Groundwater, Climate, and the Growth of River Networks

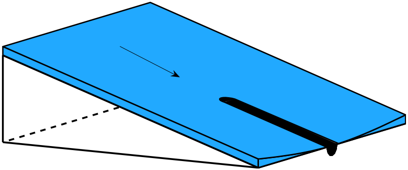
Daniel H. Rothman
Lorenz Center
Department of Earth, Atmospheric, and Planetary Sciences
Massachusetts Institute of Technology

with

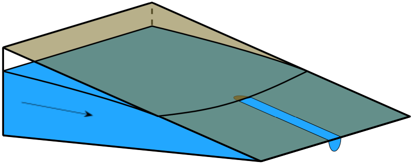
Alvaro Arredondo, Yossi Cohen, Olivier Devauchelle, Alexander P. Petroff, Hansjörg Seybold, Eric Stansifer, and Robert Yi

How are landscapes shaped by groundwater flow?

Rainfall enters rivers in either of two ways



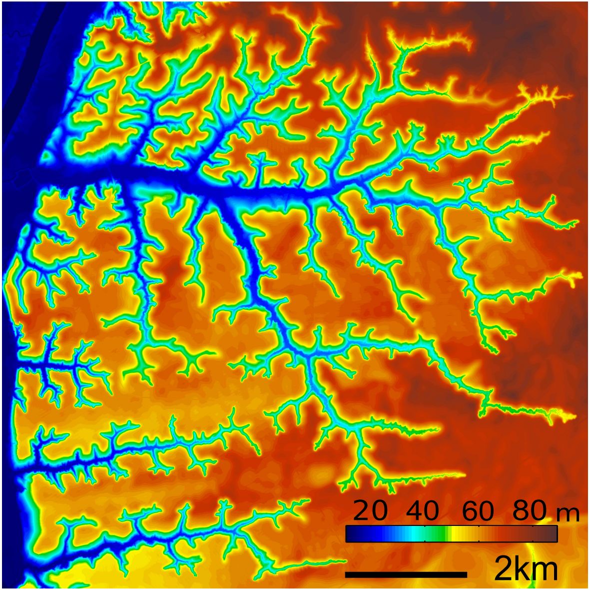
overland flow



subsurface flow

- Nearly all modern models of landscape evolution assume landscapes are eroded by overland flow (*Horton*, 1945).
- But when infiltration exceeds rainfall, runoff travels underground. Seepage to the surface can then erode channels (*Dunne*, 1980).
- How does groundwater flow shape river networks?

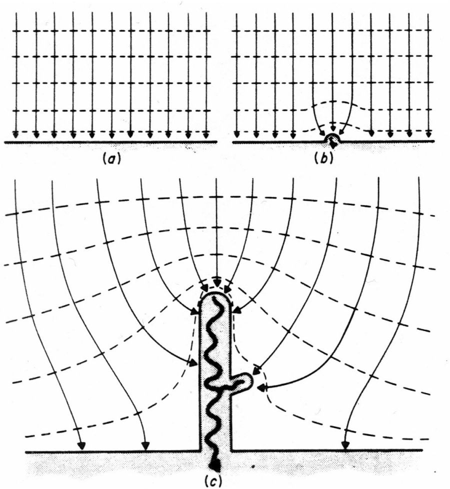
Stream network driven by groundwater seepage

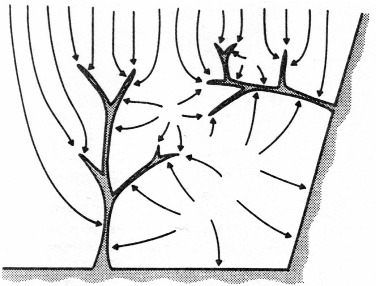


Florida Panhandle (Bristol)

Valleys cut through ancient beach sands

Conceptual model (Dunne, 1980)





Groundwater flow is focused toward channel tips, which ramify, creating a stream network.

Two-dimensional approximation of groundwater flow

- Assume the water table is approximately flat. Then the flux

$$\mathbf{q} = -K\, h \nabla h,$$

where K is the hydraulic conductivity and h is the height of the water table.

