Application of Adjoint Methods to Storm Surge Sensitivity Analysis

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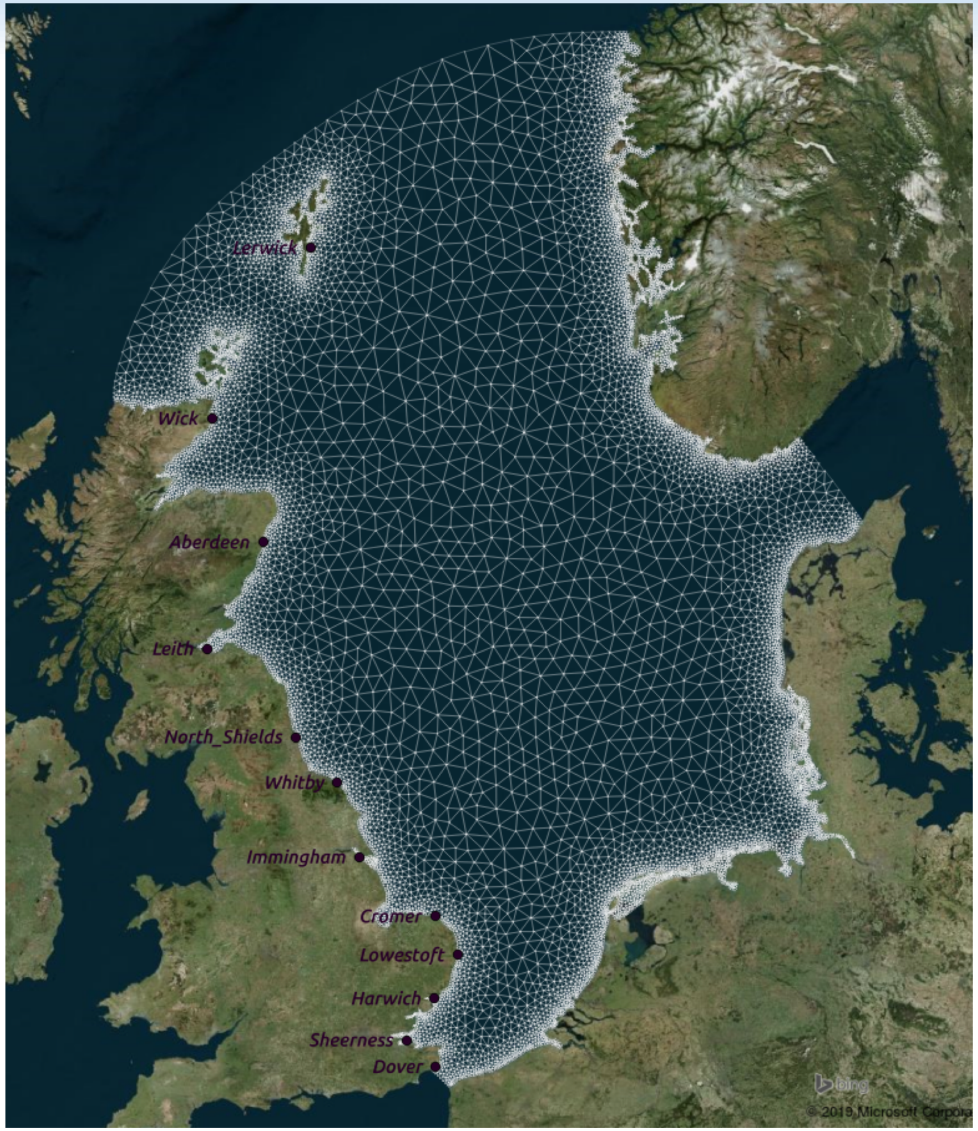
Model Setup

Modelling surges with Thetis

- Thetis is an adjoint-capable finite element coastal ocean model, solving the shallow water equations
- Python package, using Firedrake finite element framework, and PyAdjoint for adjoint code generation
- Results here use P1DG-P1DG finite element pair and Crank-Nicolson timestepper

