Clustering, Malleability and Synchronization of Hodgkin-Huxley-Type Neurons

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Summary

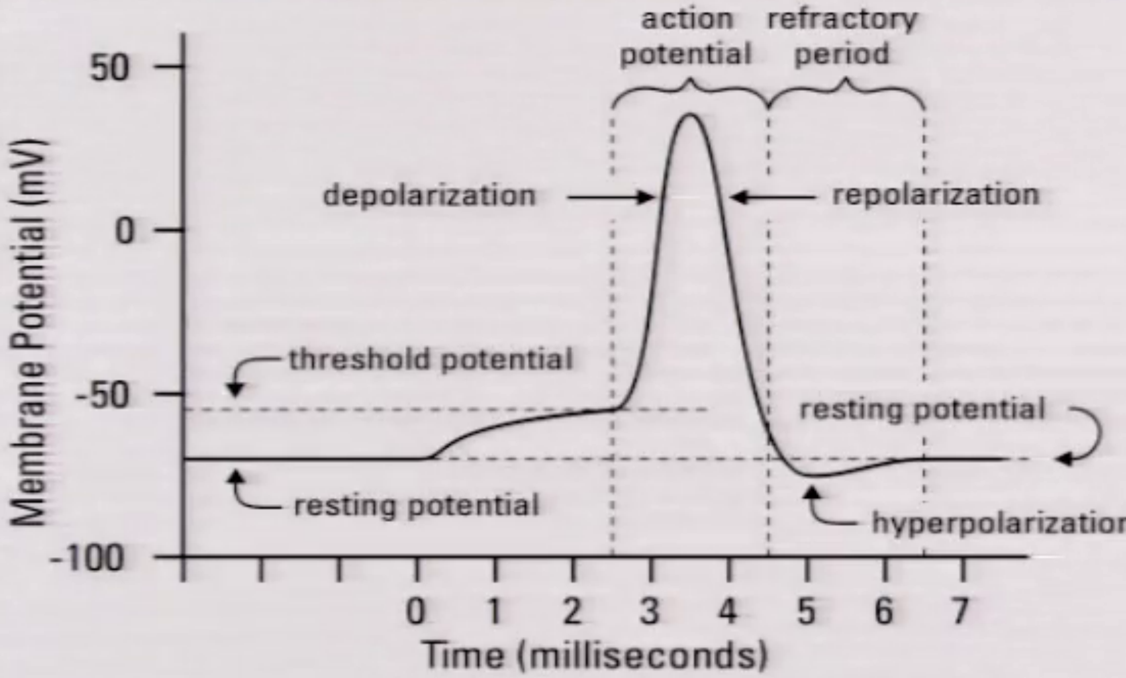
- The problem
- Details of the model
- Results and Discussion
- Conclusions

Objective

- The main idea of this work is to investigate the clustering and malleability of a large scale neuronal network of networks;
- As an example, we use as **external connections** (network to network) - the cat adjacency matrix and **internal nodes** (inside a network) - connection in a small world scheme (Newmann Watts);
- We search for synchronization of bursts and spikes, desynchronization and collective behaviors of different regions in the parameter space.

The model

The Hodgkin-Huxley model is a conductance-based set of differential equations, that reproduce the action potential in nerve cells on the giant squid.



Action Potential in a Neuron

$$I = C_m \frac{\mathrm{d}V_m}{\mathrm{d}t} + \bar{g}_K n^4 (V_m - V_K) + \bar{g}_{Na} m^3 h (V_m - V_{Na}) + \bar{g}_l (V_m - V_l)$$

$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

Here we have used a modified model called Huber-Braun equations, that is used for thermally sensitive neurons.

The model and the complete system

$$C_M \frac{dV}{dt} = -I_{Na} - I_K - I_{sd} - I_{sa} - I_L + I_{ext}$$

$$\begin{aligned} I_{Na} &= \rho \bar{g}_{Na} a_{Na} (V - E_{Na}), \\ I_K &= \rho \bar{g}_K a_K (V - E_K), \\ I_{sd} &= \rho \bar{g}_{sd} a_{sd} (V - E_{sd}), \\ I_{sa} &= \rho \bar{g}_{sa} a_{sa} (V - E_{sa}), \\ I_L &= \rho \bar{g}_L (V - E_L), \end{aligned}$$

