# The Mid-Pleistocene Transition in the Glacial Flip-Flop Model

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SIAM DS19

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Mathematics and Climate Research Network

Image courtesy of Wikimedia Commons

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“The Mid Pleistocene Transition from Budyko's Energy Balance Model” (submitted for publication Oct. 2018)

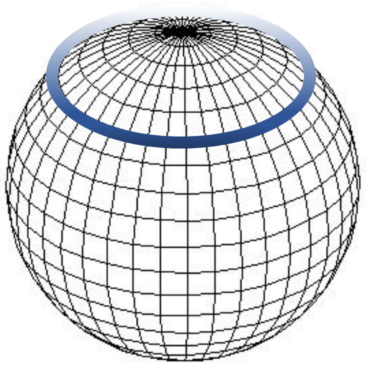
# Timeline

1969 — Budyko's EBM

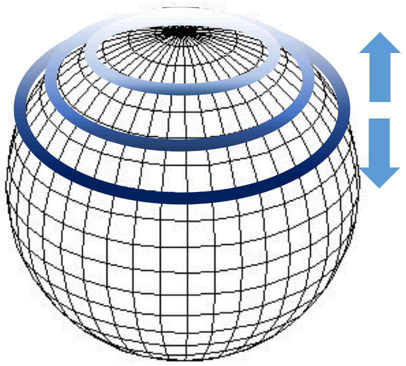
2014 — Budyko-Widiasih

2016 — Flip-Flop

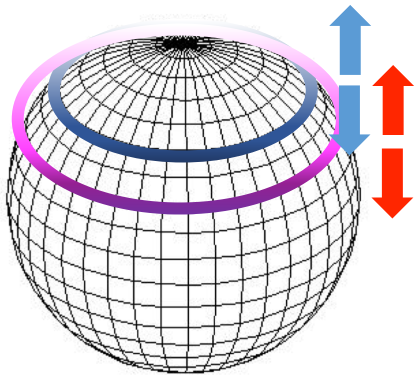
2019 — Flip-Flop (MPT?)



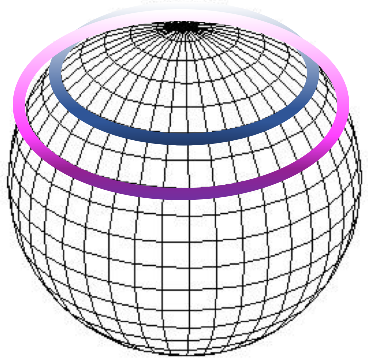
Fixed Ice Line



Dynamic Ice Line

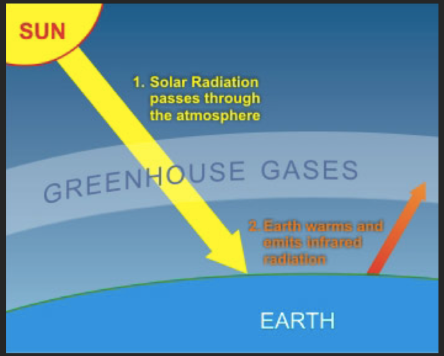


Dynamic Ice Line & Glacier's Edge



Ramped Critical Temperature for MPT

# PART 1:
# BUDYKO-WIDIASIH MODEL




# Budyko-Widiasih Model

$$\frac{\partial}{\partial t}T(t,y) = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{OLR}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\right)$$

Temperature Equation

$$\frac{d\eta}{dt} = \varepsilon\left[T(\eta) - T_c\right]$$

Ice Line Equation

E. Widiasih, R. McGehee. A Quadratic Approximation to Budyko's Ice-Albedo Feedback Model with Ice Line Dynamics, *SIAM J. Appl. Dyn. Syst.*, March 2014.

# Budyko-Widiasih Model: Temperature Equation

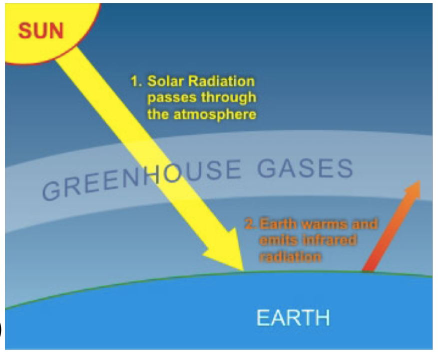
Annual average temp at latitude $y = \sin\,\theta$

$$\frac{\partial}{\partial t}T(t,y) = \frac{1}{R}\Bigg(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{Outgoing Longwave Radiation}} - \underbrace{C\,(T(y)-\overline{T})}_{\text{transport}}\Bigg)$$

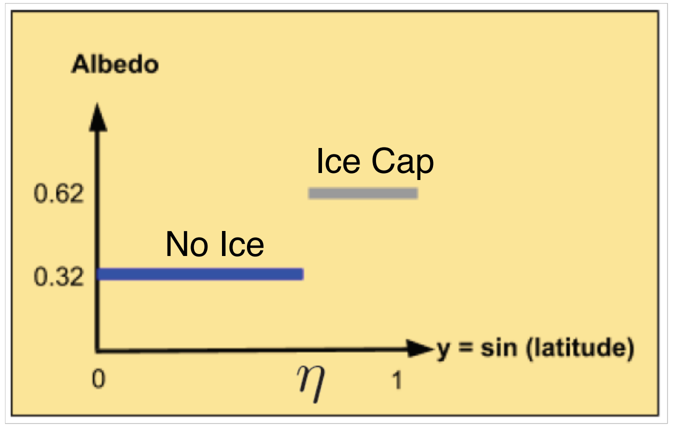
$$\frac{d\eta}{dt} = \varepsilon\,[T(\eta) - T_c]$$