- Incompressiblity yields

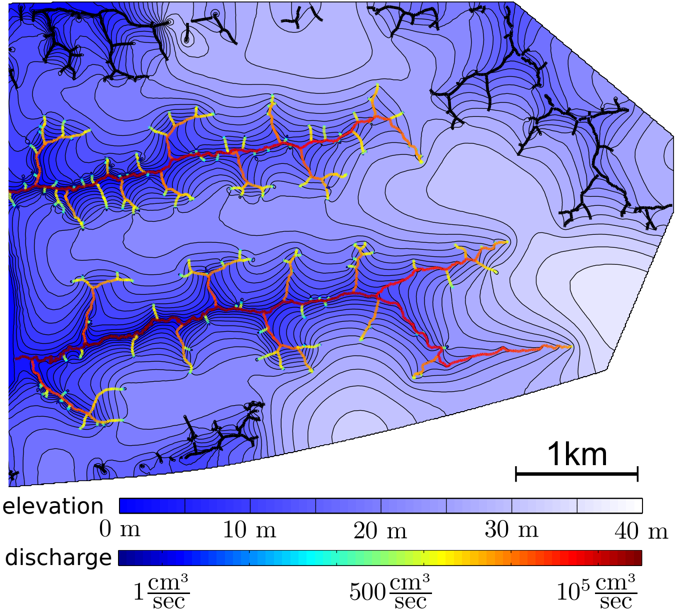
$$\nabla \cdot \mathbf{q} = R,$$

where R is the rainfall rate (minus evapotranspiration).

- The steady height $h(x, y)$ of the water table approximately satisfies the 2-D *Poisson* or *Dupuit-Forchheimer* equation

$$\nabla^2 h^2 = -2\frac{R}{K}.$$

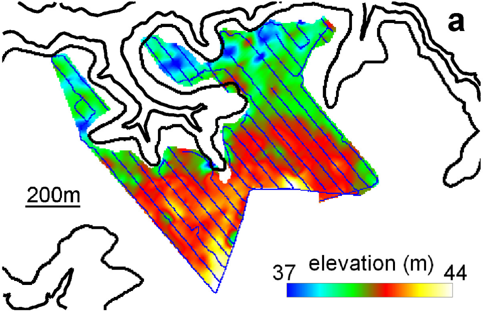
Numerical calculation of water table and discharge



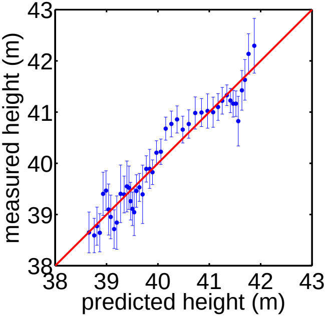
Petroff et al. (2012)

Comparison of measured and predicted water table

Ground-penetrating radar

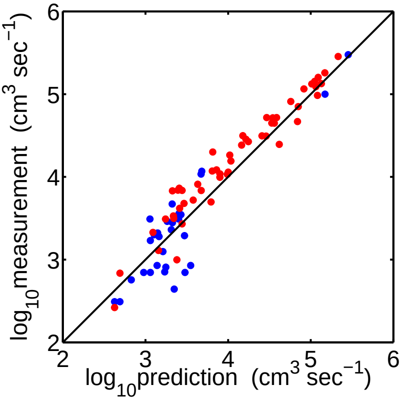


Fit to Poisson theory



Abrams et al. (2009), Petroff et al. (2011)

Comparison of measured and predicted fluxes



No adjustable parameters.

- Winter
- Spring

Conclusions:

- Water enters the network from the subsurface.
- The Poisson equation predicts subsurface flow.

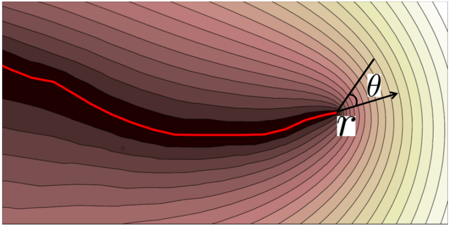
Questions

- In which direction do streams grow?
- At what angle do streams branch?
- What determines basin shape?

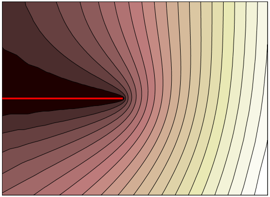
In which direction do streams grow?

Hypothesis: Streams grow in the direction predicted by the principle of local symmetry (Barenblatt & Cherepanov, 1961).