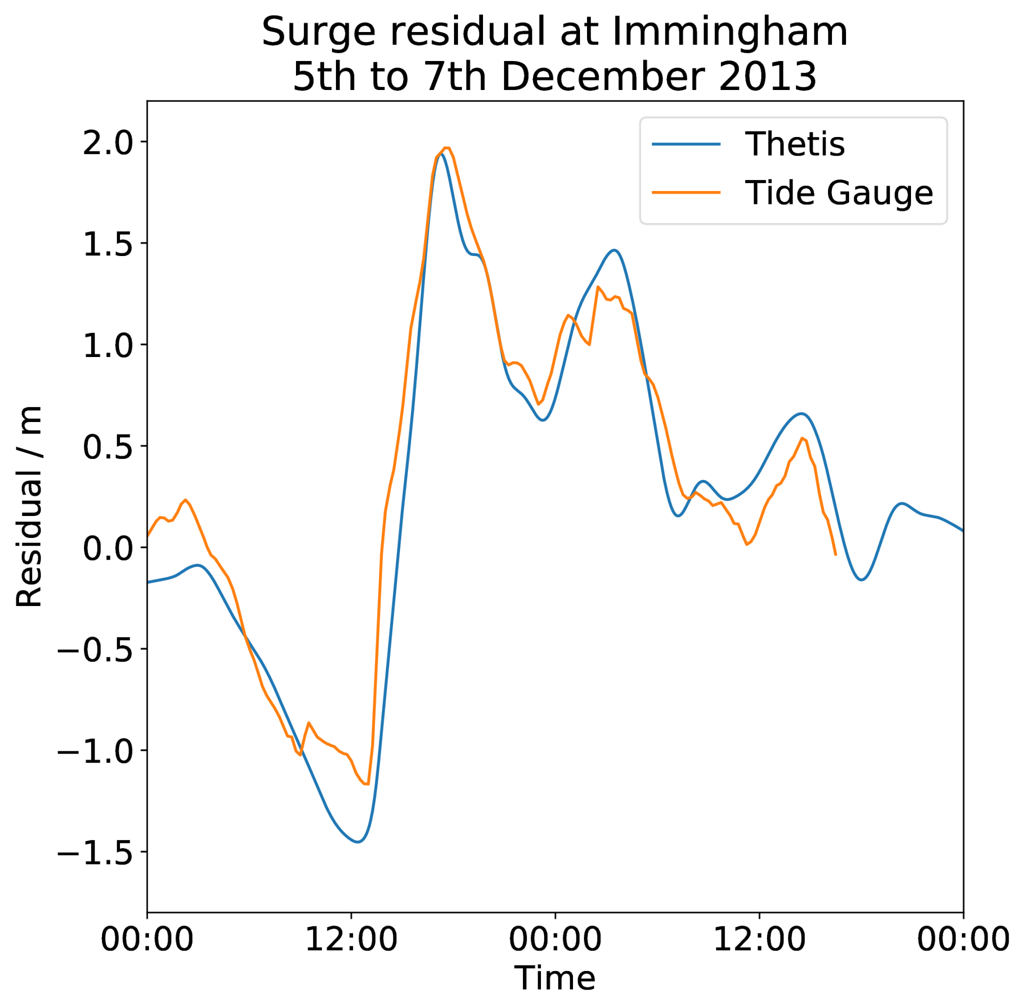
Thetis model setup: North Sea

- Mesh created with qmesh
- Models forced with tidal boundary elevation
- Wind stress and atmospheric pressure forces applied on surface, from meteorological hindcast data
- Bottom friction using Manning parameterisation



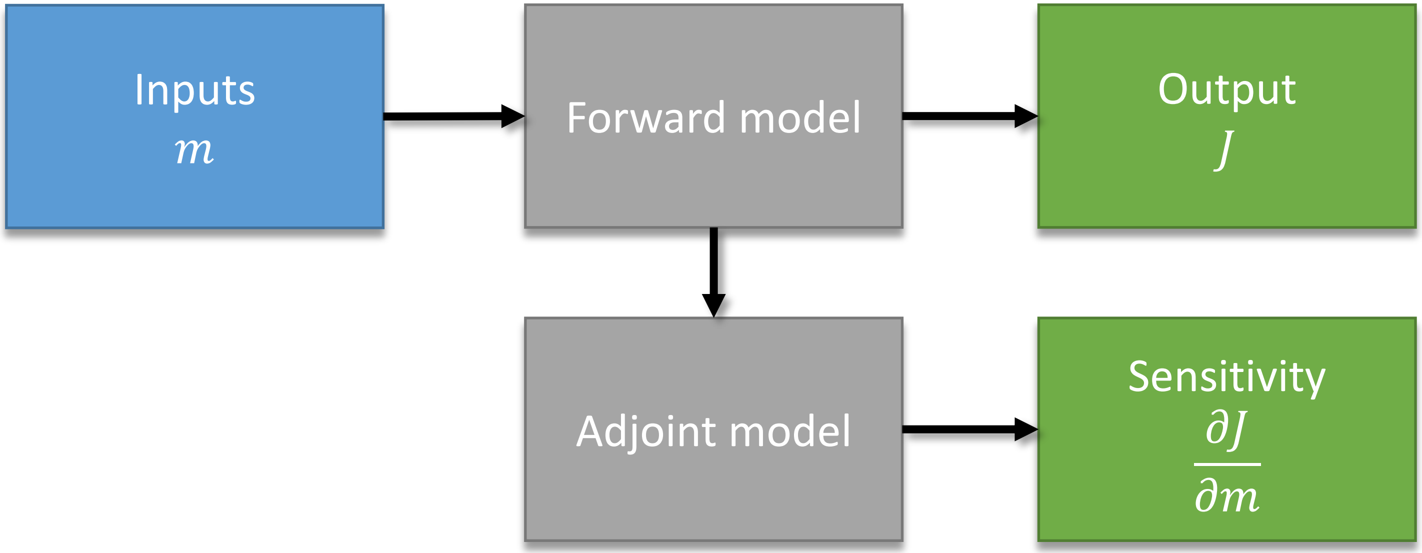
Model validation

- December 2013 storm surge event used in this study
- Tidal barrier near Immingham very close to overtopping
- 1m spread in operational forecast ensemble at 24 hour lead time
- Thetis performs well compared with tide gauge data; surge residual at Immingham shown here



Adjoint methods

- Uncertainty quantification often approached with ensembles
- Thetis surge model has adjoint; can use to obtain sensitivity of model output with respect to input, in a computationally efficient way
- Here, J is the peak surface elevation at a given location (Immingham)



