Sparsity in Multichannel Blind Deconvolution via Focusing Constraints

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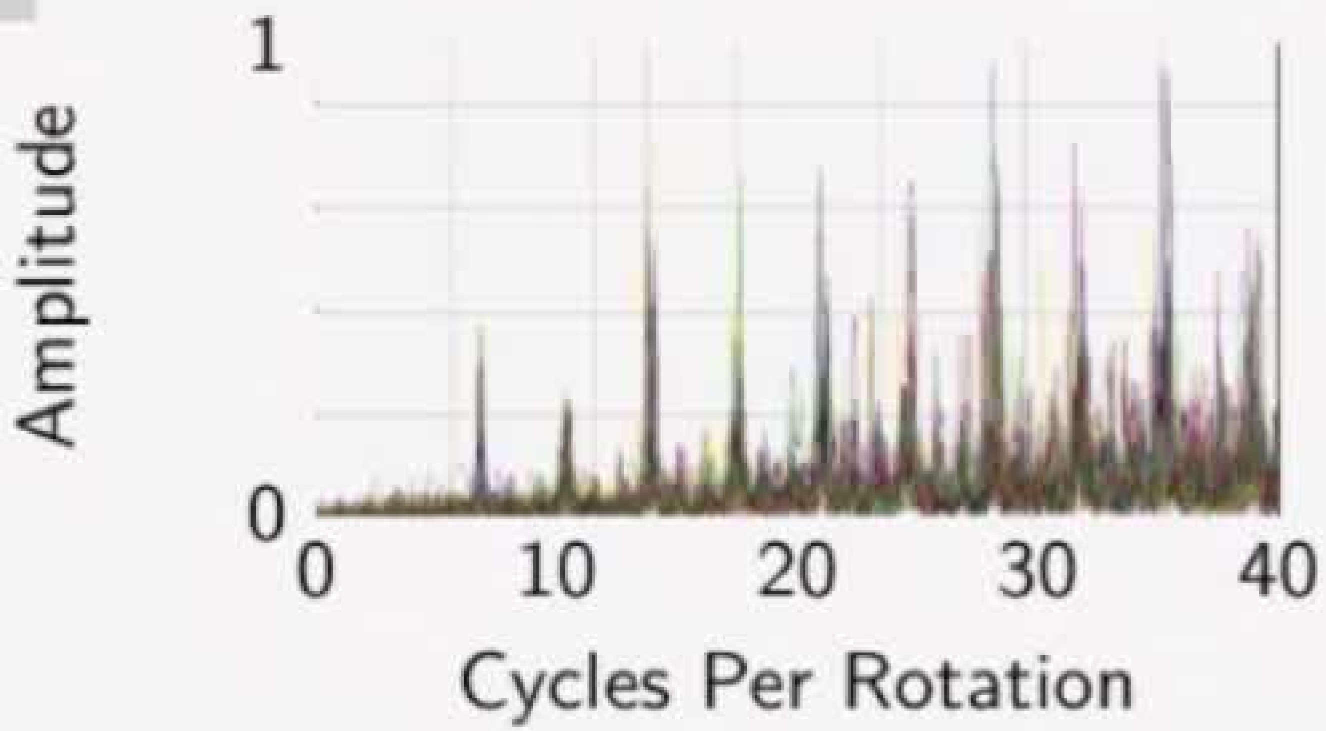
Green's Function Retrieval

- Noise source:
  - is uncontrollable and continuously inputs energy.
  - likely is heavily *correlated* (not white spectrum) in time.
- Imaging:
  - from raw records is impossible because of unknown noise signature.
  - requires the subsurface Green's function that is uncontaminated by noise.

