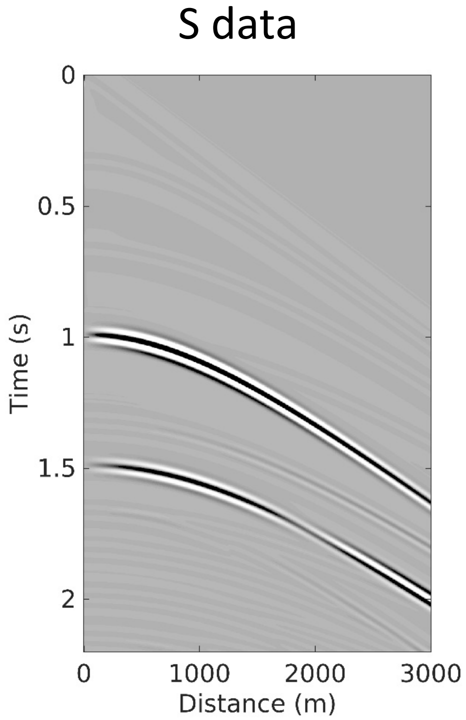
$$\nabla_\beta J_s = \left(\frac{\partial \mathbf{S}}{\partial \beta}\right)^* \bigg|_{\alpha=\alpha_0,\beta=\beta_0} (\mathbf{d_s} - \mathbf{d_{s_0}})$$
$$= -2(\nabla P_0)^* \cdot (\nabla \times \Pi_s^{-*}\delta \mathbf{d_s})$$

- Migration velocity models are often smooth
- Wave-equations reduce to fully decoupled P- and S-wave equations for their potential fields
- They can be efficiently solved using acoustic propagators

Elastic simulations in heterogeneous media







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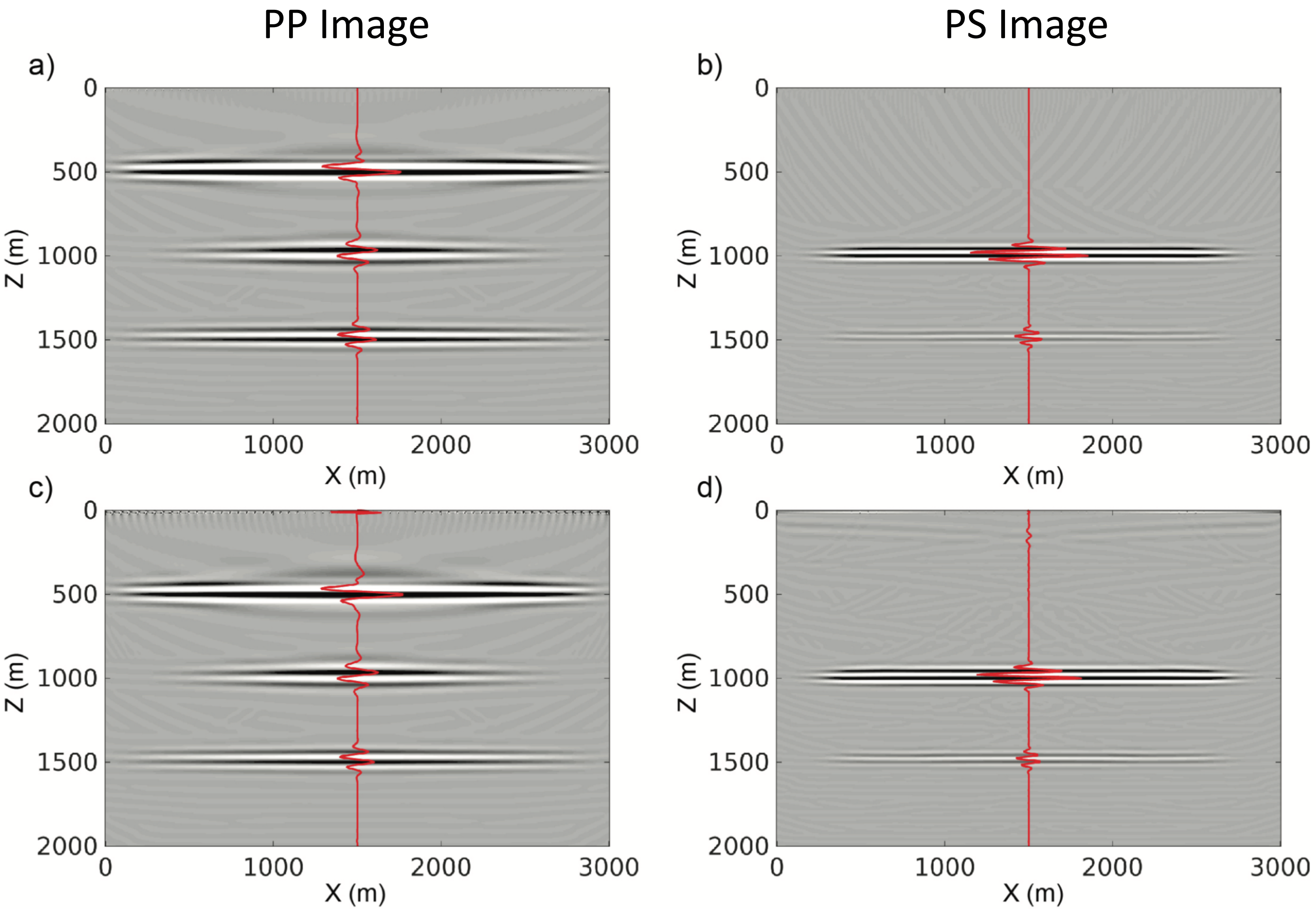
Using acoustic propagators