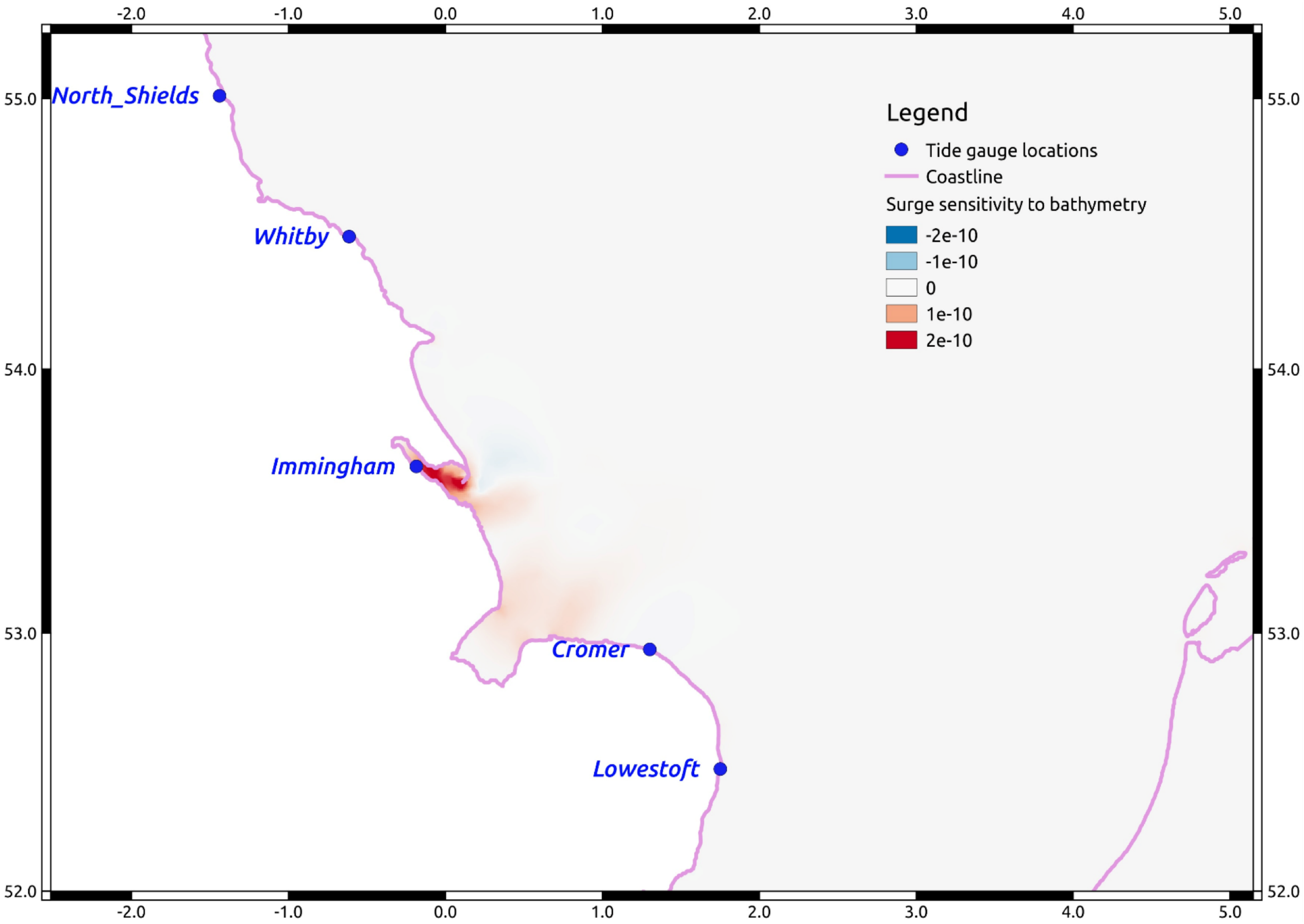
Sensitivity Analysis

Which model input has the greatest effect on model output?