Marmousi Model

Drill Noise Spectrum (Gradl et al. [2012])

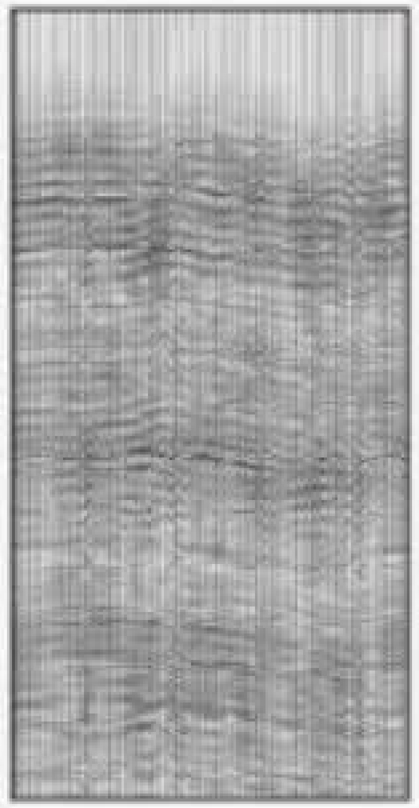


Focused Blind Deconvolution (FBD)

Marmousi Model

Raw Records

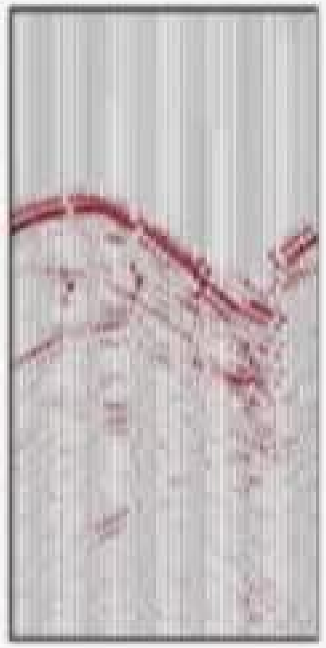
t=0s



t=7.5s

Green's Function

t=0s



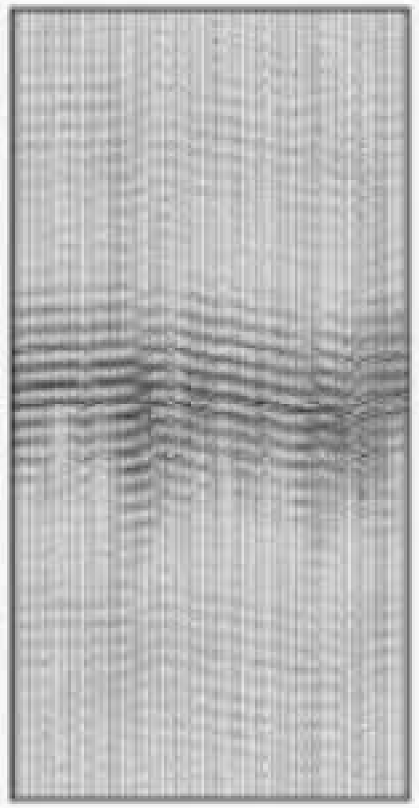
t=1s

FBD of Noise

t=0s

t=1s