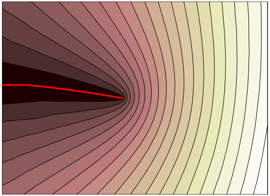
Groundwater field ϕ around a stream



$a_2 \neq 0$ (asymmetric)



$a_2 = 0$ (symmetric)

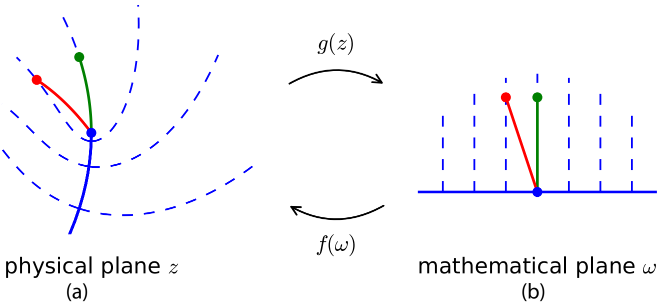


Near the tip,

$$\phi(r,\theta) = a_1 r^{1/2} \cos\frac{\theta}{2} + a_2 r \sin\theta + \mathcal{O}\left(r^{3/2}\right)$$

PLS: Streams grow in the direction for which $a_2 = 0$ (*Cohen et al., 2015, 2016*).

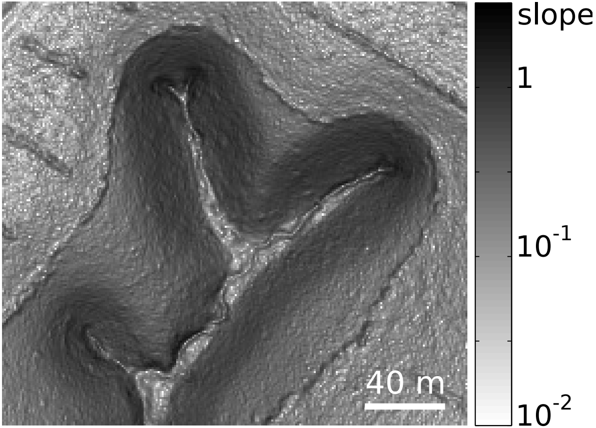
PLS ⇔ streamline growth ⇔ flux maximization



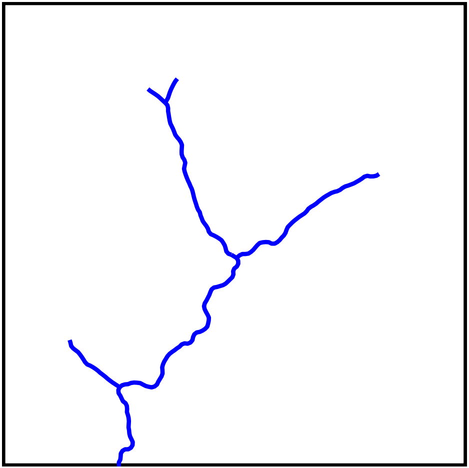
Near tips, the Poisson field becomes Laplacian ($\nabla^2\phi = 0$). Then

- Growth along the streamline entering the tip maintains local symmetry (*Cohen et al., 2015, 2016*)
- The local symmetry direction also maximizes flux to the tip as the channel grows (*Devauchelle et al., 2017*).

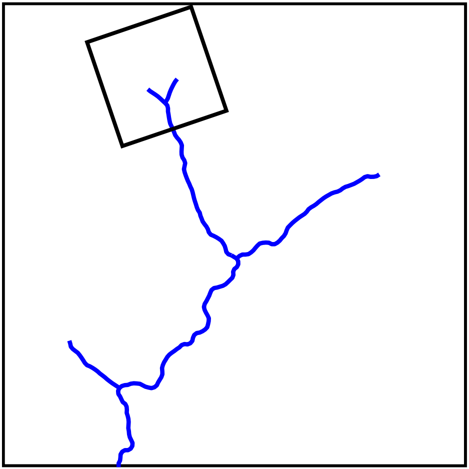
At what angle do streams branch?
(Devauchelle et al., 2012; Petroff et al., 2013)