Using elastic propagators

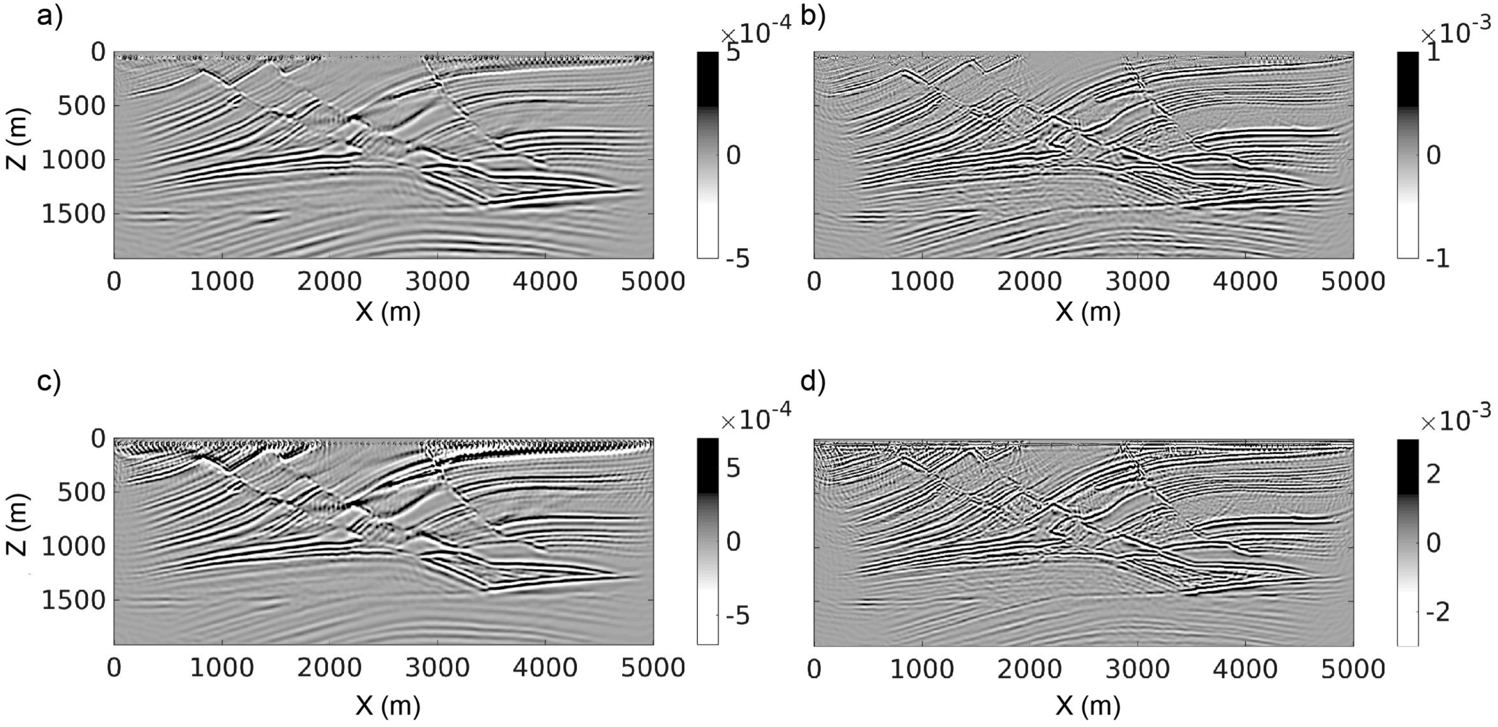


PP Image

PS Image

Using acoustic propagators

Using elastic propagators

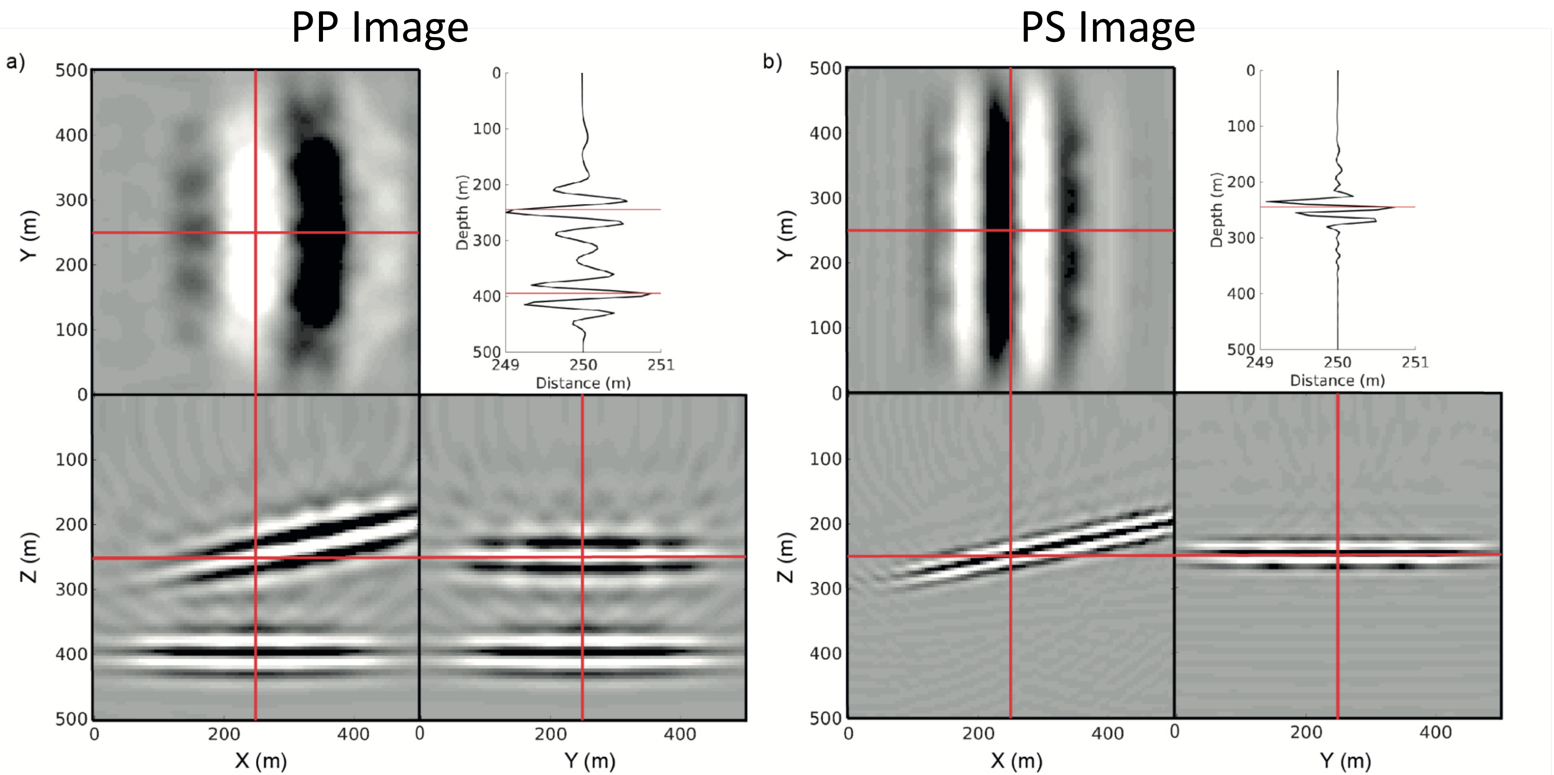


Comparison of the computational costs

Using / Cost	Acoustic propagator	Elastic propapagtors
Memory	nx*nz*3	nx*nz*3*5