Cross-correlation
With Pilot Record



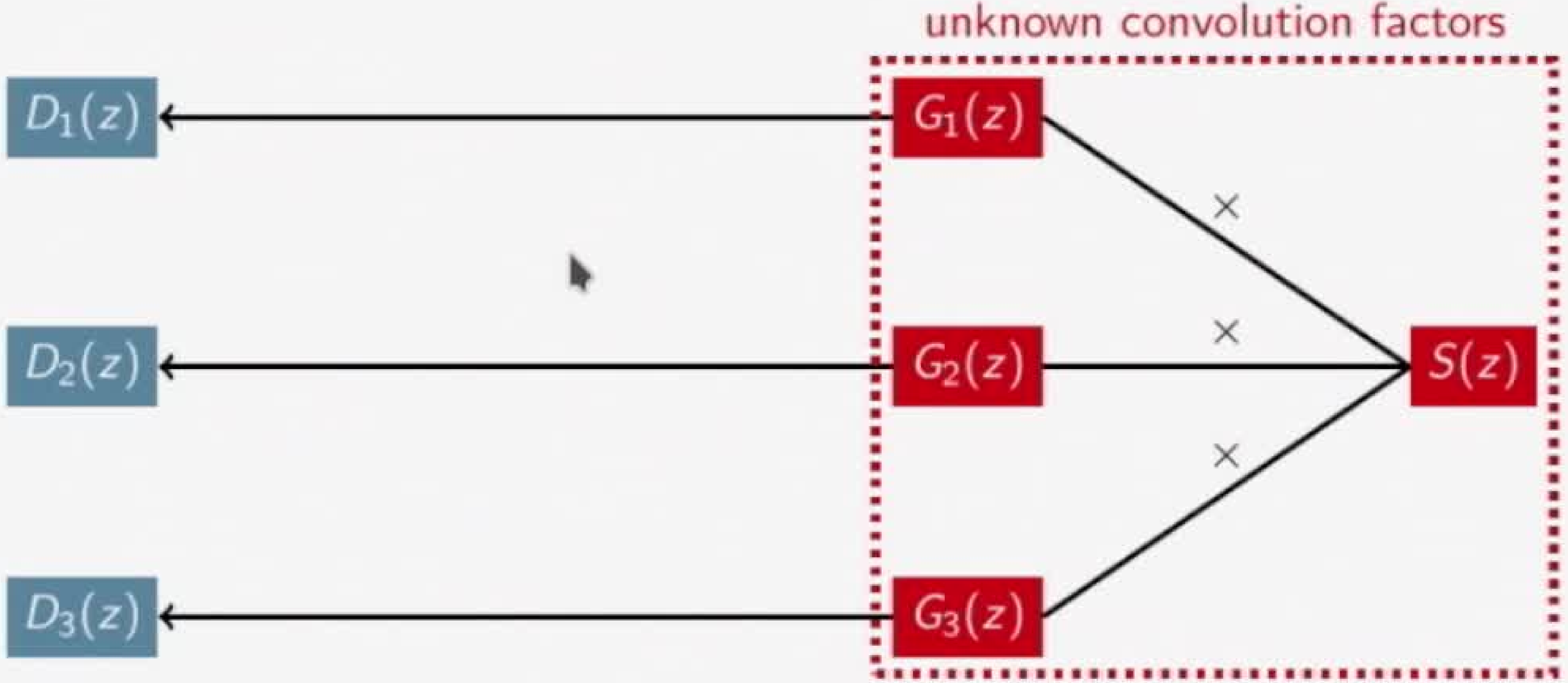
τ=-2s

τ=0s

τ=2s

Next Section

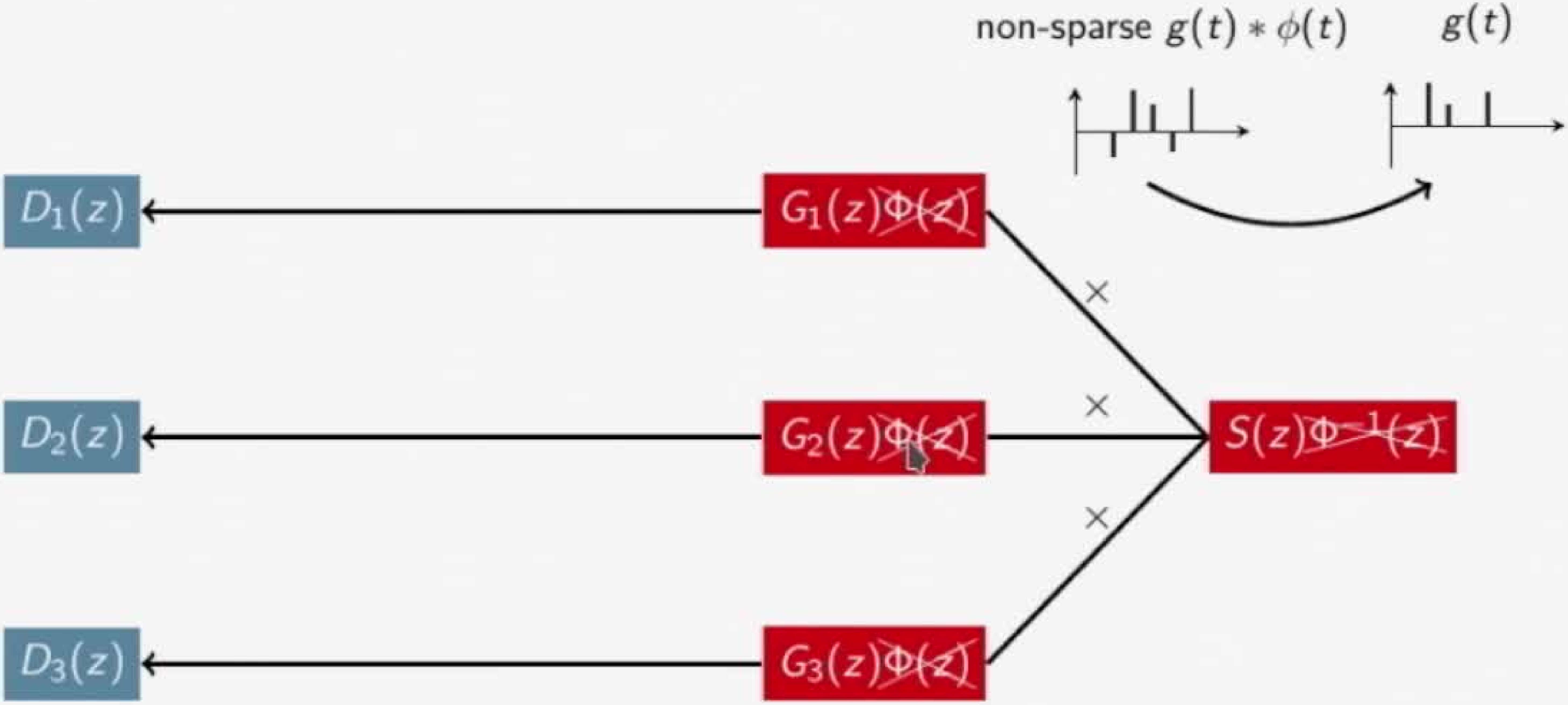