Based on the simple idea:

**Temperature Change ~ Energy IN – Energy OUT**




(Courtesy of skepticalsciences.com)

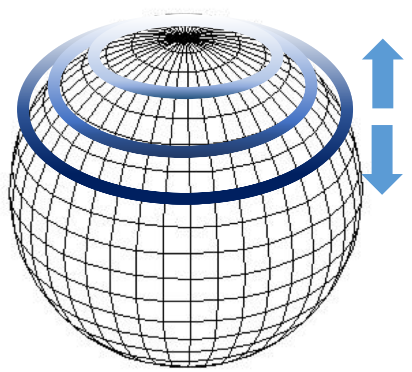


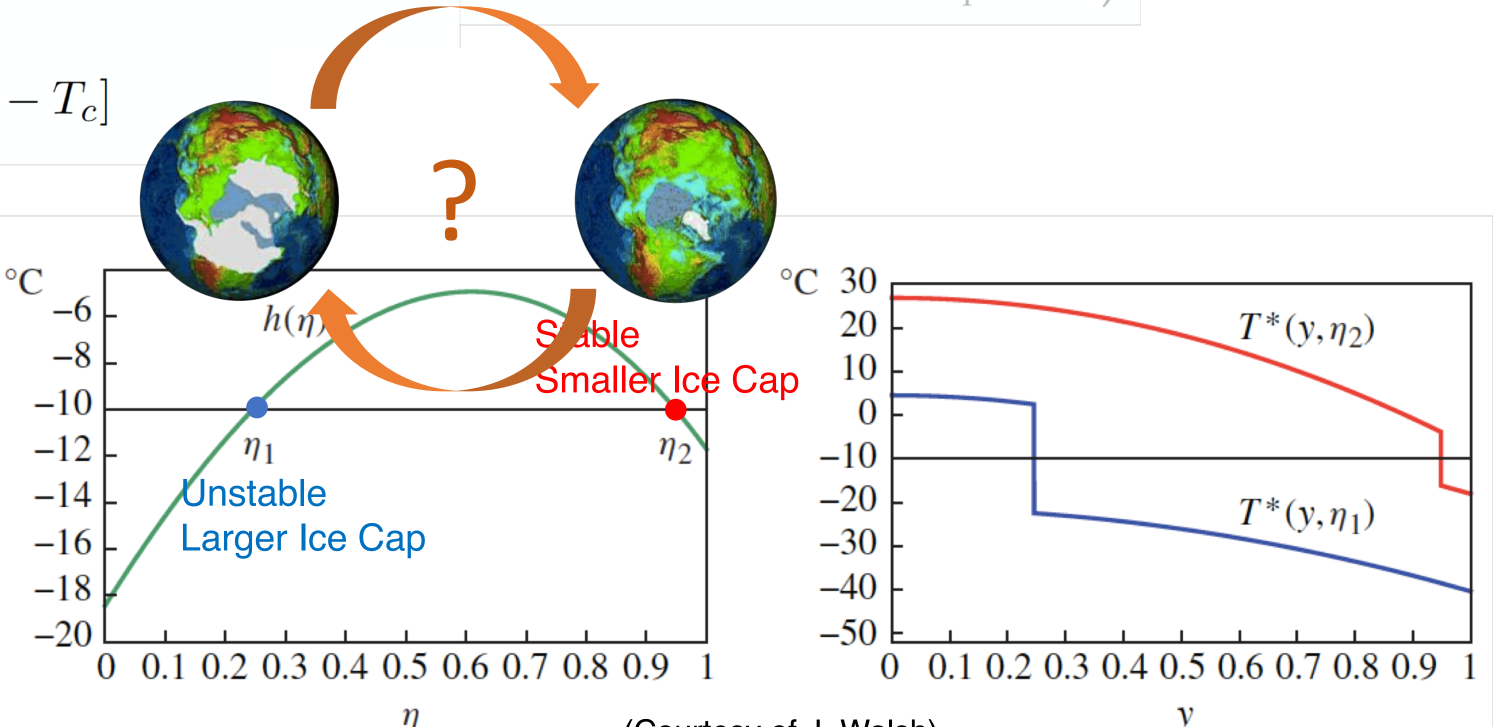

(Courtesy of E. Widiasih)

# Budyko-Widiasih Model: Ice Line Dynamics

$$\frac{\partial}{\partial t}T(t,y) = \frac{1}{R}\Big(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{OLR}} - \underbrace{C\,(T(y)-\overline{T})}_{\text{transport}}\Big)$$

$$\frac{d\eta}{dt} = \varepsilon\,[T(\eta) - T_c]$$



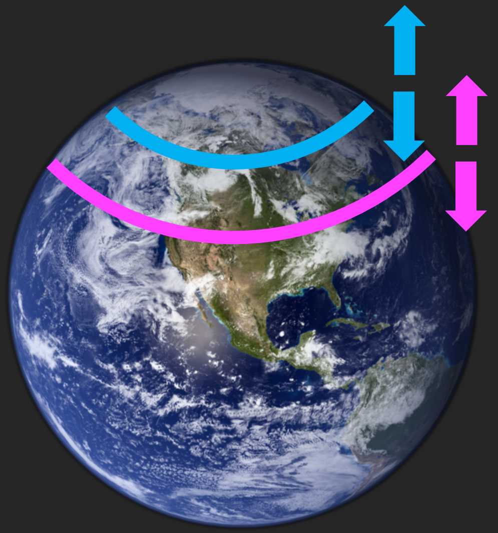



(Courtesy of J. Walsh)

