A Survey of Recent Advances in Storm Surge Simulations

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Slides: available upon request.

Storm Surges: A Global Problem

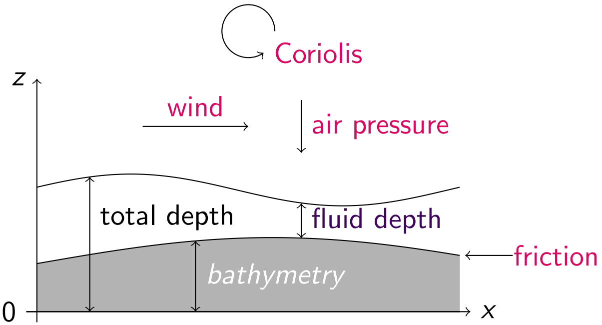
Scituate, Massachusetts, USA

Lahinch, Co. Clare, Republic of Ireland

Schedule of this Minisymposium

- 3:00-3:20
  Application of Adjoint Methods to Storm Surge Sensitivity Analysis
  Simon C. Warder
- 3:25-3:45
  Storm Surge Modeling in Support of Response and Recovery Efforts
  Clint Dawson
- 3:50-4:10
  A Cyberinfrastructure for Storm Surge Modeling
  Clint Dawson
- 4:15-4:35
  A Green's Function Approach to Efficient Shallow Water Uncertainty Quantification
  Will D. Mayfield

Mathematical Problem Description: 2D Shallow Water Equations



Projection on to 1D

Movement:

- Advection
- Pressure Gradients

Forces:

- Earth Rotation
- Wind Stress
- Atmospheric Pressure
- Topography

Mathematical Problem Description: 2D Shallow Water Equations

A 2D DG Model for Non-Linear Shallow Water Equations

$$\begin{bmatrix} h \\ h\mathbf{u} \end{bmatrix}_t + \nabla \cdot \begin{bmatrix} h\mathbf{u} \\ h\mathbf{u} \otimes \mathbf{u} + \frac{g}{2} h^2 \mathbf{I}_2 \end{bmatrix}$$

$$= - \begin{bmatrix} 0 \\ gh\nabla b + fh\mathbf{u} - \rho^{-1} \left(\boldsymbol{\tau} + h\nabla p_A + \frac{gn^2 \|\mathbf{u}\|_2}{h^{7/3}} h\mathbf{u} \right) \end{bmatrix}$$

Prognostic variables:

- h depth of the fluid layer
- $h\mathbf{u}$ depth-averaged horizontal momentum

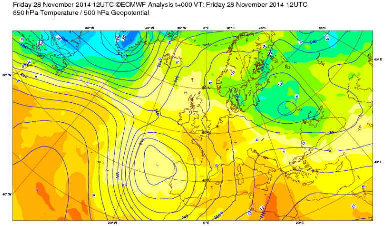
The Source Terms

$f h\mathbf{u}$ – Coriolis Force

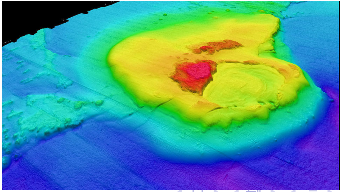
Kelvin's Tide Machine, 1872

$$\frac{\left(\frac{gn^2\|\mathbf{u}\|_2}{h^{7/3}} h\mathbf{u}\right)}{\rho}$$ – Bottom Friction

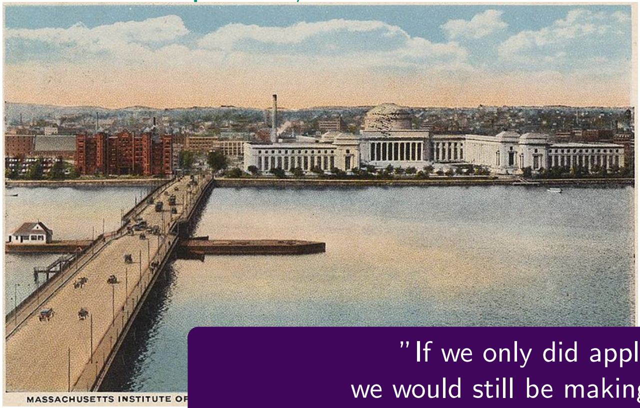
$$\frac{(\tau + h\nabla p_A)}{\rho}$$ – Wind & Pressure



$gh\nabla b$ – Bathymetry



The Need For Improved/ New Models



"If we only did applied research,
we would still be making better spears."