Convolutional Model For Three Channels



Next Subsection

Convolutional Model For Three Channels

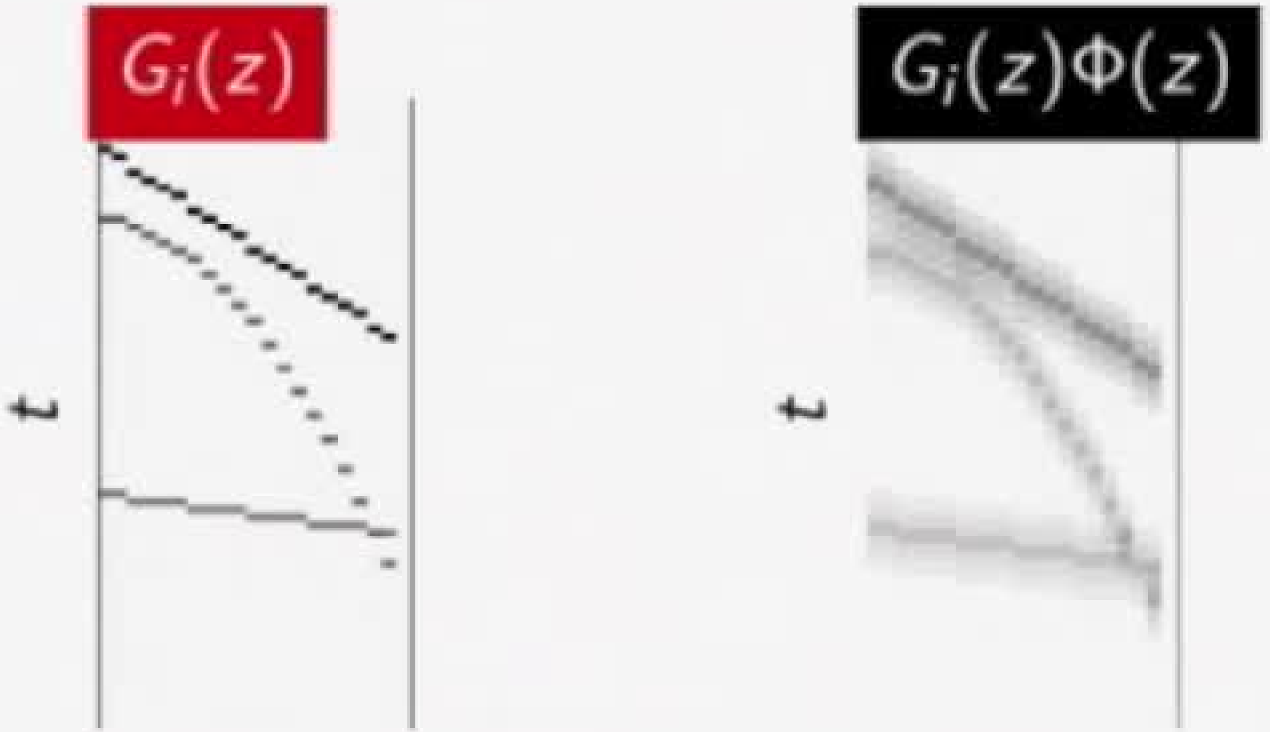
We Need Constraints to Remove Common Roots