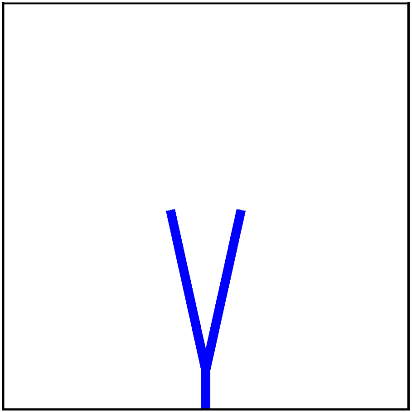
A set of bifurcated tips



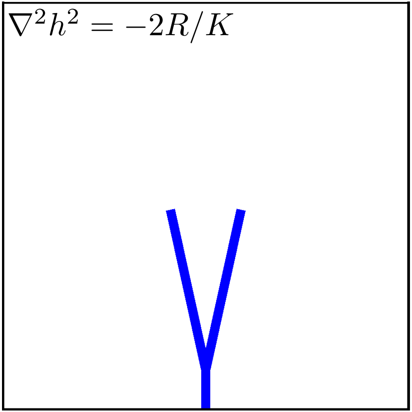
Zoom in on a bifurcation



An idealized bifurcation



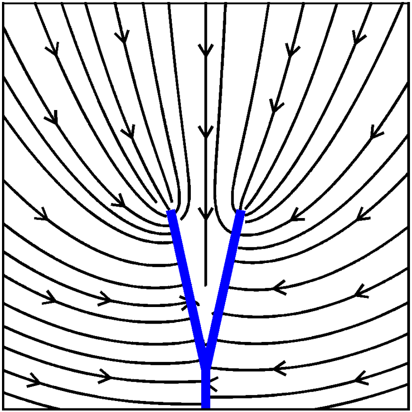
Near tips, the Poisson field becomes Laplacian



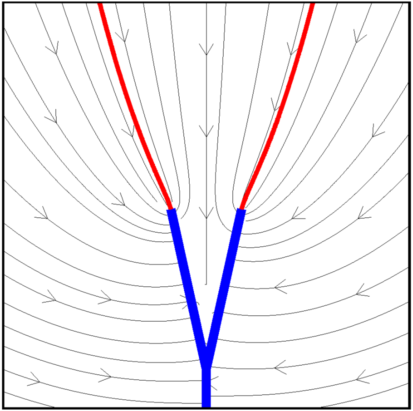
Precipitation contributes negligibly to subsurface flux near tips:

$$\nabla^2 h^2 \simeq 0$$

Solve for the flow field

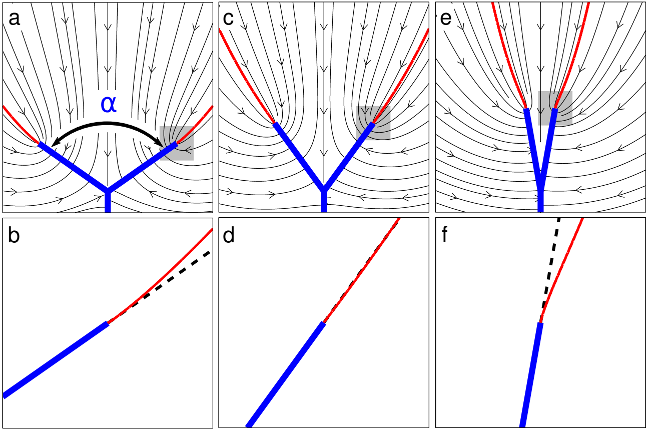


Assume growth follows the streamline



Equivalently, assume the growth direction maintains local symmetry, or that it maximizes flux to tips.

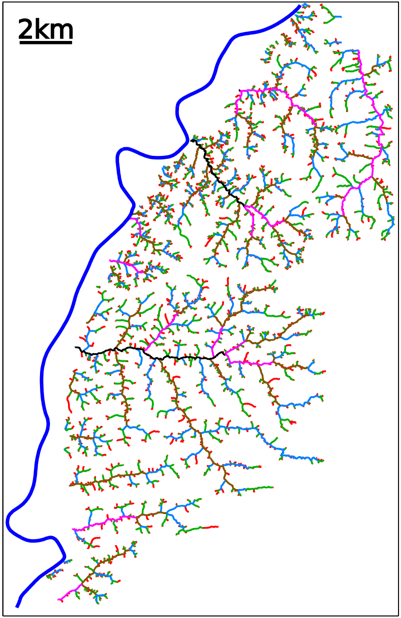
Growth leads to a stable angle α^*



$$\alpha^* = \frac{2}{5}\pi = 72°$$

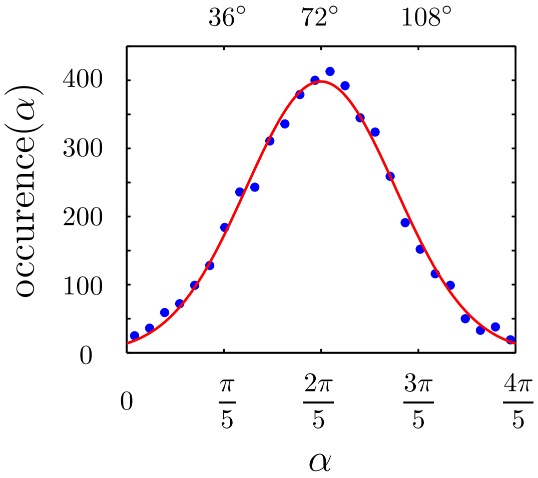
See also *Hastings* (2001); *Carleson & Makarov* (2002)

Measure 4966 branching angles, at all scales



- Approximate channel segments by straight lines.
- Measure angle between the two upstream branches at all junctions.