$T(\eta) > T_c$: Ice line moves poleward
$T(\eta) < T_c$: Ice line descends

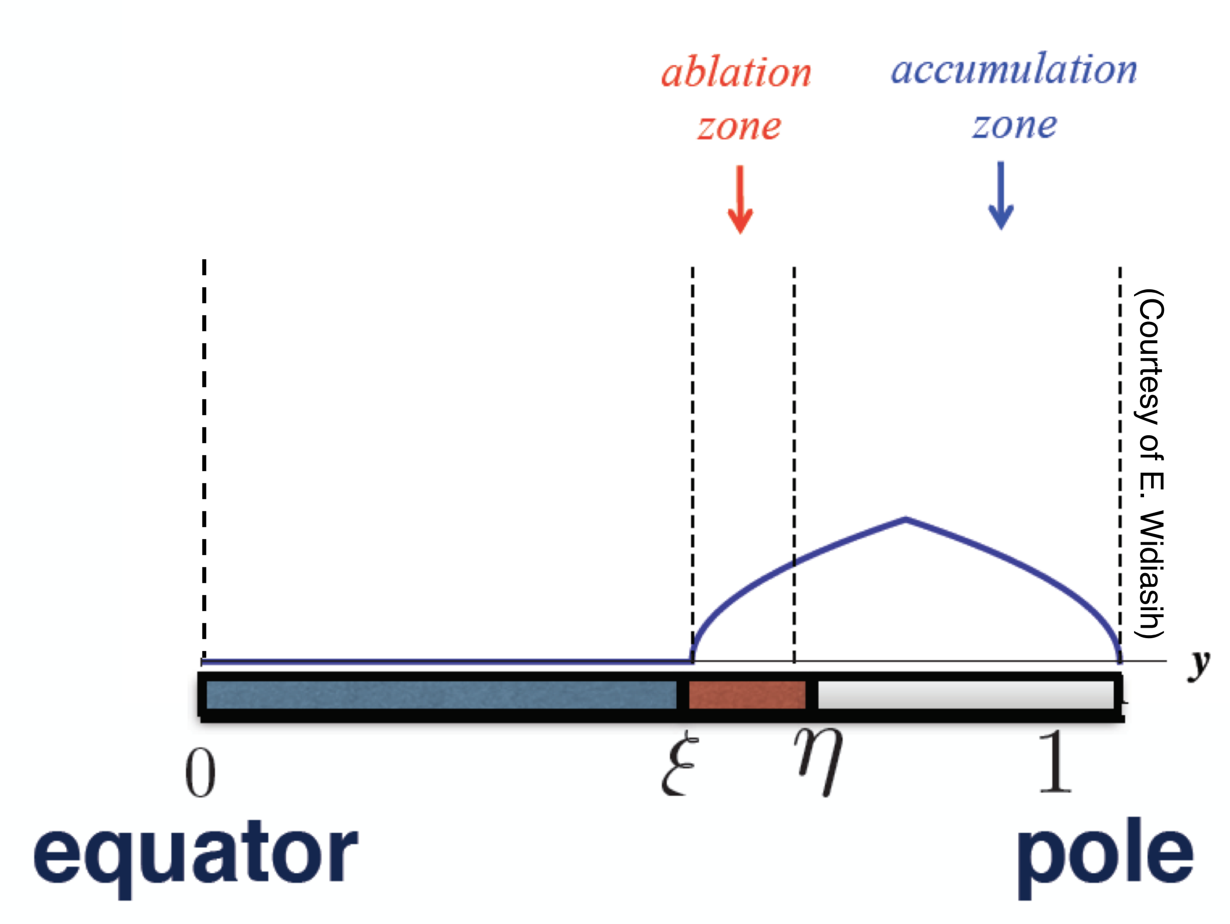
PART 2:

# GLACIAL FLIP-FLOP MODEL



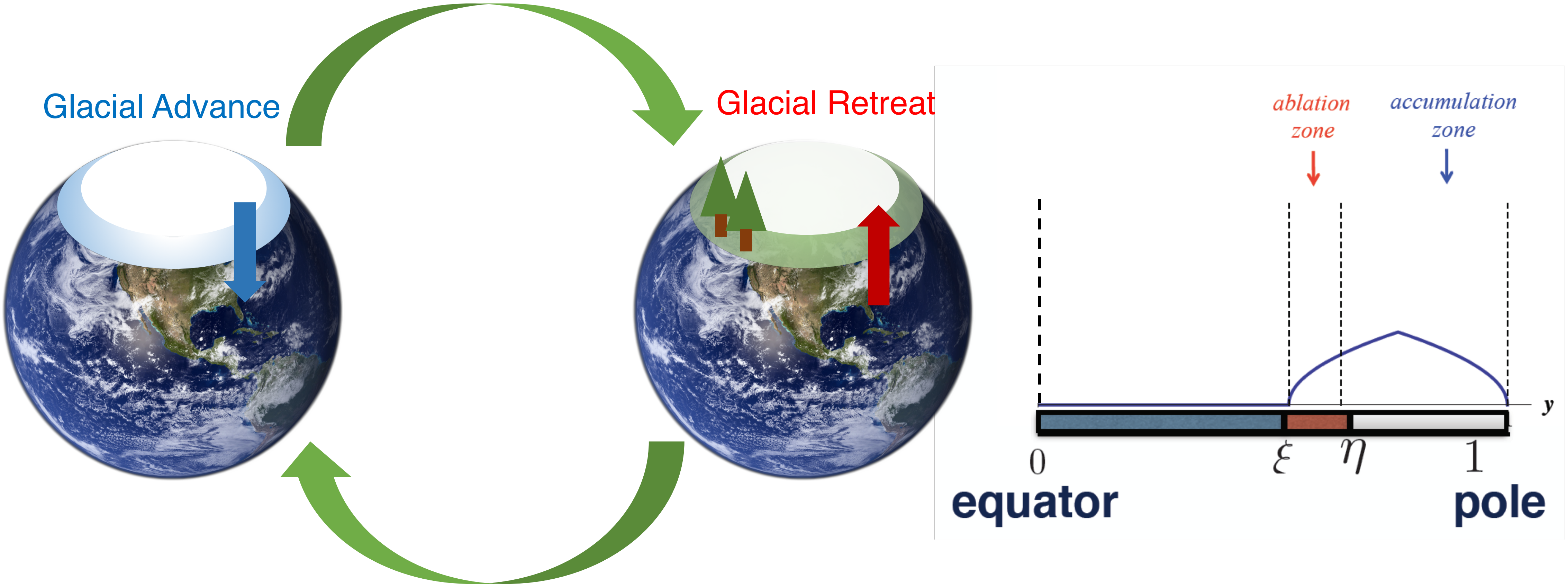
# From EBM to Flip-Flop Model: Variables

| | |
|---|---|
| **Widiasih's Snow Line $\eta$** | Snow/ice cover remains throughout the year |
| **(NEW) Glacier's Edge Variable $\xi$** | Snow accumulating to become glacier, or thinner retreating ice |




J. Walsh, E. Widiasih, J. Hahn, R. McGehee. Periodic orbits for a discontinuous vector field arising from a conceptual model of glacial cycles. *Nonlinearity* 29.6 (2016): 1843.

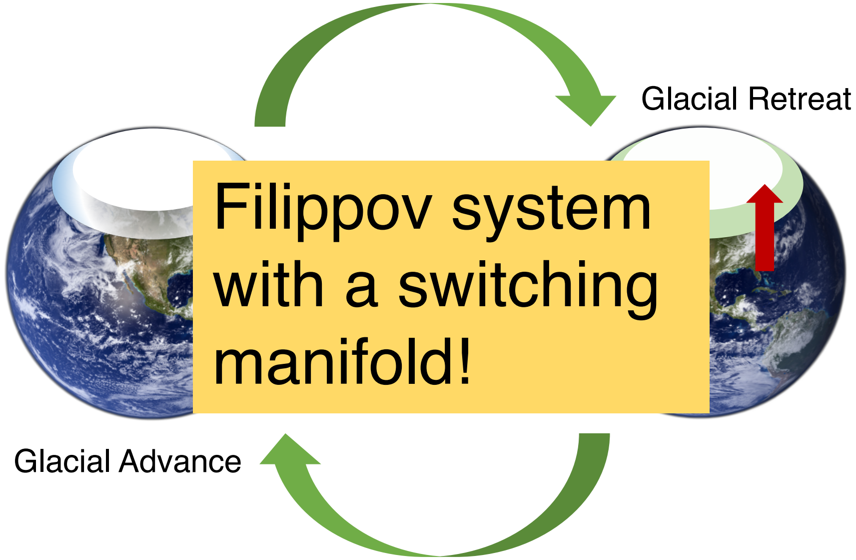
# Flip-Flop Model's States: Motivation



# Flip-Flop Model's States: Glacial Advance & Retreat

## Glacial Advance

- Global conditions favor ice sheet decay
- Ablation < accumulation
- $T_c \; = \; -10℃$




## Glacial Retreat

- Global conditions favor ice sheet decay
- Ablation > accumulation
- $T_c \; = \; -5.5℃$ (can vary)

$$\frac{\partial}{\partial t} T(t,y) = \frac{1}{R}\left( \underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{Absorbed insolation}} - \underbrace{(A+BT(y))}_{\text{OLR}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}} \right)$$