George Smoot

Incomplete List of Current Storm Surge Research Models

- Firedrake-Thetis
  (OHSU & Imperial College London – thetisproject.org)
- ADCIRC
  (UNC, Notre Dame & UT Austin et al. – adcirc.org)
- GeoClaw (ClawPack)
  (UW & Columbia et al. – http://www.clawpack.org/geoclaw)
- FVCOM (UMASSD-WHOI– http://fvcom.smast.umassd.edu/)
- Mike 21 (DHI – https://www.mikepoweredbydhi.com)
- SLIM-ocean (UC Louvain – https://www.mikepoweredbydhi.com/)
- amatos2d+StormFlash2d (U Hamburg, FU Berlin & UCD – amatos.info)
- <insert yours here>

Common model components

What most of us have in common

- Mesh-based Approaches
- Finite Volumes, Finite Elements and/or Discontinuous Galerkin
- Explicit Timestepping
- Wetting and Drying (Riemann solver, filter, slope limiter,...)

Discontinuous Galerkin Model

A 2D DG Model for Non-Linear Shallow Water Equations

- Solves the 2D Non-Linear Shallow Water Equations
- Linear Lagrange Polynomials

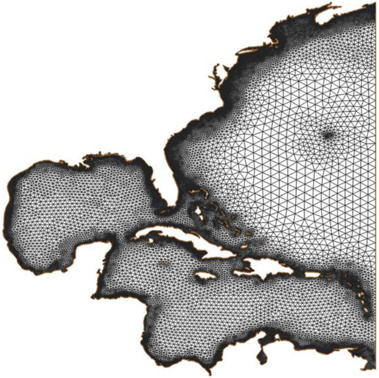
$$h(x) = \sum_{k=1}^{3} h_k(t)\varphi_k(\mathbf{x}), \quad h\mathbf{u}(x) = \sum_{k=1}^{3} h\mathbf{u}_k(t)\varphi_k(\mathbf{x})$$

- Explicit Time Integration (Runge-Kutta)

$$\frac{d\mathbf{U}_k}{dt} = \widetilde{\mathbf{H}}(\mathbf{U}_k), \qquad \mathbf{U} = (h, h\mathbf{u})^\top$$

- Slope Limiter (positivity preserving, well-balanced)
- Conforming, Triangular & Adaptive Meshes (`amatos2d`)

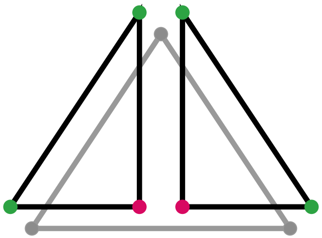
Mesh-Based Methods: Non-Uniform Meshes



Highly optimised mesh for North West Atlantic and Gulf of Mexico (stolen from: adcirc.org)

- Geometric flexibility: Triangles, rectangles, (high-order) polynomial
- Uniform errors can't be achieved on uniform meshes
  $\Rightarrow$ non-uniform meshes
- Measure numerical convergence?
- Automated refinement based on sensitivity indicators

Adaptive Mesh Refinement with amatos



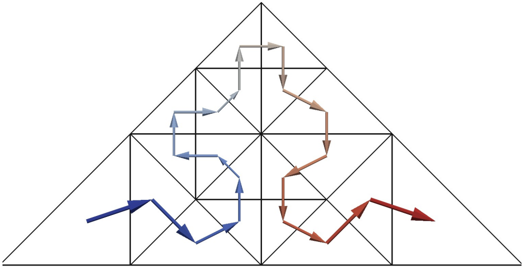
- Dynamically adaptive and triangular grids (Behrens et al., 2005).
- Bisection → fine triangles
- Heuristic refinement indicators η_Ω, e.g. topography gradient at time t.

$$\Delta_k \leftarrow \text{number}$$

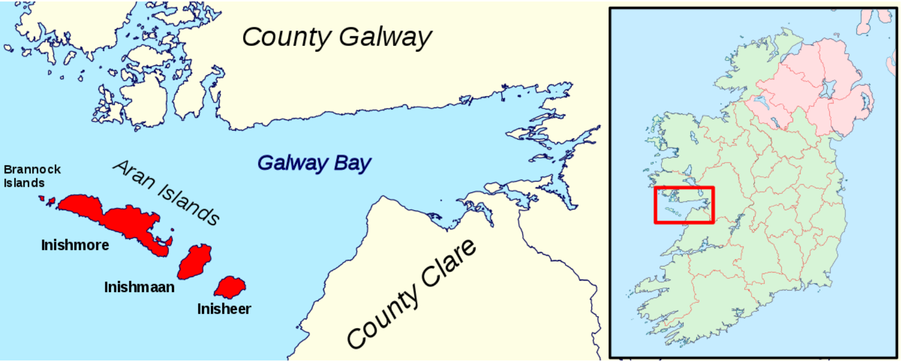
- Tolerances $0 \leq \theta_{\text{crs}} < \theta_{\text{ref}} \leq 1$ determine the fraction of the domain to be modified