Floating-point operations	O(nx*nz)	O(nx*nz*5)
# of simulations	2	1

Memory saving up to 80%, run time saving 60%
Run time saving up to 80%, memory saving 60%

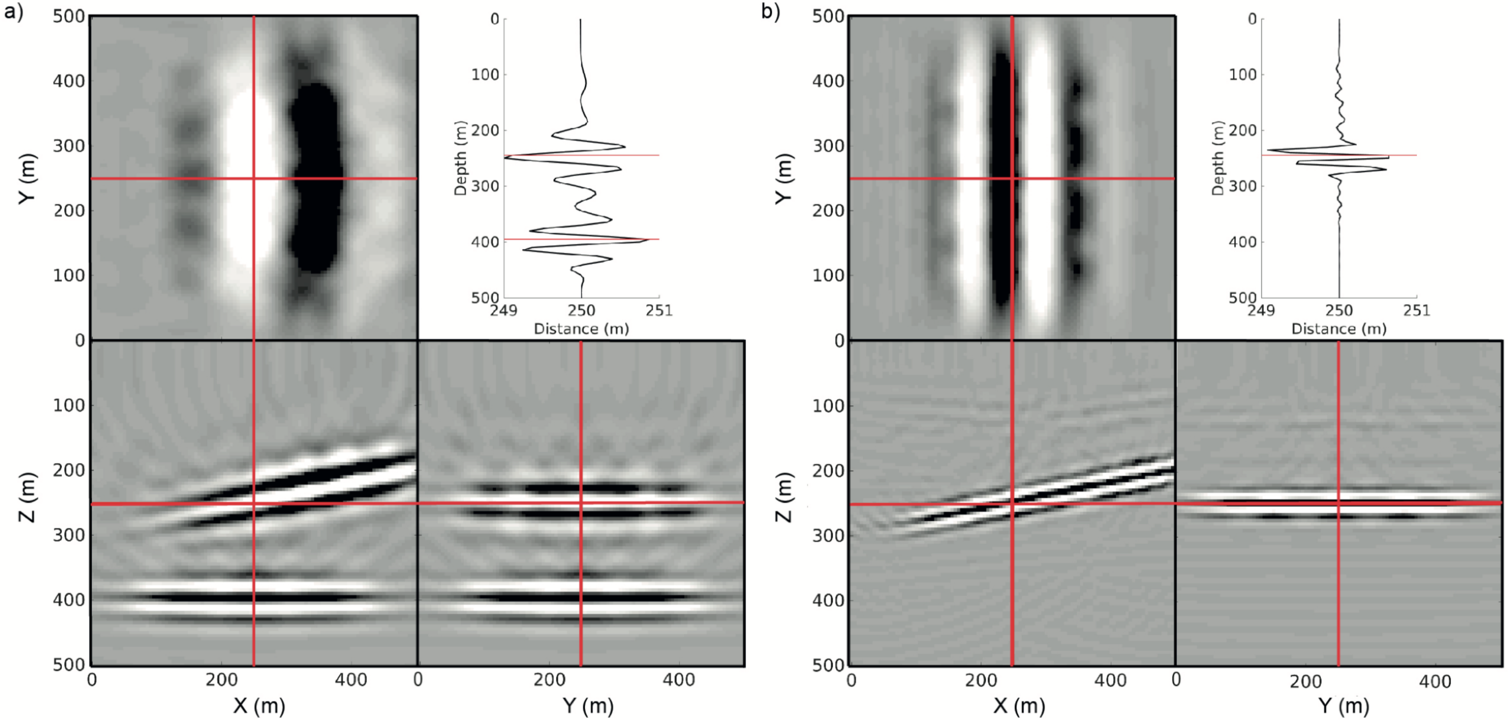
Elastic imaging in 3D using acoustic prop.



Elastic imaging in 3D using elastic prop.