$$\frac{d\eta}{dt} = \varepsilon\left[T(\eta) - T_c\right]$$

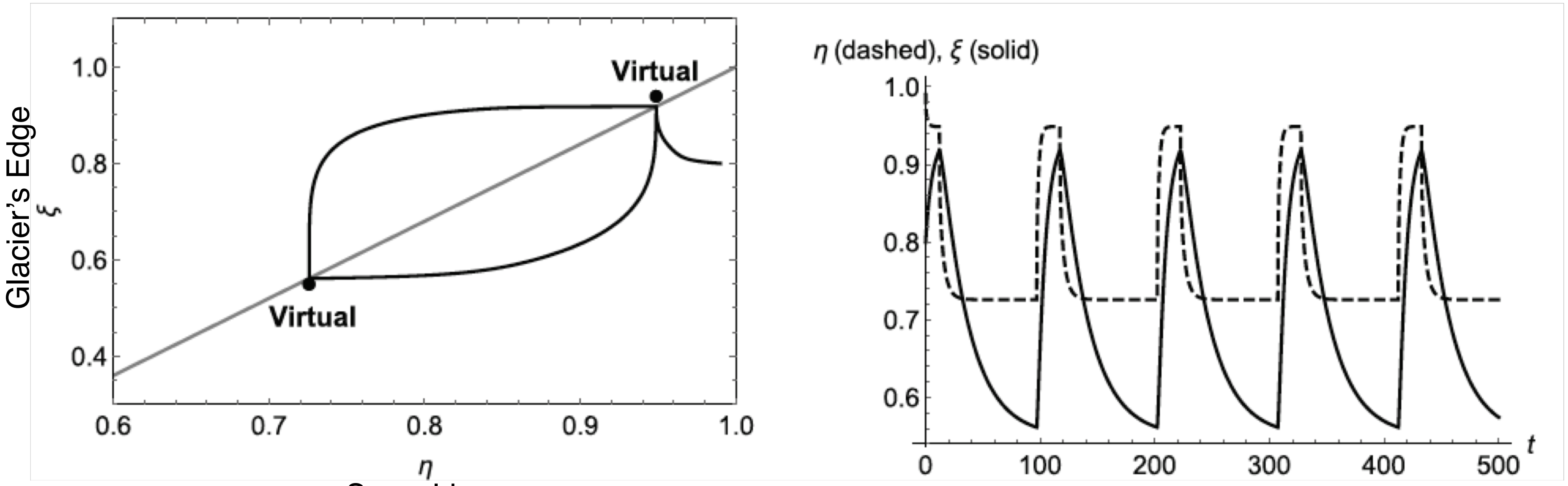
$$\frac{d\xi}{dt} = \epsilon\left[b(\eta-\xi) - a(1-\eta)\right]$$

Switching between states at Ablation rate = Accumulation rate

# Flip-Flop Model's Dynamics

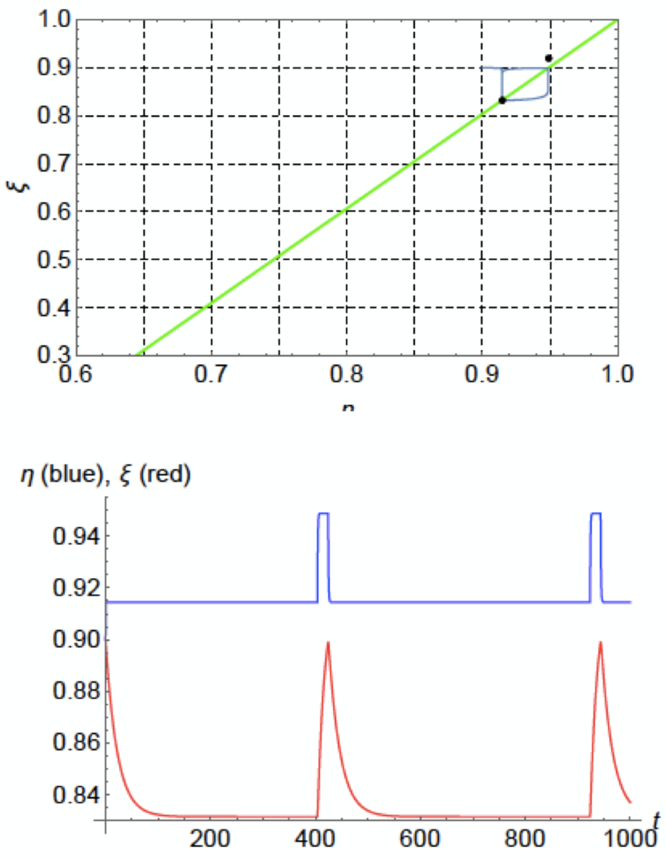
**Theorem(WWHM-2016):**

With the standard set of parameters, the Filippov system of the Flip-Flop model admits a unique, attracting periodic orbit



# Creating Desired Cycles from Flip-Flop

(Courtesy of E. Widiasih)




Blue: Ice Line
Red: Glacier's Edge

$T_{c_r}$ = $T_c$ for glacial ret.
$T_{c_a}$ = $T_c$ for glacial adv.

Conjecture: $|T_{c_r} - T_{c_a}|$ drives the orbit's period and amplitude.

# Adding Milankovitch Forcing

(Courtesy of E. Widiasih)

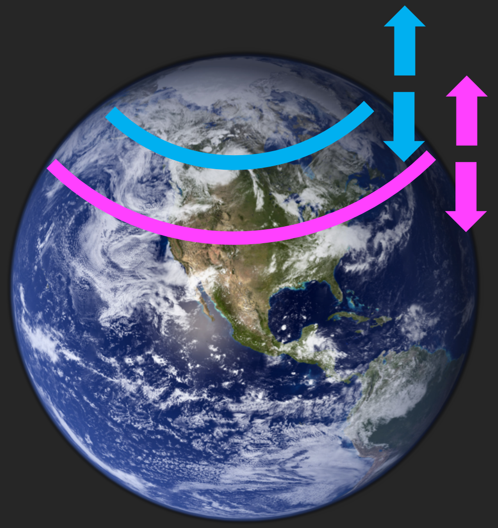

Does the Flip-Flop have a mechanism to capture MPT?