Adaptive Mesh Refinement - Advantages

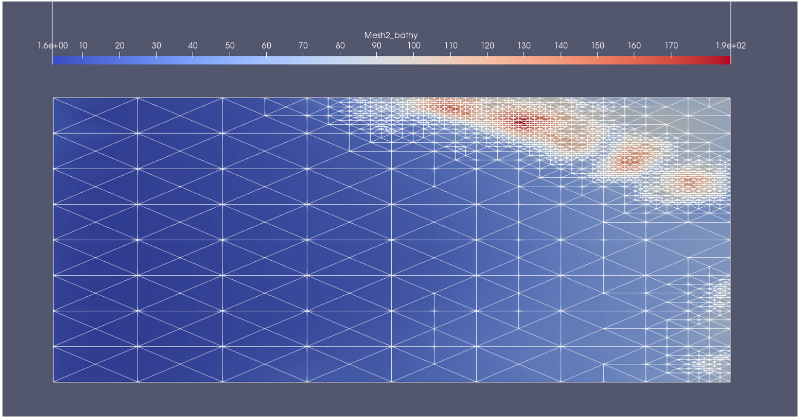
- Cache-efficient space-filling curve-ordering of elements (Behrens & Bader, 2009)
- Conformity of the mesh, i.e. no hanging nodes
- Mesh generation decoupled from numerics



Adaptive Mesh Refinement: Example



Adaptive Mesh Refinement: Example



The Aran Islands

Mesh details: 2166 triangles; small triangles $\rightarrow$ large bathymetry variation

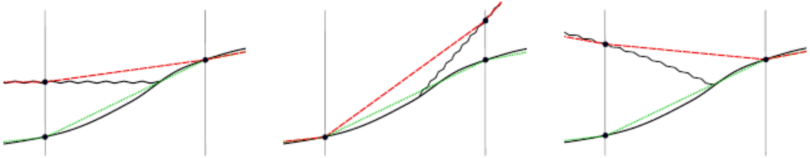
Wetting & Drying: Different Approaches

- Redistribution of mass within cells ⇒ positivity of the mean fluid depth & mass conservation (Bunya et al., 2009, Ern et al., 2008, Xing et al, 2013).
- Neglect gravity terms to remove artificial gradients (Bunya et al., 2009, Gourgue et al., 2009)
- Thin layer approach to obtain well-defined velocities (Bunya et al., 2009)
- Transformation of the bathymetry (Gourgue et al., 2009, Kärnä et al., 2011)
- Generalised minmod TVB slope limiter (Cockburn et al., 1998) → only apply in cells without wetting and drying
- Wetting and drying using information from edge (Barth & Jesperson, 1989) or node neighbours (Kuzmin, 2013)

Velocity-Based Slope Limiter

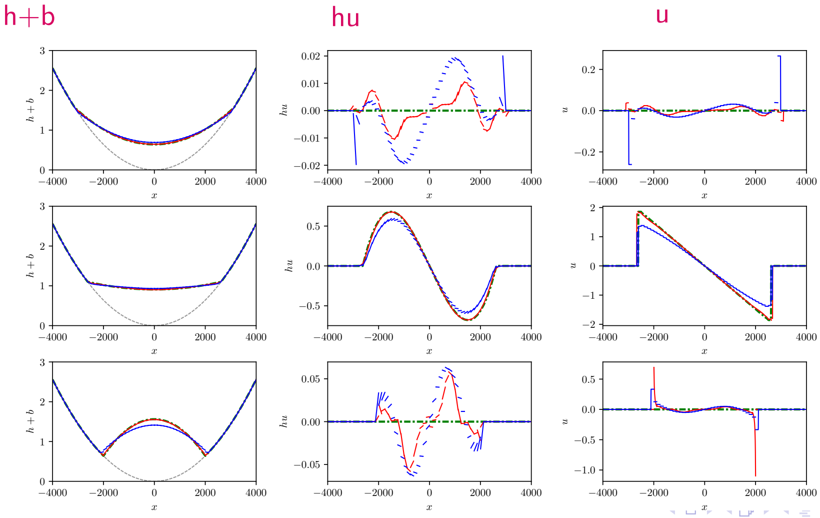
(Vater, B & Behrens, 2017)