ℓ_1 Minimization Is Not Sufficient

$\Rightarrow$ minimize ℓ_1 norm (Kazemi & Sacchi [2014], Nose-Filho et al. [2015], Liu et al. [2016]):

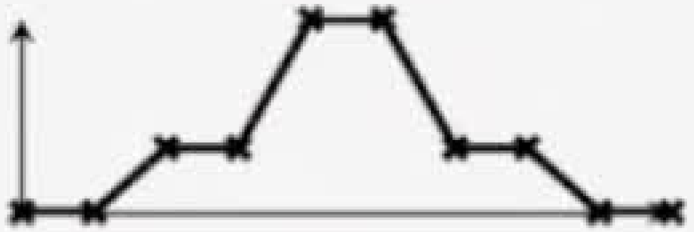
$$\min \sum_i \|g_i\|_1 \quad \text{s.t.} \quad d_i = g_i * s.$$



$$\sum_i \|g_i\|_1 = \sum_i \|g_i * \phi\|_1$$

$$\text{whenever} \int_t \phi(t) = 1$$

$\phi(t)$



Next Section

Convolutional Model For Three Channels

Interferometric Convolutional Model For Three Channels

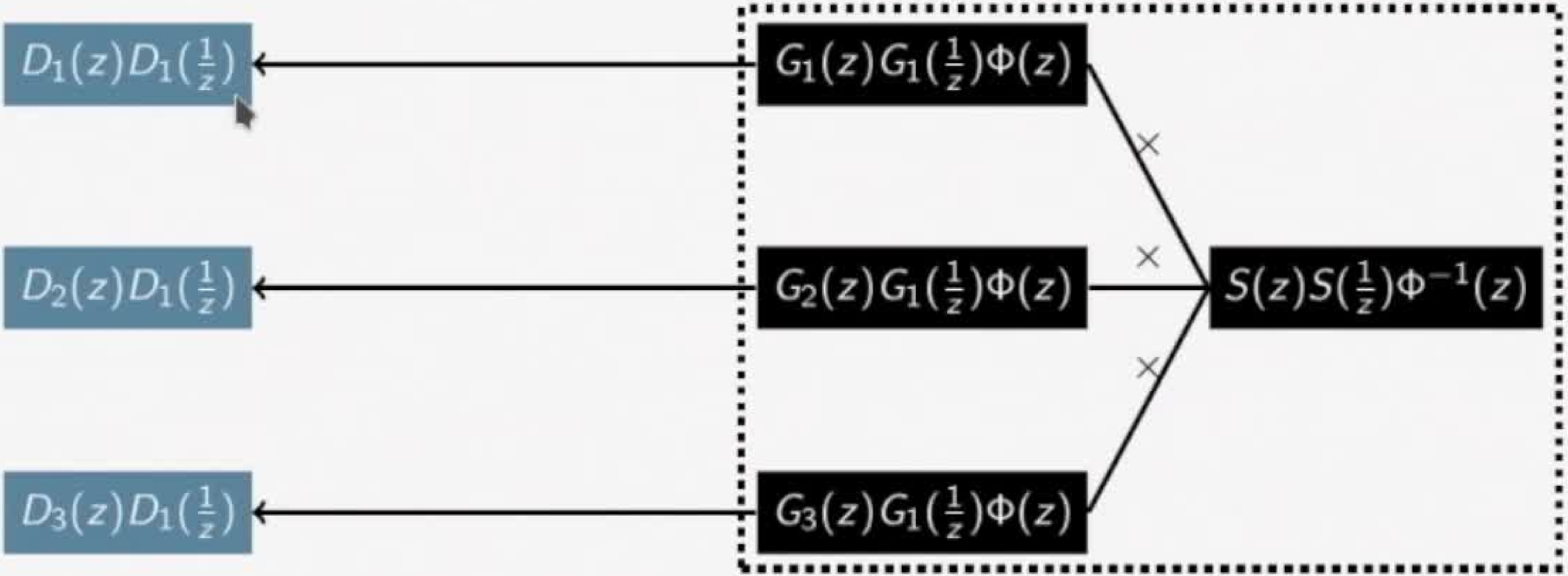
Interferometric Blind Deconvolution

Definition

$$\arg\min \sum_{k,l} \sum_{\tau} \{\{d_k \otimes d_l\}(\tau) - \{\{s \otimes s\} * \{g_k \otimes g_l\}\}(\tau)\}^2 + \underbrace{\alpha \sum_{k} \sum_{\tau} t^2 \{g_k \otimes g_k\}^2(\tau)}_{\text{Maximally White}}$$

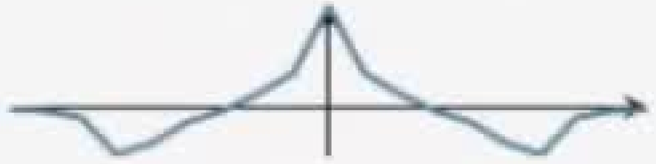
$\Phi(z)$ in Interferometric Blind Deconvolution