$T_{c_r}$ = -9, $T_{c_a}$ = -10

PART 3:

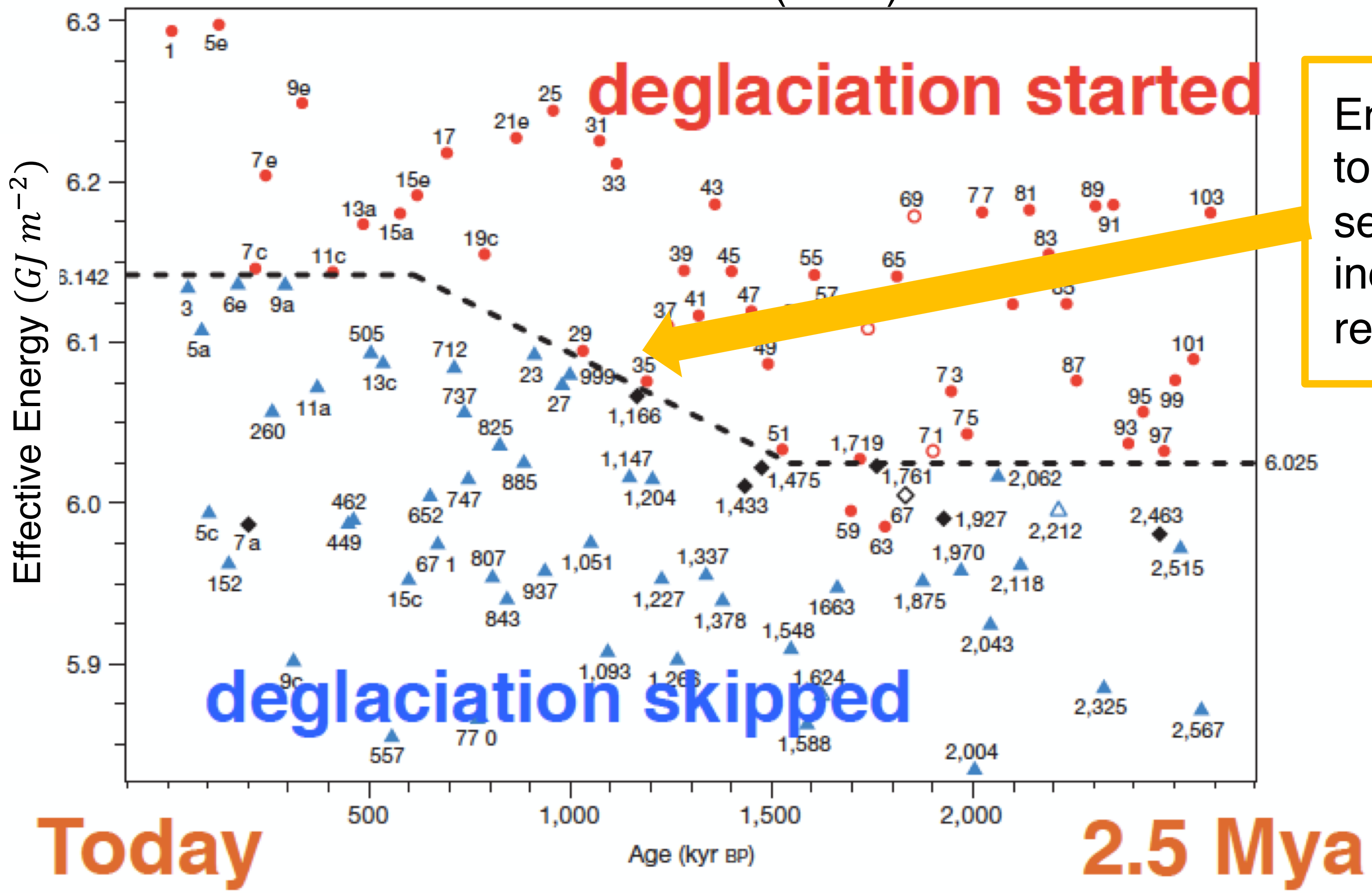
# GLACIAL FLIP-FLOP MODEL:

**MPT edition**



# Motivation to tweak Flip-Flop's structure for MPT

Tzedakis et al. (2017)