Histogram of all 4966 angles



Prediction:
$\alpha^* = \frac{2}{5}\pi = 72°$

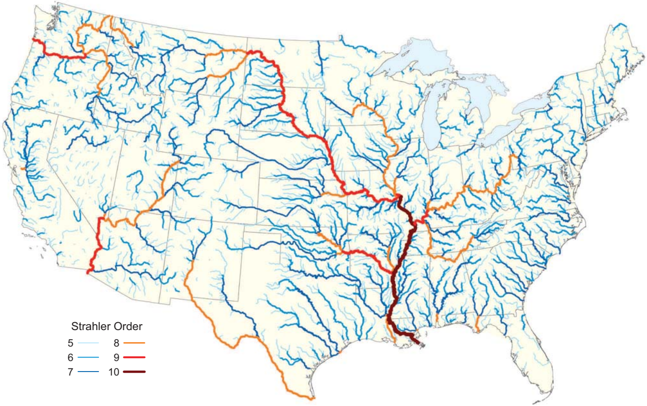
Measurement:
$\langle\alpha\rangle = 71.88° \pm 0.75°$

Sources of variance:
network interactions,
inhomogeneities,
random forcing,...

Neither the mean nor the variance are scale dependent.

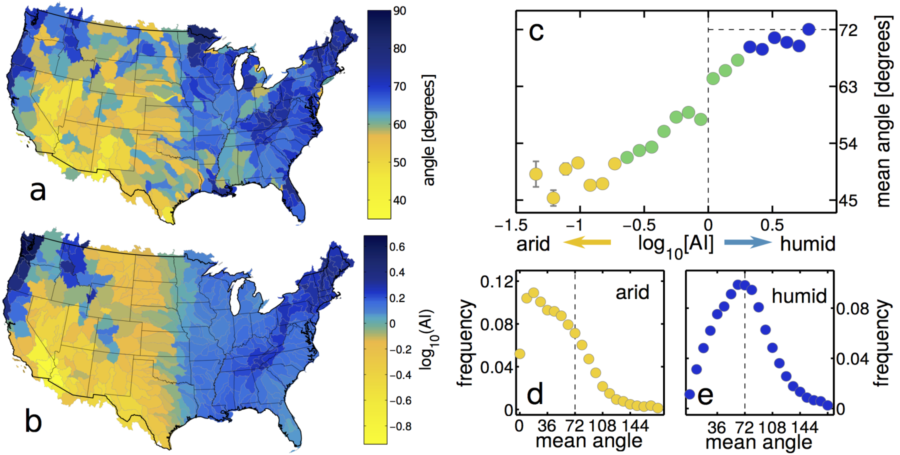
How widespread is the $2\pi/5$ bifurcation? (*Seybold et al., 2017*)

- We measure branching angles throughout the continental United States, using drainage networks mapped in the NHDPlus Version 2 database.



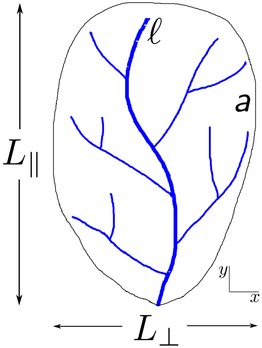
- 934,207 stream junctions.

Correlation between angle and aridity index AI



As climate becomes more humid, branching angles $\rightarrow 72^{\circ}$.