$\Phi(z)$ is real and non-negative when $|z| = 1$

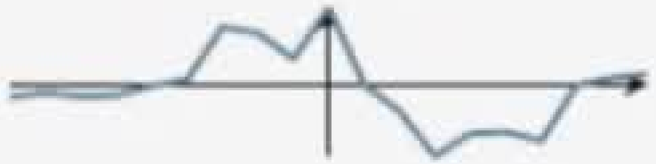


Undesired Solution 2: Defocusing Due To $\Phi(z)$

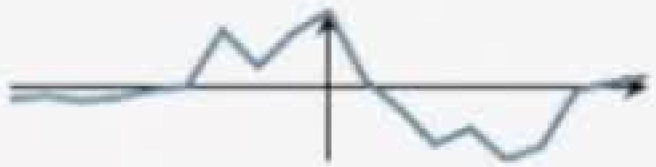
$\{d_1 \otimes d_1\}(\tau)$



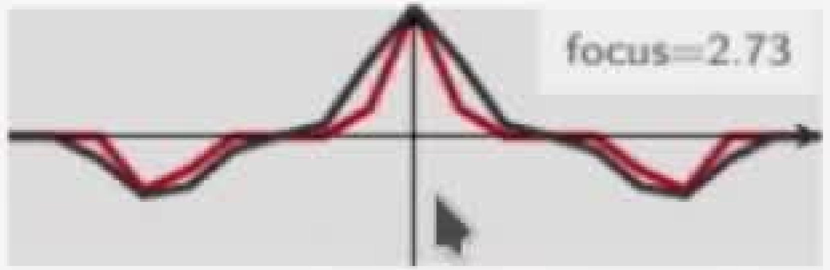
$\{d_1 \otimes d_2\}(\tau)$



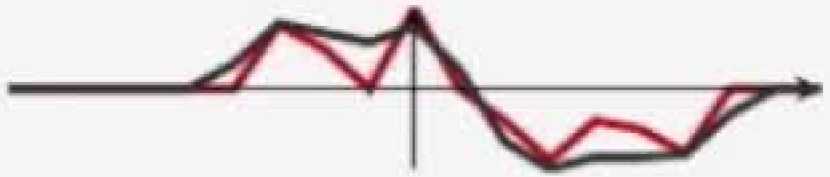
$\{d_1 \otimes d_3\}(\tau)$



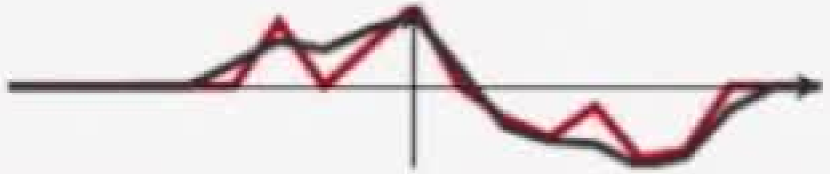
$\{g_1 \otimes g_1\}(\tau)$



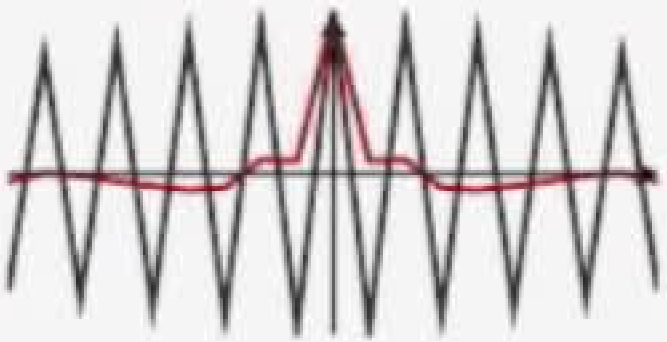
$\{g_1 \otimes g_2\}(\tau)$



$\{g_1 \otimes g_3\}(\tau)$

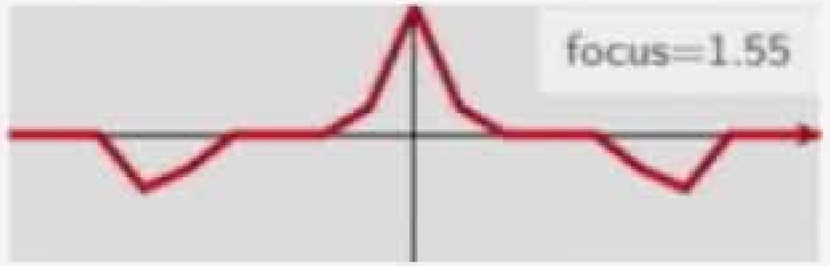


$\{s \otimes s\}(\tau)$



Focusing · · ·

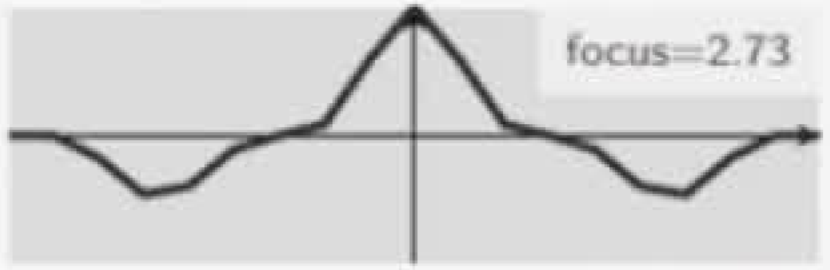
$G_1(z)G_1(\frac{1}{z})$



⇒ Focusing objective:

$$\sum_k \sum_\tau t^2\{g_k \otimes g_k\}^2(\tau).$$

$G_1(z)G_1(\frac{1}{z})\Phi(z)$

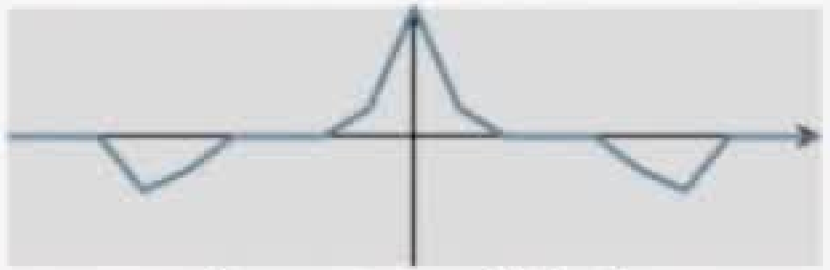


⇒ Theorem (Bharadwaj, Demanet & Fournier 2018):
lowest value of the focusing objective iff $\phi(t) = \delta(t)$ among all ϕ that cause indeterminacy.

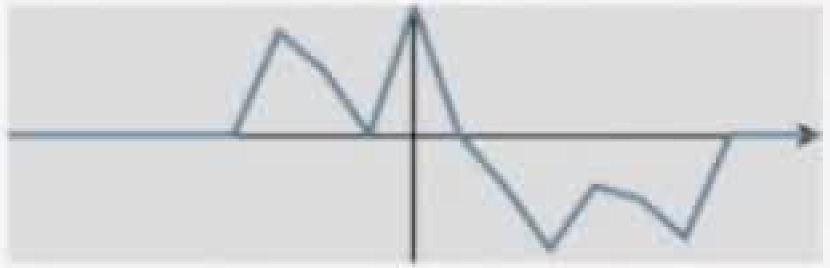
Phase Retrieval

Undesired Solution 1: Green's Function Not Front-loaded

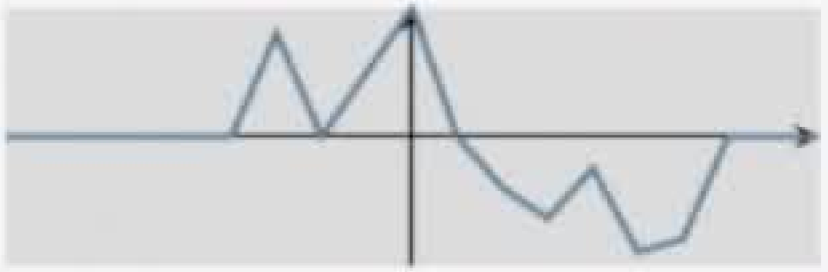
$\{g_1 \otimes g_1\}(\tau)$



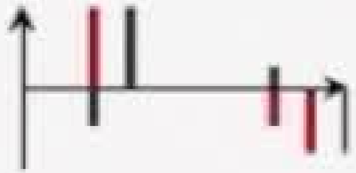
$\{g_1 \otimes g_2\}(\tau)$



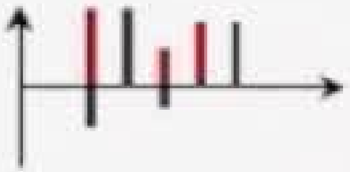
$\{g_1 \otimes g_3\}(\tau)$



$g_1(t)$

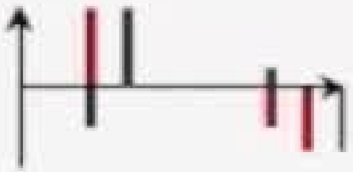


$g_2(t)$



$g_3(t)$

$g_1(t)$



Undesired Green's Function: Unknown Phase Due to $\Phi(z)$

Conclusions

Focused Blind Deconvolution:

- ➔ achieves data-driven Green's function retrieval from the multi-channel seismic data of a noisy source.
- ➔ and it doesn't demand either velocity model or unrealistic noisy-source assumptions.

Two Focusing Constraints:

- ➔ resolve the indeterminacy due to the amplitude spectrum of $\phi(t)$, by choosing the Green's functions that are maximally white.
- ➔ resolve the indeterminacy due to the phase spectrum of $\phi(t)$, by choosing the Green's functions that are maximally front-loaded.

Acknowledgements

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equinor