PP Image

PS Image



Comparison of the computational costs

Using / Cost	Acoustic propagator	Elastic propapagtors
Memory	nx*ny*nz*3	nx*ny*nz*3*9
Floating-point operations	O(nx*ny*nz)	O(nx*ny*nz*9)
# of simulations	4	1

Memory saving up to 88.9%, run time saving 55.6%
Run time saving up to 88.9%, memory saving 55.6%

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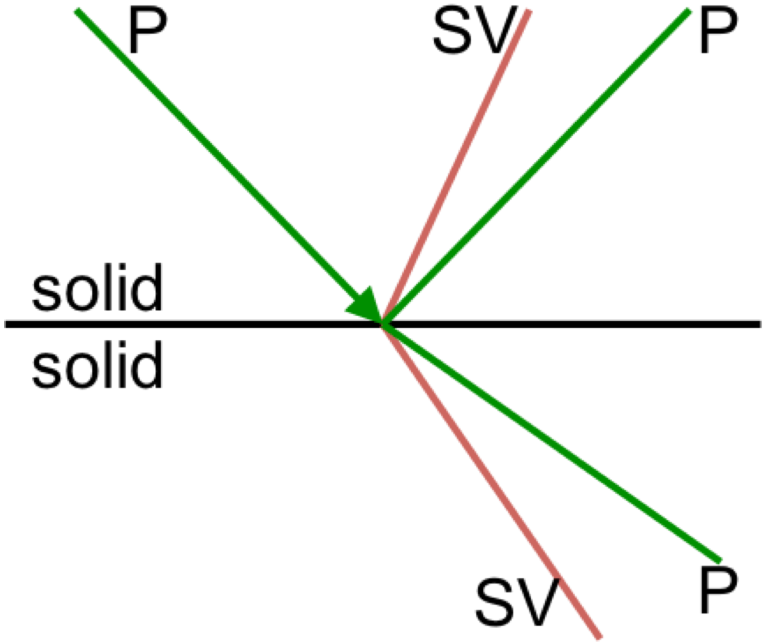
Source free converted imaging



Imaging as an inverse problem

- Match the modeled S-wave data with the recorded S-wave data with higher-order terms

$$J_s(\alpha, \beta) = \frac{1}{2}||\mathbf{d_s} - \mathbf{d_{s_o}}||_2^2$$

- Converted PS-image

$$\nabla_\beta J_s = \left(\frac{\partial \mathbf{S}}{\partial \beta}\right)^* \Bigg|_{\alpha=\alpha_0, \beta=\beta_0} (\mathbf{d_s} - \mathbf{d_{s_o}})$$
$$= -2\,(\nabla P_0)^* \cdot (\nabla \times \Pi_s^{-*} \delta \mathbf{d_s}) - 2\,\boxed{(\nabla \delta P)^*} \cdot \boxed{(\nabla \times \Pi_s^{-*} \delta \mathbf{d_s})}.$$

$$I_{sfps} = \int_t dt \; \mathrm{grad}\left(\boxed{\text{Backward propagated “scattered”P-wavefield}}\right) \times \mathrm{curl}\left(\boxed{\text{Backward propagated “scattered” S-wavefield}}\right)$$

Du et. al., Geophysics, 2019

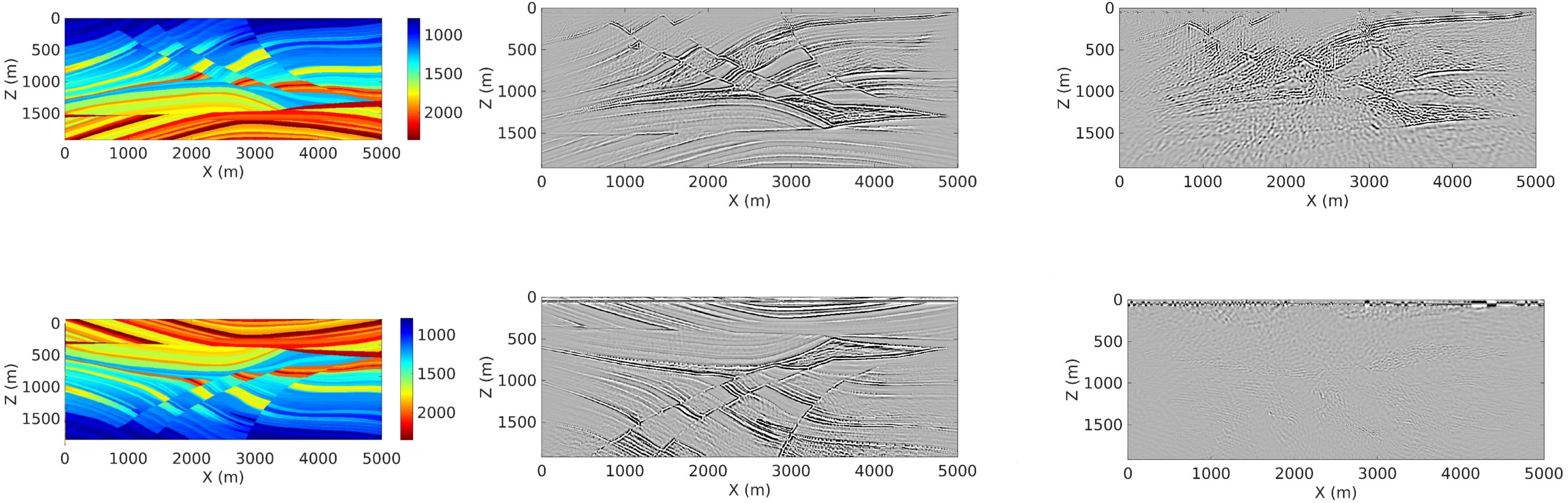
Velocity imprints by elastic propagators

$$\ddot{P} - \alpha\nabla^2 P = P\nabla^2\alpha + 2\nabla\alpha\cdot\nabla P - 2P\nabla^2\beta \boxed{- 2\nabla\beta\cdot\nabla\times\mathbf{S}} + \nabla\cdot\mathbf{f}$$

$$\ddot{\mathbf{S}} - \beta\nabla^2\mathbf{S} = \nabla\beta\cdot\nabla\mathbf{S} - (\nabla\beta)\times(\nabla\times\mathbf{S}) + \boxed{2(\nabla\beta)\times(\nabla P)} + \nabla\times\mathbf{f}$$