- Remove artificial gradients (Flux Modification):



- Limiting of fluid depth using nodal (Kuzmin, 2010) or edge-based (Barth & Jesperson, 1989) limiters and positivity preserving treatment a la (Bunya et al., 2009).
- Limiting of momentum using nodal (Kuzmin, 2010) or edge-based (Barth & Jesperson, 1989) limiters and extrapolate in-cell velocities and minimize.

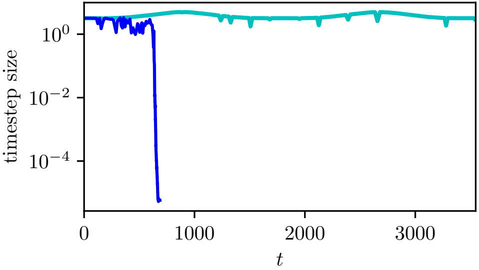
Long Wave Resonance in a Paraboloid Basin



Slope Limiters & Timestep Restrictions

Discontinuous Galerkin Methods are CFL restricted!

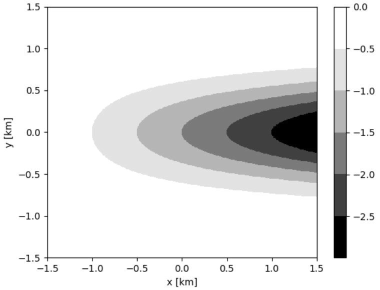
Comparison of Two Limiting Strategies (Vater, B & Behrens, 2019)

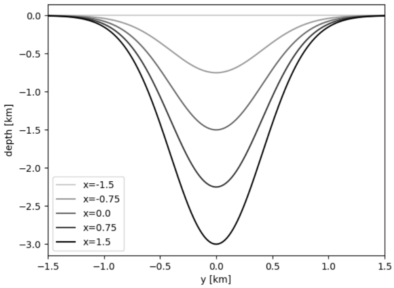


Time step Δt over time by keeping $\mathrm{cfl} = 0.2$. Results with velocity-based limiting of the momentum (cyan) and with direct limiting in momentum (blue).

Hypothetical Embayment with Tidal Inflow

Lake at Rest on a square domain Ω with symmetric bathymetry.





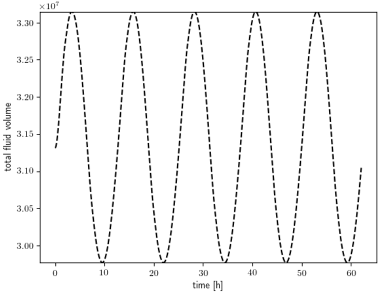
$\Leftarrow$ Tidal Inflow on the right

$$h(\mathbf{x}, t) = \sin\left(\frac{2\pi t}{T}\right),$$

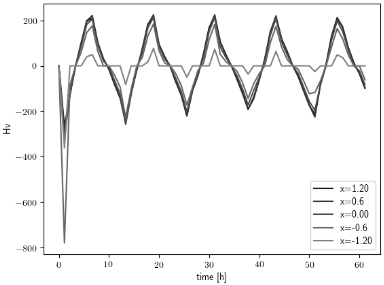
Bathymetry cros^{3}ections at $x \in [\pm 1.5, \pm .75, 0]$

Mass Conservation & Fluxes

Total Fluid Volume



Integrated Flux $H\mathbf{v}$ over cros^{3}ections



Non-Destructive Slope Limiter

Filters out over-and undershoots in partially dry cells based on the most realistic approximation for the numerical velocity.


part of the collaborative project :
Understanding Extreme Near-shore Wave Events
through Studies of Coastal Boulder Transport
UCD DUBLIN
QUEEN'S UNIVERSITY BELFAST
WILLIAMS
Funded by the US-Ireland R&D Programme
Science Foundation Ireland
sfi
For what's next
NSF
National Science Foundation
Department for the Economy
Image by Peter Cox

Summary & Conclusion

- Storm surge models mostly employ 2D finite volume or finite element based discretisations
- Mesh creation and optimisation is still a vast area of research
- Wetting and drying requires additional processing of the model data
- Sensitivity studies and uncertainty quantification needed for improved simulations

Thank you!/ Go raibh maith agaibh!