$$\begin{aligned} \frac{da_{Na}}{dt} &= \frac{\phi}{\tau_{Na}} (a_{Na,\infty} - a_{Na}), \\ \frac{da_K}{dt} &= \frac{\phi}{\tau_K} (a_{K,\infty} - a_K), \\ \frac{da_{sd}}{dt} &= \frac{\phi}{\tau_{sd}} (a_{sd,\infty} - a_{sd}), \\ \frac{da_{sa}}{dt} &= \frac{\phi}{\tau_{sa}} (-\eta I_{isd} - \gamma a_{sa}), \end{aligned}$$

$$\rho = \rho_0^{\frac{(T - T_0)}{\tau_0}}, \qquad \phi = \phi_0^{\frac{(T - T_0)}{\tau_0}}.$$

$$\begin{aligned} a_{Na,\infty} &= \frac{1}{1 + \exp[-s_{Na}(V_i - V_{0Na})]}, \\ a_{K,\infty} &= \frac{1}{1 + \exp[-s_K(V_i - V_{0K})]}, \\ a_{sd,\infty} &= \frac{1}{1 + \exp[-s_{sd}(V_i - V_{0sd})]}, \end{aligned}$$

$$C_M \frac{dV_i^{(j)}}{dt} = -I_{i,Na}^{(j)} - I_{i,K}^{(j)} - I_{i,sd}^{(j)} - I_{i,sa}^{(j)} - I_{i,L}^{(j)} + I_{i,ext}^{(j)},$$

$$I_{i,IN}^{(j)} = g_{IN} \sum_{k=1}^{N} A_{ik}^{(j)} r_k^{(j)}(t)(V_{syn} - V_k^{(j)}),$$

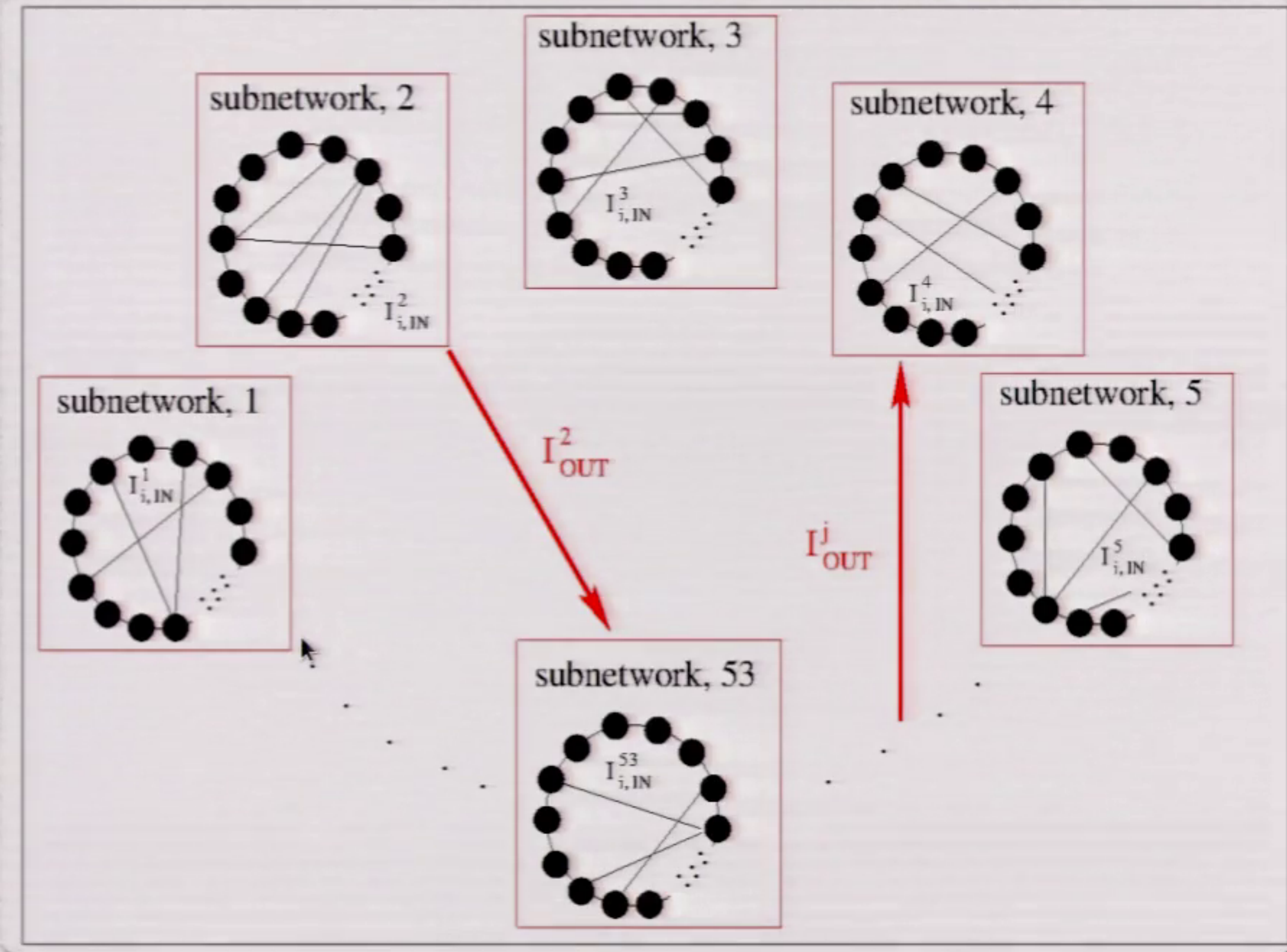
$$I_{i,ext}^{(j)} = I_{i,IN}^{(j)} + I_{OUT}^{(j)}.$$