Sensitivity of peak surge residual to bathymetry

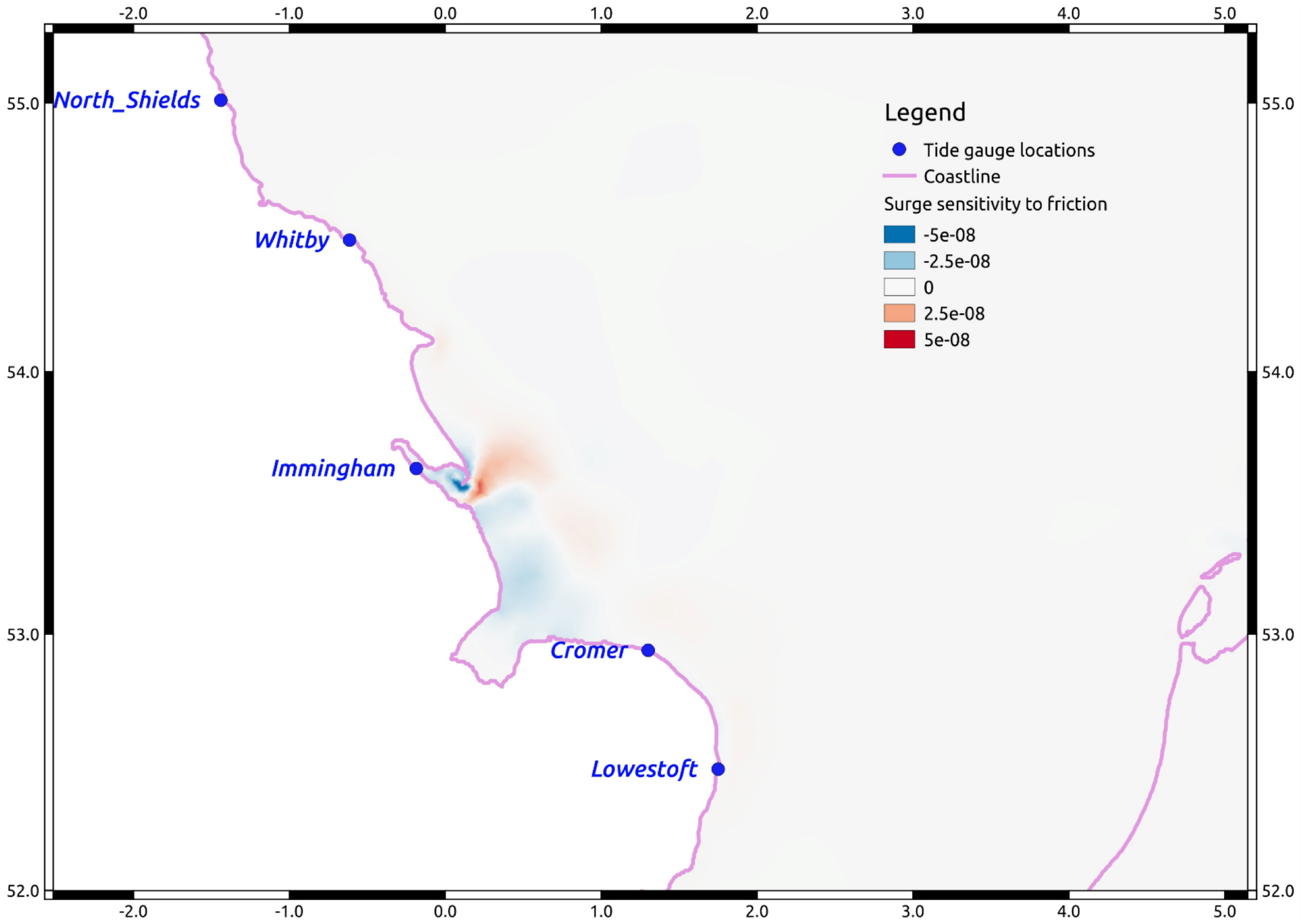
- Effect localised around surge observation location

- 5% uncertainty in bathymetry produces 2.6 cm uncertainty in peak surge residual



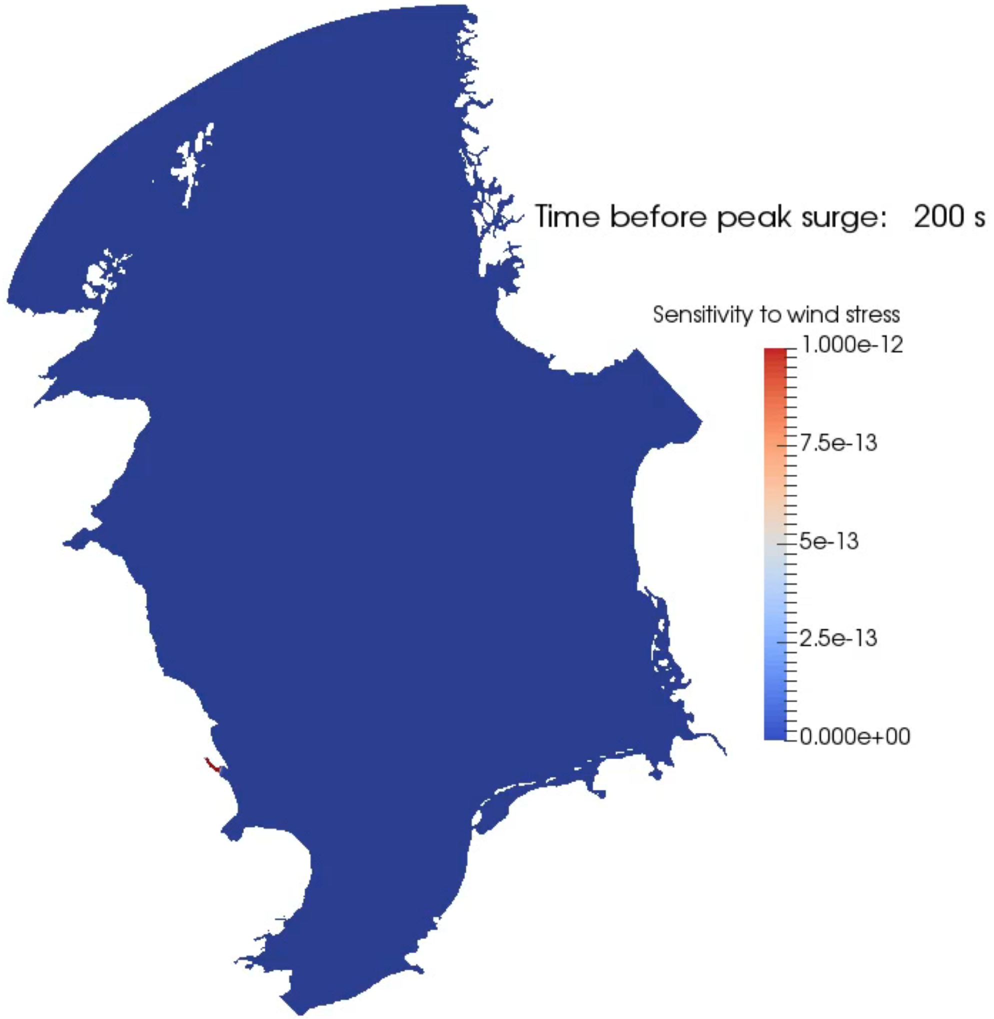
Sensitivity of peak surge residual to bottom friction coefficient