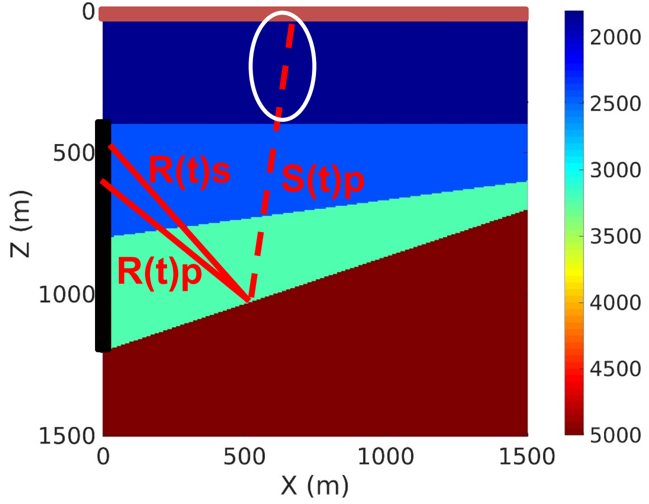
Elastic Propagator

Acoustic Propagator

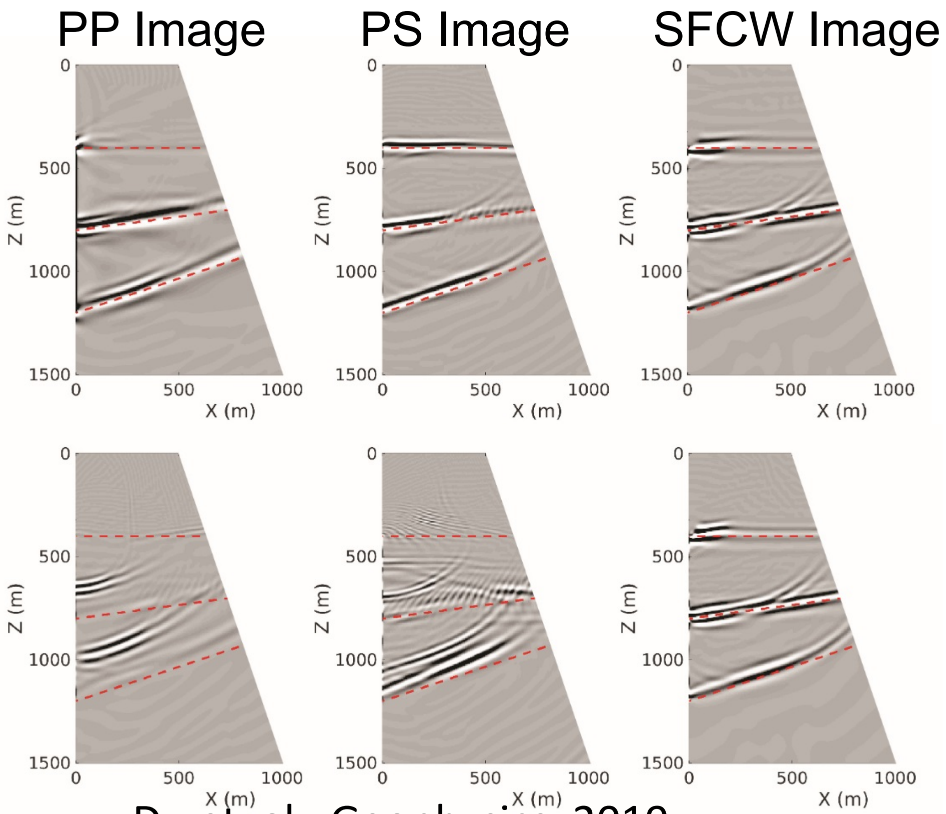


(Du et al., 2018)