$$I_{OUT}^{(j)} = g_{OUT} \sum_{\ell=1}^{S} A_{j,\ell}^{(j)} M(j),$$

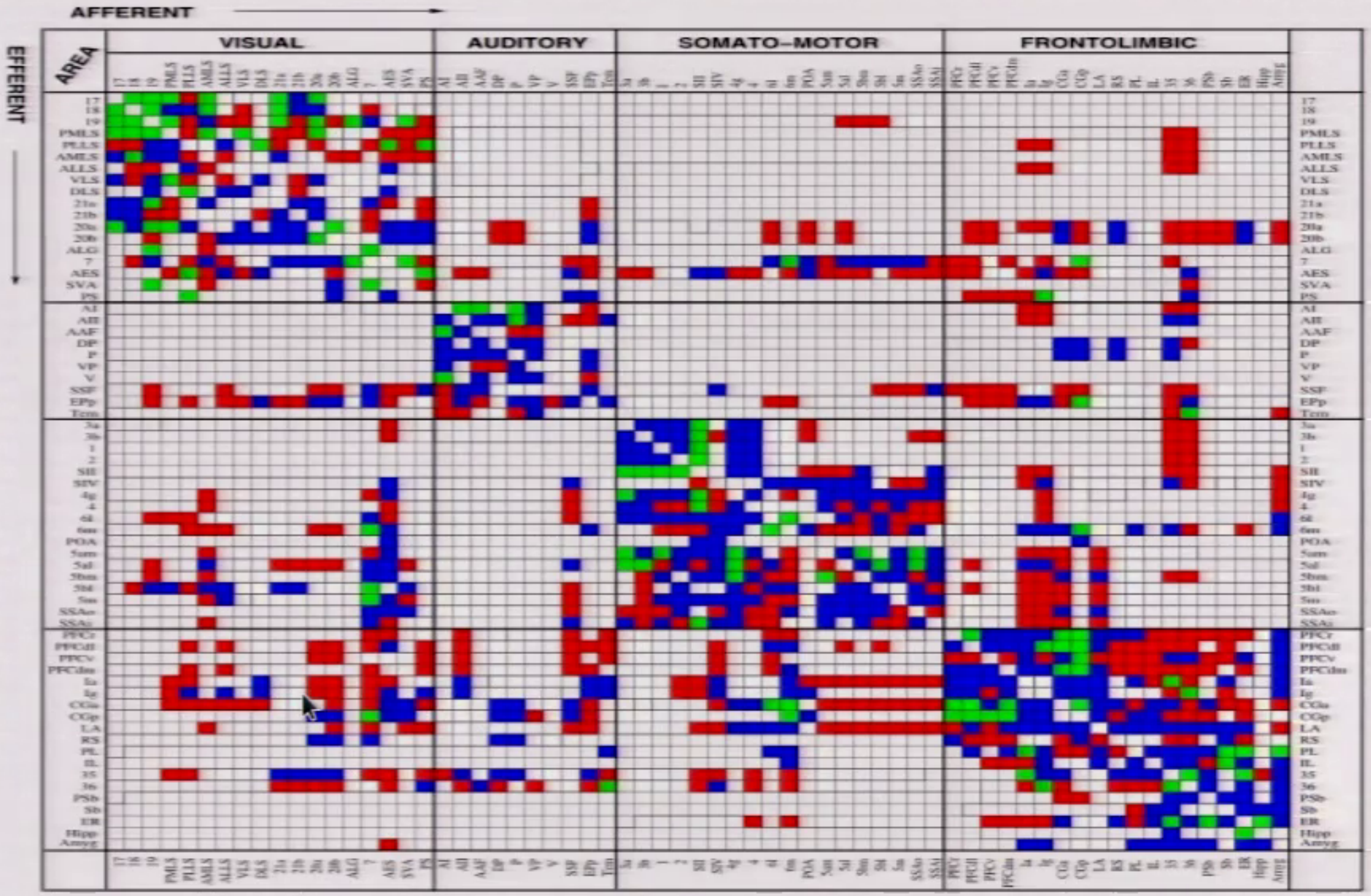
$$\frac{dr_k^{(j)}}{dt} = \left(\frac{1}{\tau_r} - \frac{1}{\tau_d}\right) \frac{1 - r_k^{(j)}}{1 + \exp(-V_k^{(j)} + V_0)} - \frac{r_k^{(j)}}{\tau_d},$$