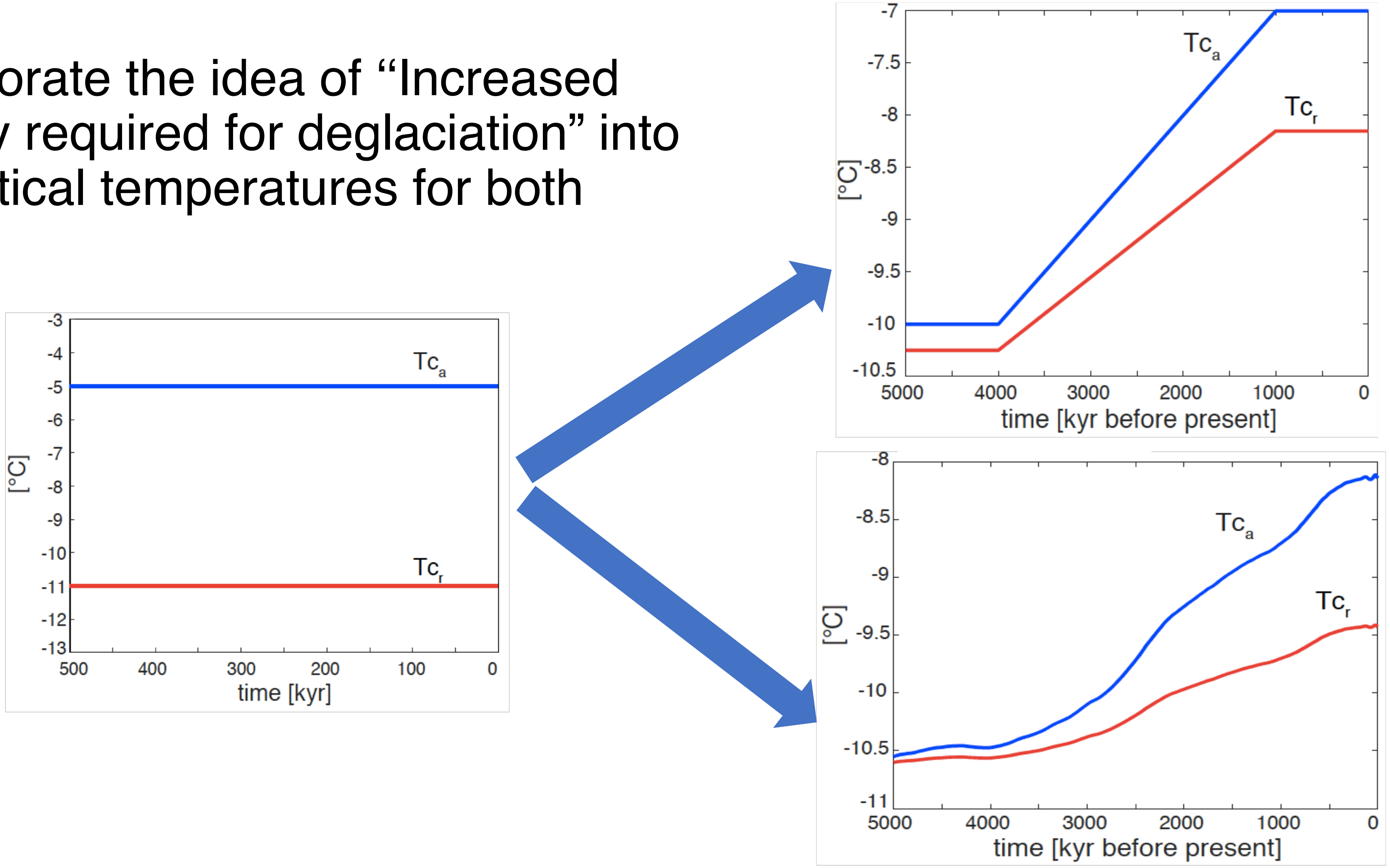



All points plotted are 'caloric summer half-year insolation peaks' at 65° North

Energy required to deglaciate seem to have increased in the recent years

# Motivation to tweak Flip-Flop's structure for MPT

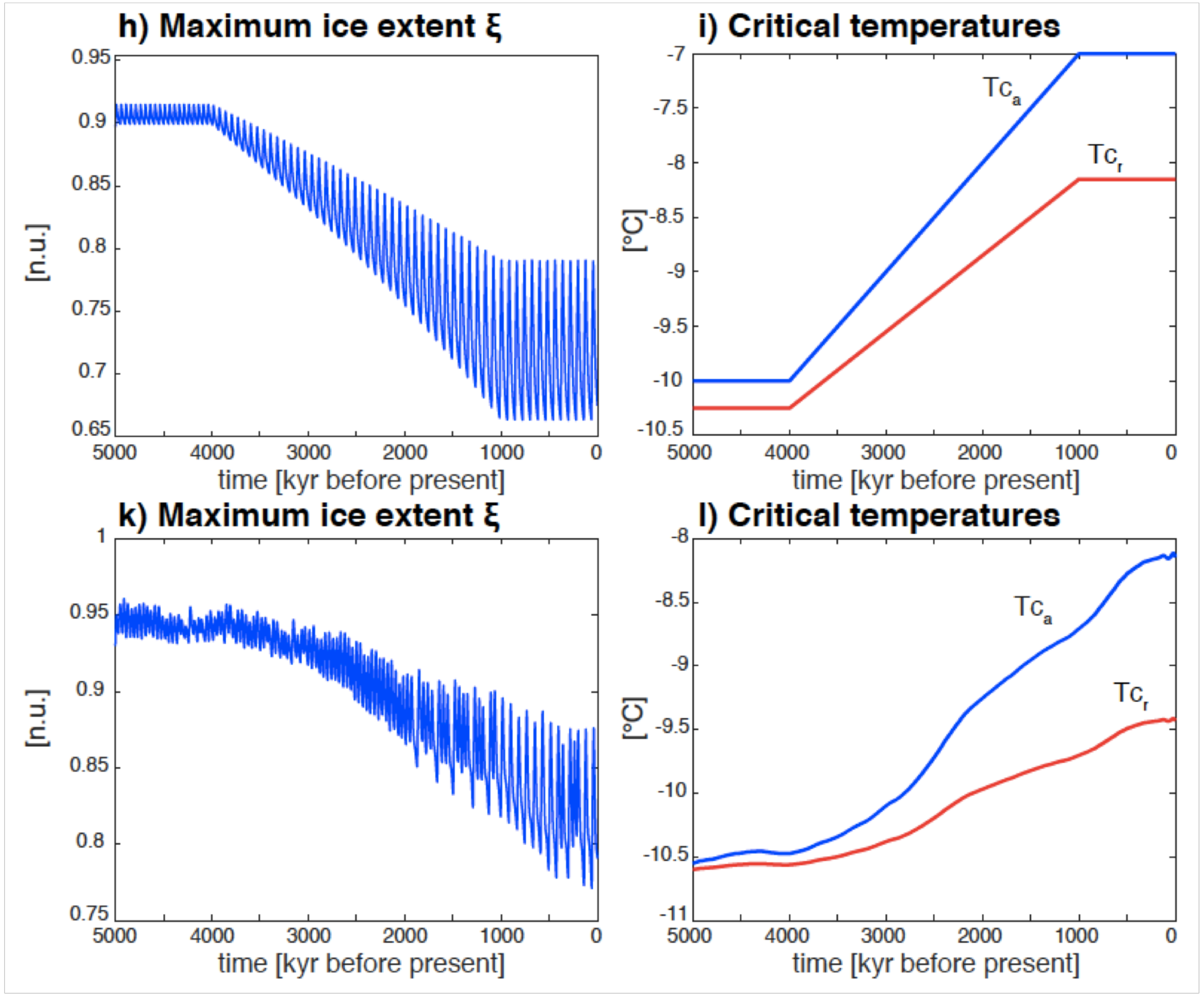
- Incorporate the idea of "Increased energy required for deglaciation" into our critical temperatures for both