Toy VSP imaging example

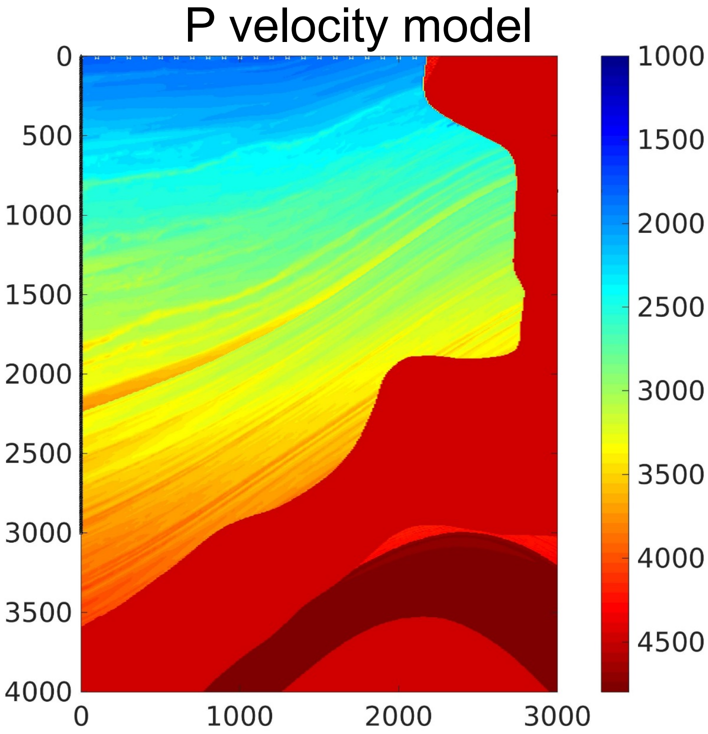


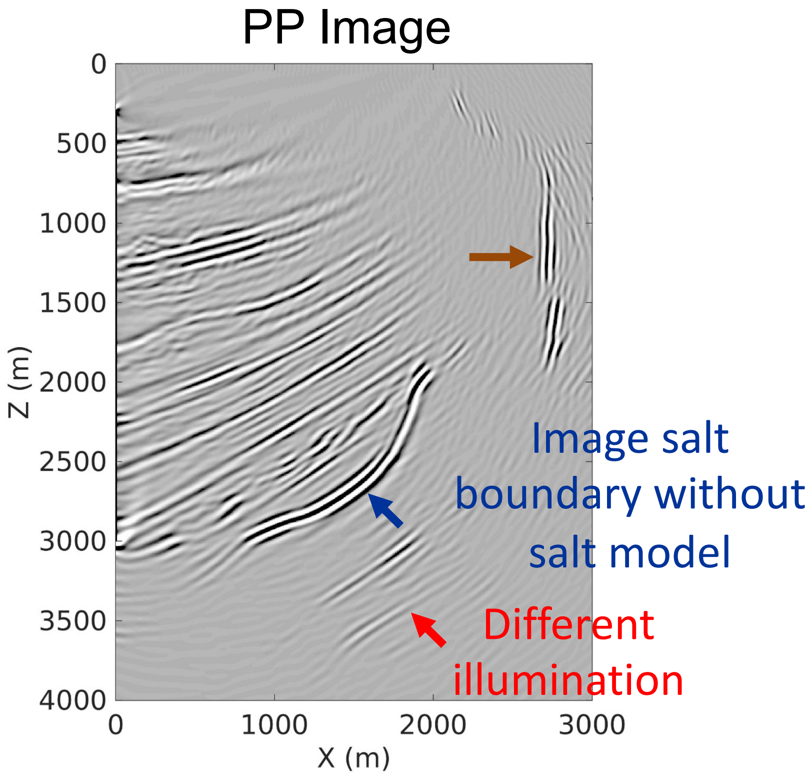
Too-slow near surface velocity



Du et. al., Geophysics, 2019

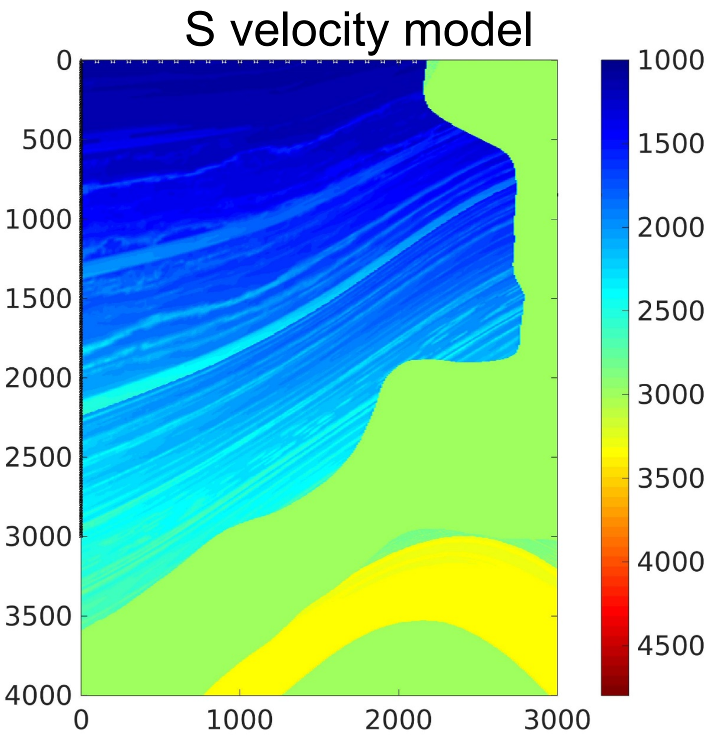
Near-salt SEAM model

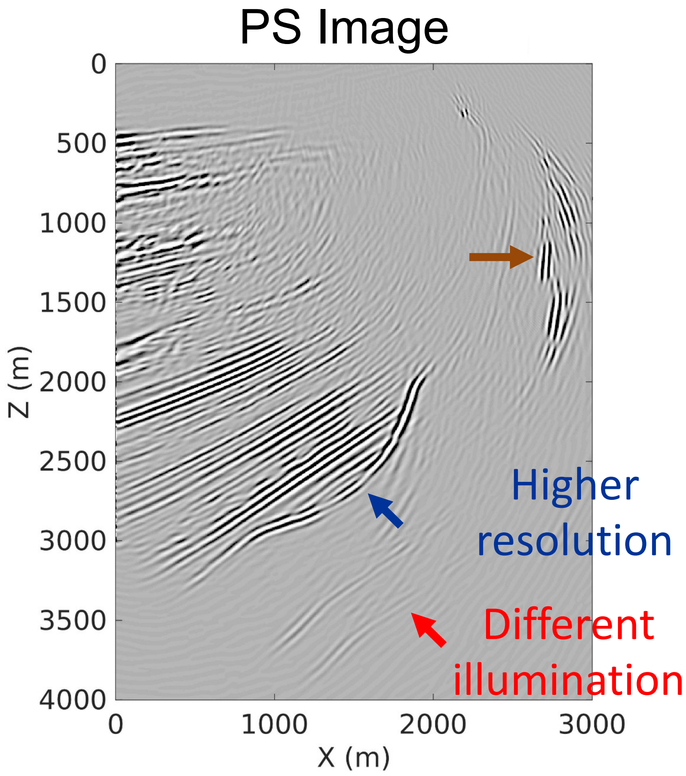