- Effect localised around surge observation location
- 5% uncertainty in bottom friction coefficient produces 5.3 cm uncertainty in peak surge residual
- Model tuning parameter – no empirical data, but can be calibrated



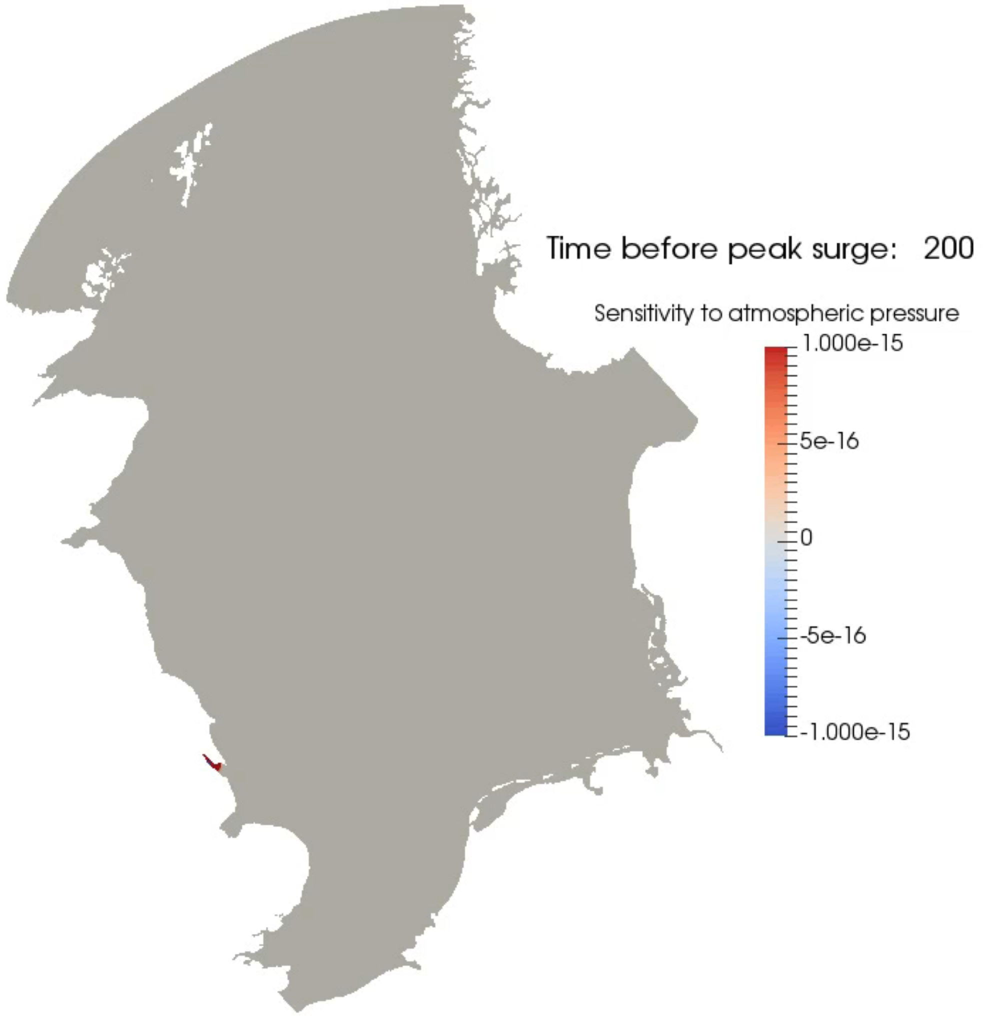
Sensitivity of peak surge residual to wind stress

- Wind stress is time varying
- Perturbations due to wind stress travel at approximately the shallow water wave speed
- Sensitivity pattern is like shallow water wave, spreading out from observation location backwards in time
- 5% uncertainty in wind stress magnitude produces 6.2 cm uncertainty in peak surge residual



Sensitivity of peak surge residual to atmospheric pressure

- Sensitivity to atmospheric pressure follows similar pattern to wind stress
- Effect of atmospheric pressure on surge residual is very small compared with wind stress
- 5% uncertainty in atmospheric pressure anomaly produces 0.1 cm uncertainty in peak surge residual



Comparison of sources of uncertainty

- Uncertainty in peak surge residual due to 5% uncertainty in inputs:

Bathymetry: 2.6 cm

Wind stress: 6.2 cm

Bottom friction coefficient: 5.3 cm

Atmospheric pressure: 0.1 cm

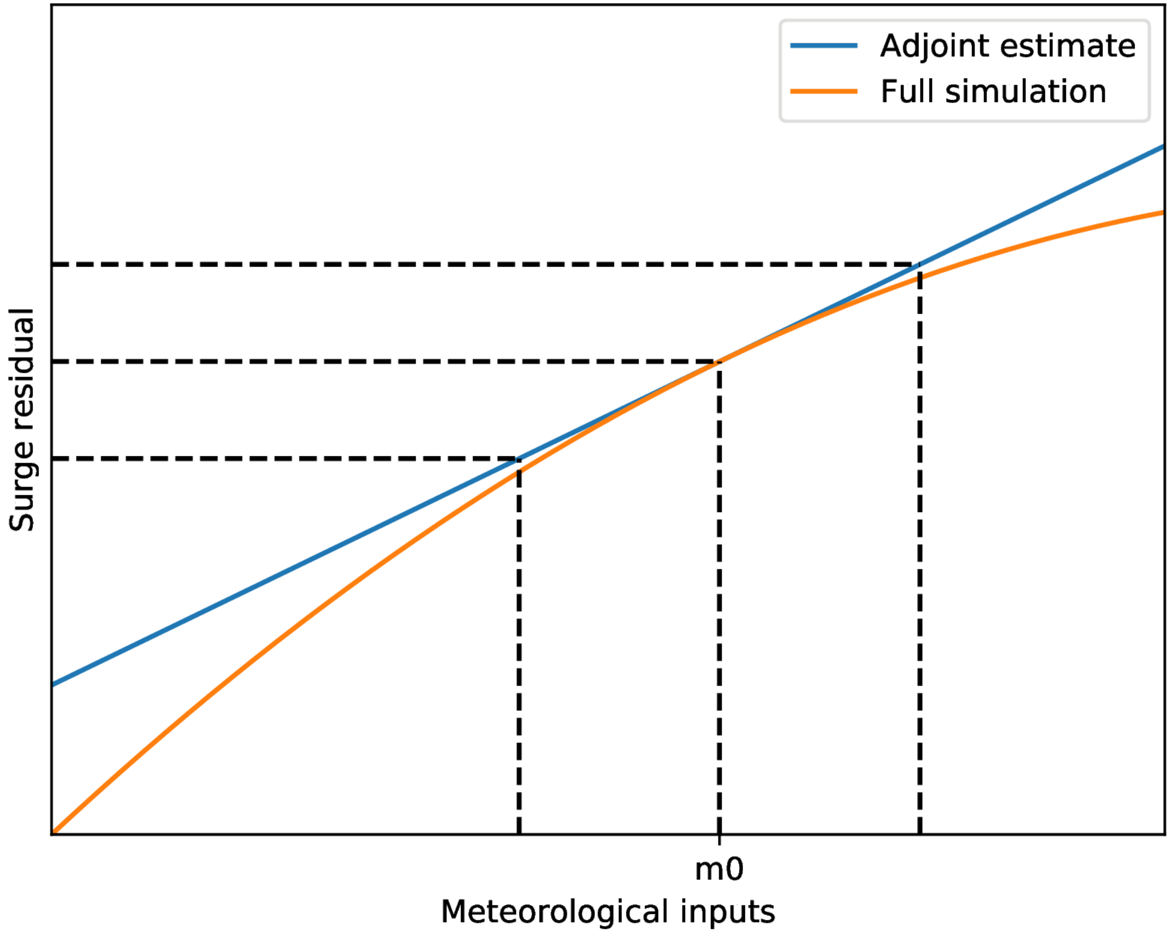
- Uncertainty due to bottom friction and bathymetry can be reduced by calibration/data assimilation
- Uncertainty due to tidal boundary condition is part of future work; contribution to surge uncertainty depends on strength of tide-surge interaction

Other Adjoint Applications

How can adjoint models assist forecasts, and what else can we learn?