# Flip-Flop Model Simulation with modified $T_c$

Simulation 1

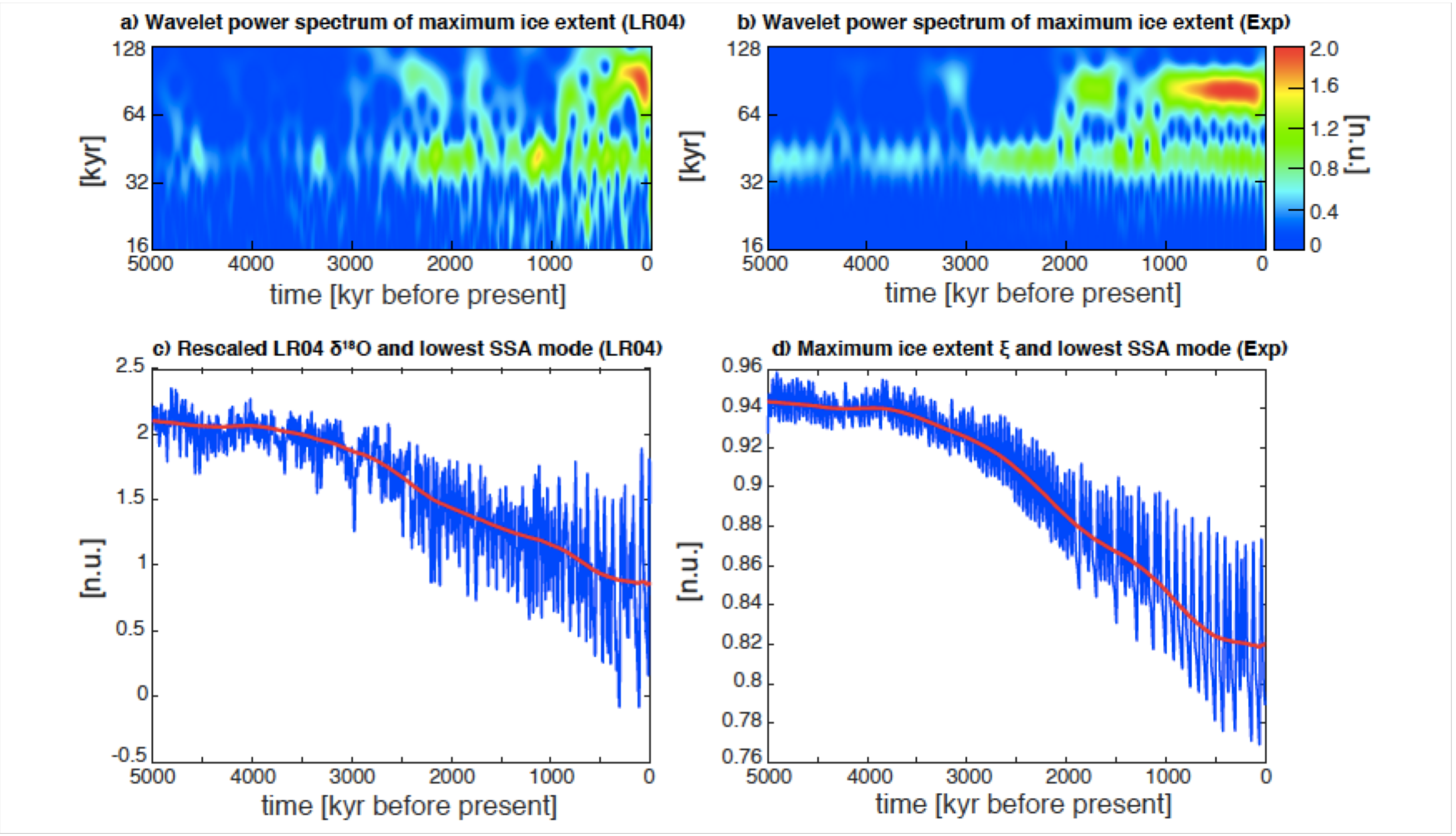
Simulation 2



# Checking for MPT in Simulation

LR04 Data

Simulation



# Conclusion

- A simple glacial cycle model based on Budyko's energy balance model is used to simulate the glacial cycles over the past 5 million years
- When a critical parameter of the model is linked to a stack of benthic 18O, the model simulates a realistic Mid Pleistocene Transition
- The critical parameter is the ice forming critical temperature, capturing a connection between the ice-albedo feedback and the temperature-accumulation ablation feedback.

# References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# Thank you!