Du et. al., Geophysics, 2019

Near-salt SEAM model





Du et. al., Geophysics, 2019

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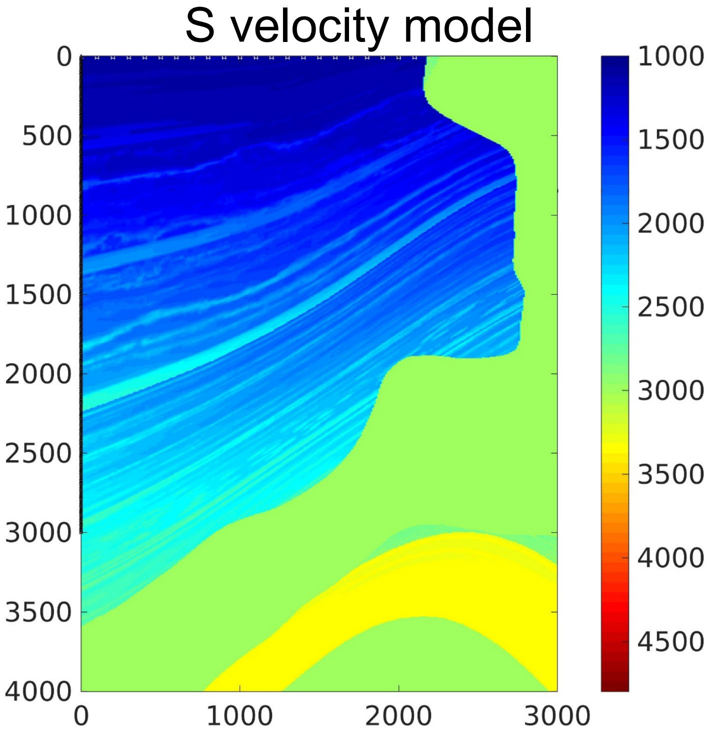
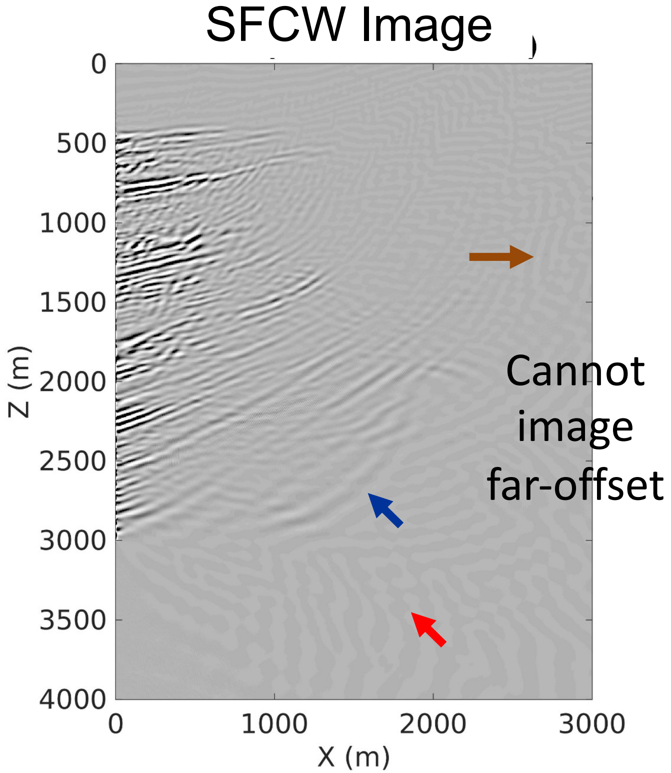
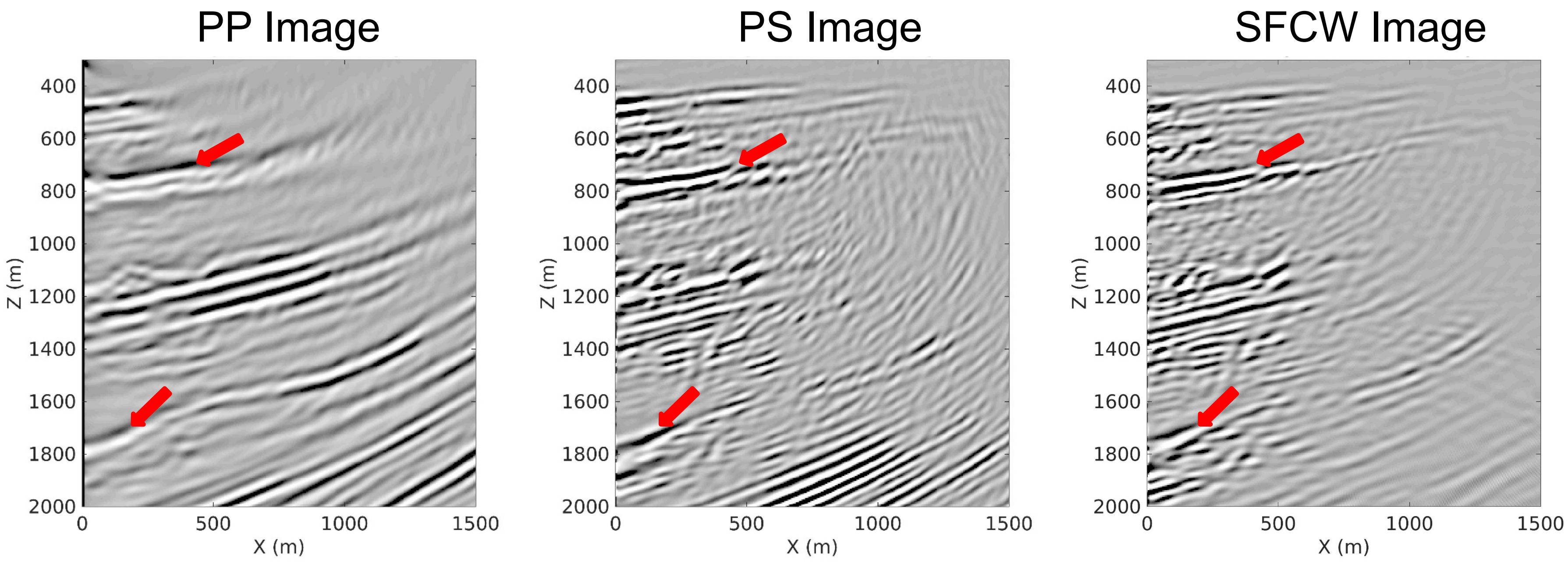