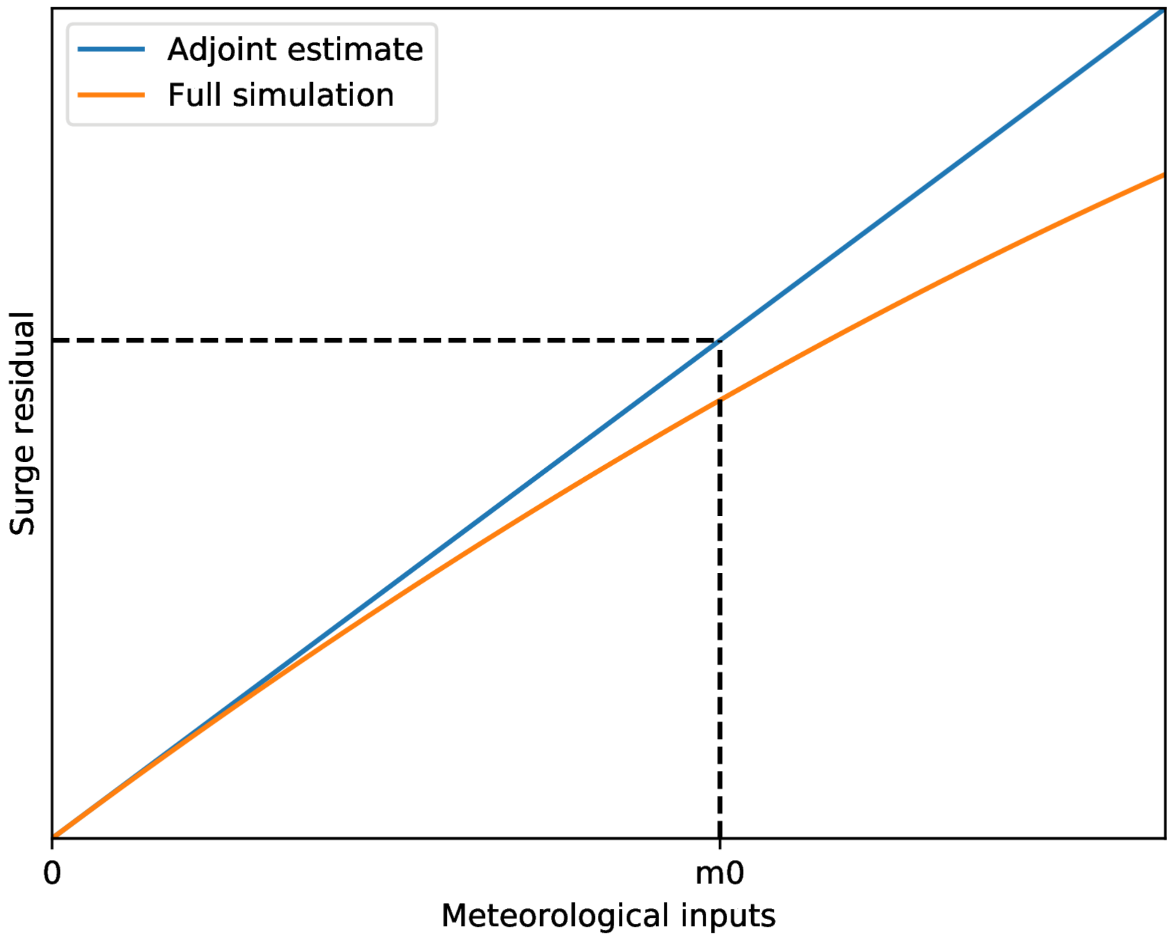
Applications of adjoints

- If ensemble of meteorological forecasts consists of “small” deviations about the deterministic forecast, adjoint can be used to propagate uncertainty through surge model
- $J(m_0 + \Delta m) \approx J(m_0) + \Delta m \left(\frac{\partial J}{\partial m}\right)_{m_0}$
- Only viable when only one output of interest (J)



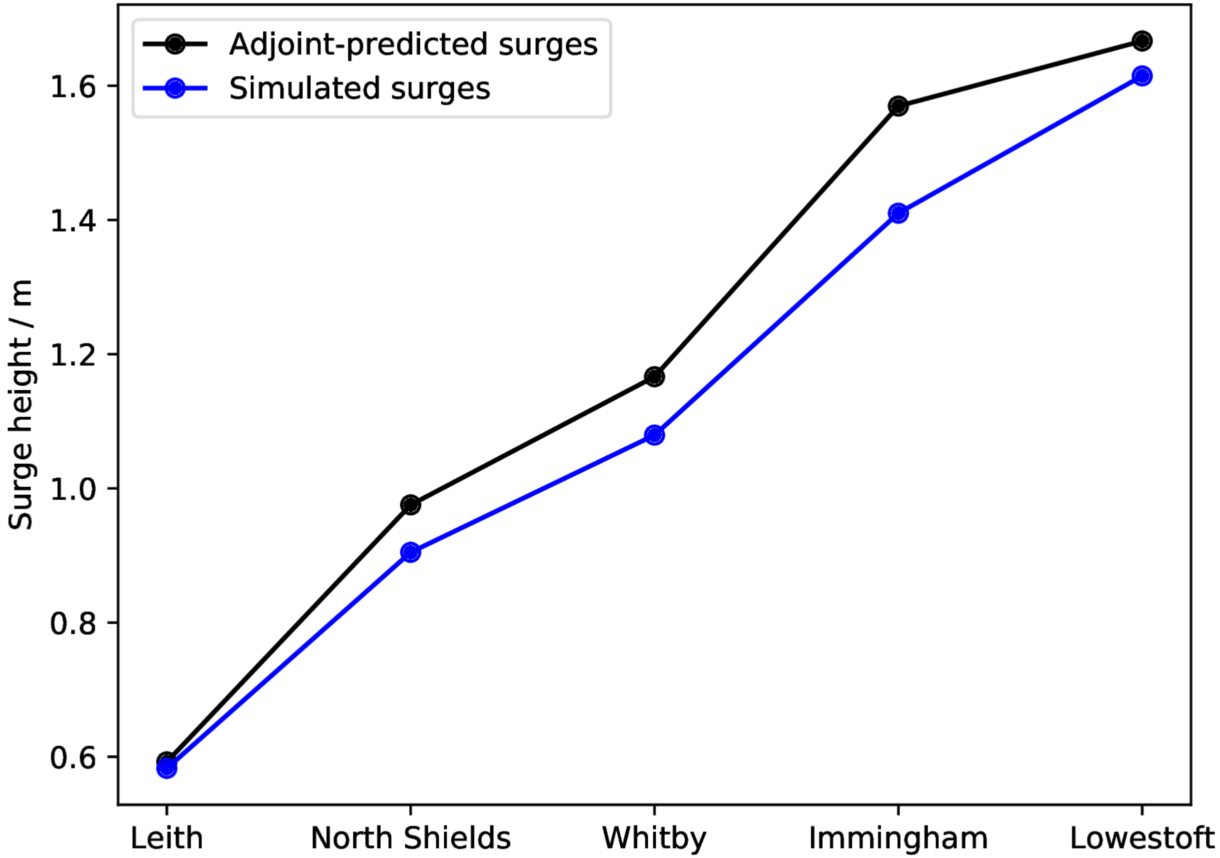
Applications of adjoints

- For "small" surges, entire surge may fall into linear response regime
- Neglects part of tide-surge interaction
- $J(m_0) \approx m_0 \left(\frac{\partial J}{\partial m}\right)_{m=0}$
- How bad is this approximation?