Shapes of river networks (Yi et al., 2018)



Empirical scaling laws:

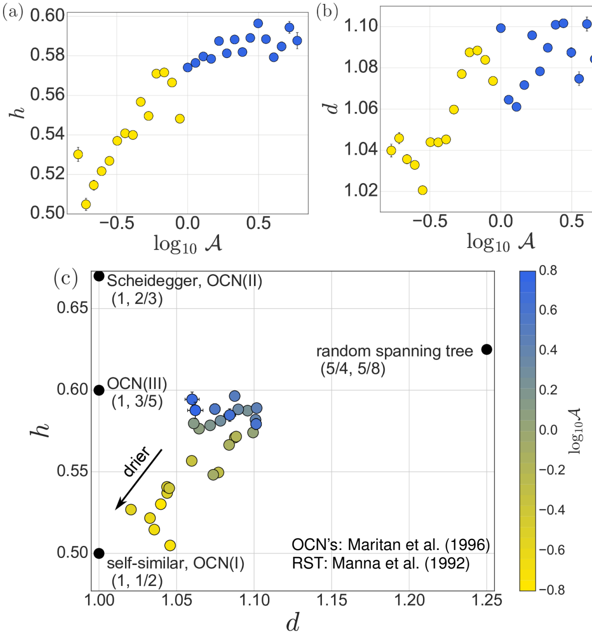
$$\ell \propto a^h, \quad h \simeq 0.6$$
$$\ell \propto L_{\parallel}^d, \quad d \simeq 1.1$$

(Hack, 1957; Tarboton et al., 1988)

Shape: *The aspect ratio* $L_{\perp}/L_{\parallel}$

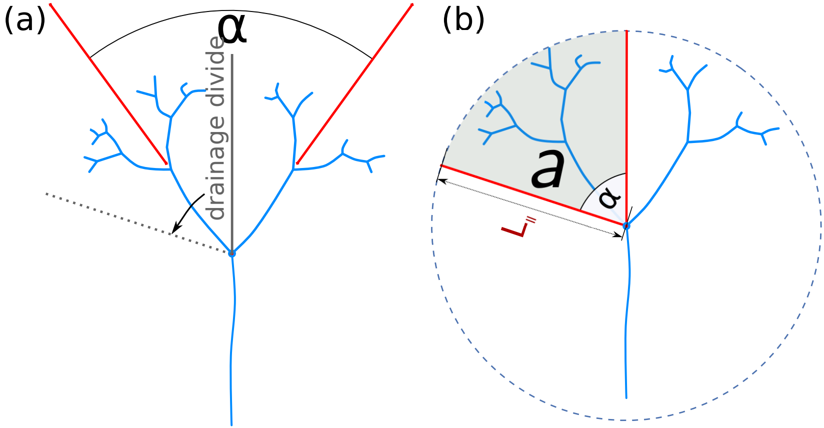
- $h > 1/2$ implies that $L_{\perp}/L_{\parallel}$ depends on basin size.
- $h = 1/2$ implies that shapes are independent of size.

Aridity and the exponents *h* and *d*



$h \to 1/2,\ d \to 1$ as climate becomes drier.

Hypothesis: $L_\perp/L_\parallel$ depends on the junction angle α



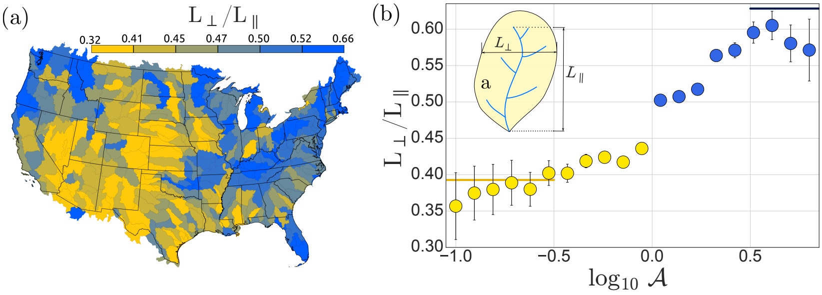
Basin area $a = \alpha L_\parallel{}^2/2$. Assuming $a \sim L_\parallel L_\perp$,

$$\frac{L_\perp}{L_\parallel} = \frac{\alpha}{2}.$$

Climatic limits of basin shape

$$\textit{Wet:}\ \ \alpha = 72^\circ \Rightarrow \frac{L_\perp}{L_\parallel} = \frac{\pi}{5} = 0.63$$

$$\textit{Dry:}\ \ \alpha \simeq 45^\circ \Rightarrow \frac{L_\perp}{L_\parallel} \simeq \frac{\pi}{8} = 0.39$$



Conclusions

Stream networks in humid climates exhibit quantitative consequences of their interaction with groundwater.

- Junction angles are relatively wide, tending toward $2\pi/5 = 72^\circ$.
- Their shapes are also relatively wide; smaller basins tend toward an aspect ratio of $\pi/5 = 0.63$.

In these ways, the flow of water *underground* determines the organization of streams *overground*.