Du et. al., Geophysics, 2019

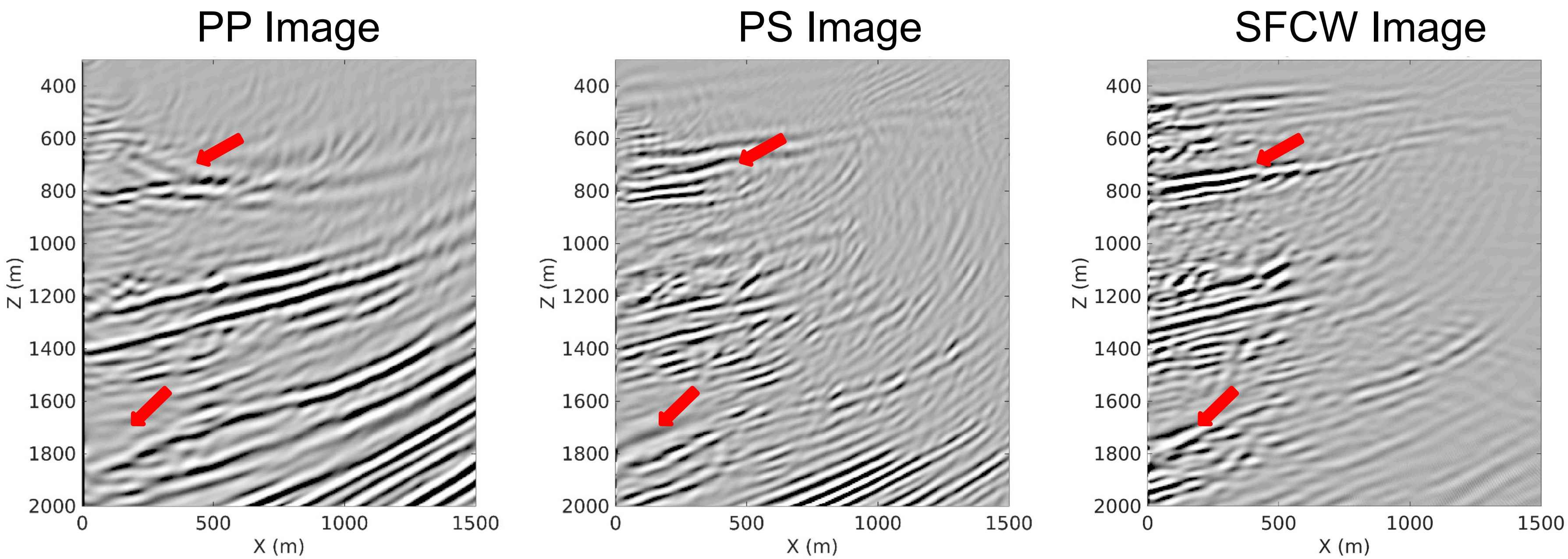
Near wellbore imaging

- accurate near surface velocity



Near wellbore imaging

- Too-fast near surface velocity



- The events in PP and PS images are pushed down by faster migration velocities.
- The overburden velocity error has stronger impact on the shallower layers.

Discussions and conclusions

- We derive a new set of coupled, but separated wave equations for P- and S-wave propagation
- This work provides a straightforward interpretation of elastic wave physics and a rigorous theoretical basis for the elastic image conditions
- Better interpretation of the PP and PS images based on fundamental wave physics

Discussions and conclusions

- Advantages of using acoustic propagators for elastic imaging
  - Lower memory and computational cost
  - Free of the artifacts caused by the unphysical wave mode conversion:

    1. Artifacts near the receiver locations
    2. Imprints of S-wave velocity model – “in-situ” mode conversions

Limitations

- Constant density assumption
  - P- and S-waves are fully coupled at all density discontinuities
  - Images are contaminated with density contrasts
- P- and S-data separation in the recorded data
  - Potential data are needed for this formulation
  - Inverse problem to solve for the separated fields

Acknowledgements

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Complete set of equations for constant density media

$$\underbrace{\ddot{P} - \alpha\nabla^2 P}_{\text{P propagation}} = \underbrace{P\nabla^2\alpha + 2\nabla\alpha\cdot\nabla P}_{\text{P scatter at Vp constrast}} + \underbrace{2(\nabla\nabla\beta\cdot\nabla\nabla(\nabla^{-2}P) - P\nabla^2\beta)}_{\text{P scatter at Vs contrast}}$$
$$- \underbrace{2[\nabla\beta\cdot\nabla\times\mathbf{S} + \nabla\nabla\beta\cdot\nabla\nabla\times(\nabla^{-2}\mathbf{S})]}_{\text{SP mode conversion at Vs contrast}} + \underbrace{\nabla\cdot\mathbf{f}}_{\text{source}}, \qquad (26)$$

$$\underbrace{\ddot{\mathbf{S}} - \beta\nabla^2\mathbf{S}}_{\text{S propagation}} = \underbrace{2\nabla\beta\times\nabla P + 2\nabla\nabla\beta\star\nabla\nabla(\nabla^{-2}P)}_{\text{PS mode conversion at Vs contrast}} + \underbrace{\nabla\beta\cdot\nabla\mathbf{S} - (\nabla\beta)\times(\nabla\times\mathbf{S})}_{\text{S scatter at Vs constrast}}$$
$$- \underbrace{\nabla\nabla\beta\star\{\nabla\nabla\times(\nabla^{-2}\mathbf{S}) + [\nabla\nabla\times(\nabla^{-2}\mathbf{S})]^T\}}_{\text{S scatter at Vs contrast}} + \underbrace{\nabla\times\mathbf{f}}_{\text{source}}. \qquad (27)$$