Testing linearity for 2013 event

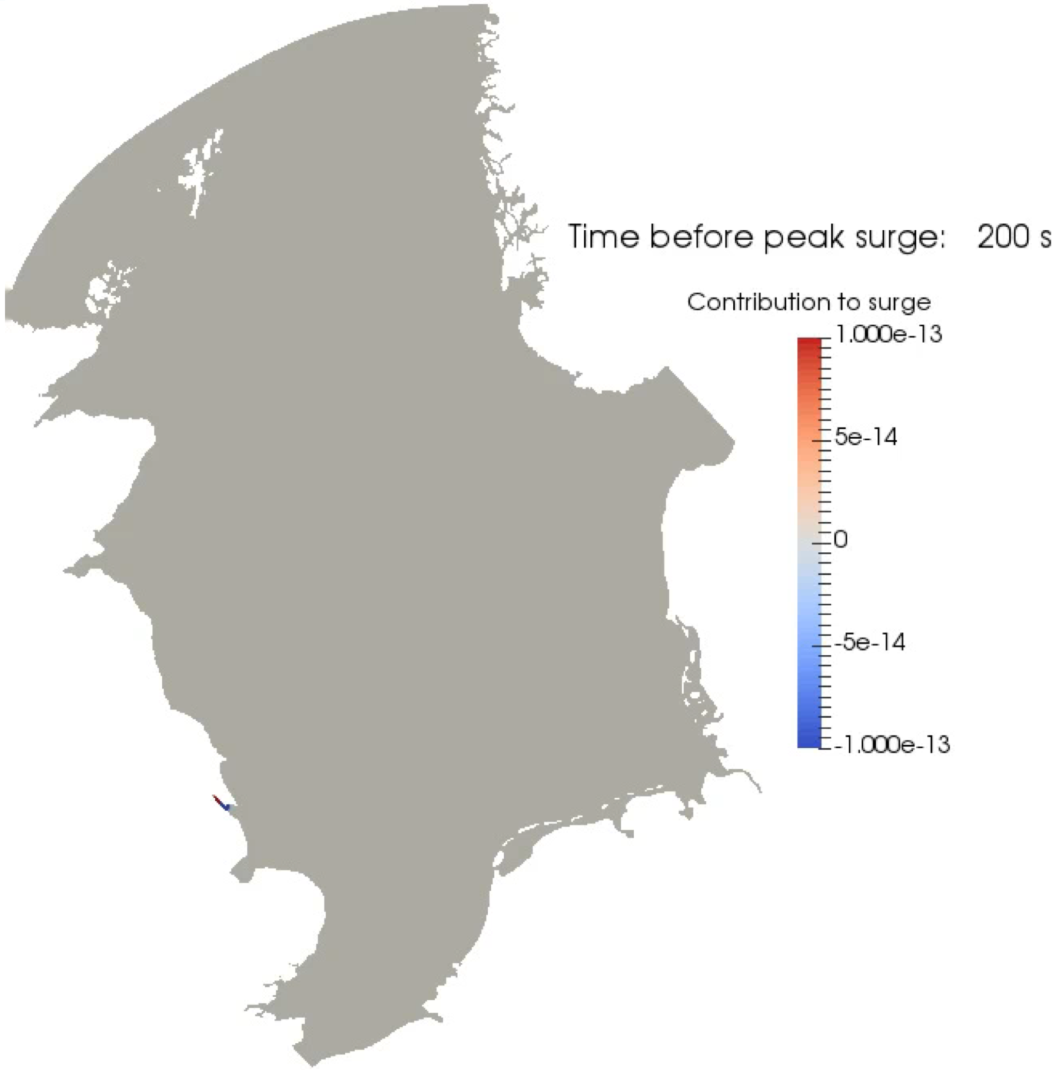
- 2013 was an extreme event
- Discrepancy between linear-response surge heights and fully simulated surges can reach around 20 cm
- Not useful for forecasting. However, still some skill…



Insight into surge generation

$$J \approx \int_{\Omega} \int_0^T m(t, \boldsymbol{x}) \left(\frac{\partial J}{\partial m(t, \boldsymbol{x})} \right)_{m=0} dt\, d\boldsymbol{x}$$

- Inner product of sensitivities and forcings is a function of space and time
- Shows where sensitivities and forcings combine to enhance or diminish net surge
- Reveals properties of storm that lead to surge, e.g. some regions of high winds might make surprisingly small contribution to surge, and vice versa



Conclusions

- Uncertainty in surge predictions has been analysed using an adjoint surge model
- For a given % uncertainty in model inputs, wind stress has the greatest effect on surge predictions
- Bottom friction also contributes large uncertainty, and its effect is localised around the observation region of interest
- Using adjoint sensitivities about zero forcing, insight can be gained into where and when surge is developed, i.e. where winds enhance or diminish net surge at a given location and time

Thank you for your attention

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