$$M(j) = \frac{1}{N} \sum_{i=1}^{N} V_i^{(j)}$$

The entire system comprehends
53x256 = 13568 HB systems!!!

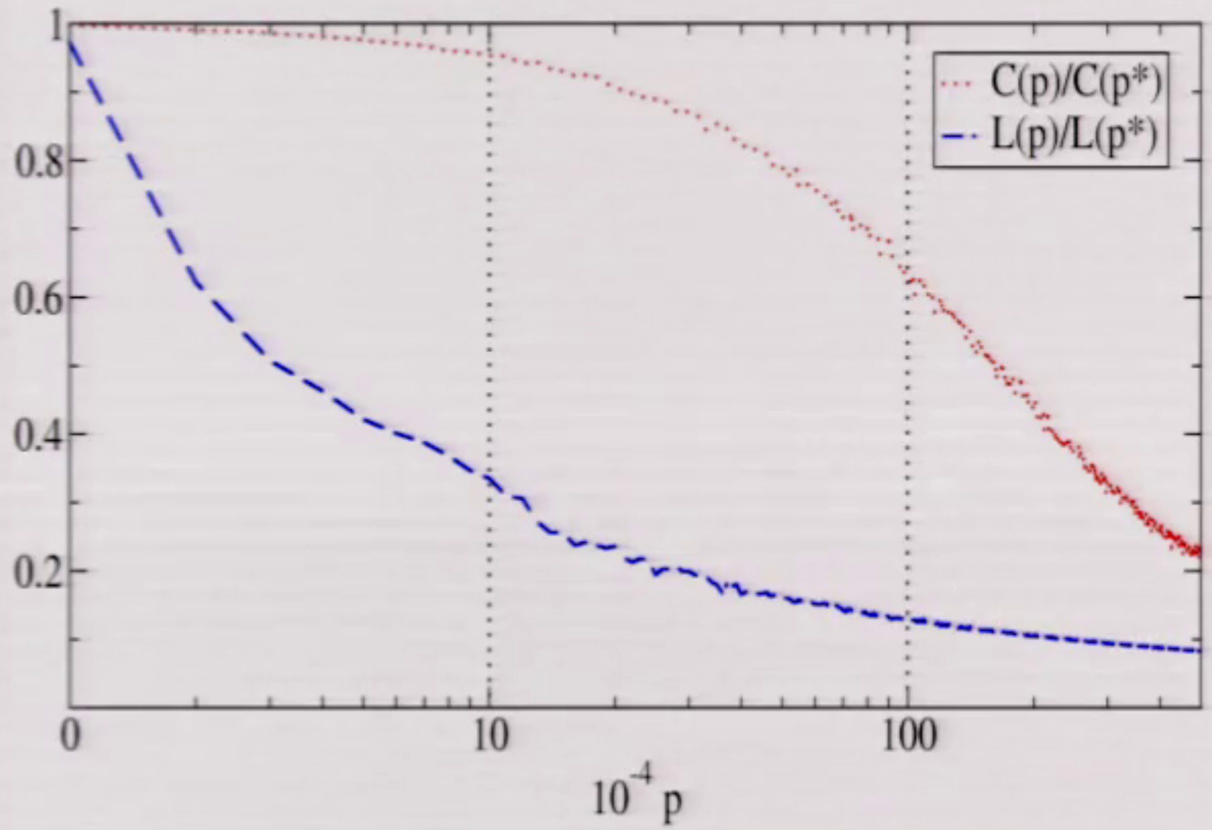


subnetwork, 3
$I^3_{i,IN}$
subnetwork, 2
$I^2_{i,IN}$
subnetwork, 4
$I^4_{i,IN}$
subnetwork, 1
$I^1_{i,IN}$
I^2_{OUT}
I^j_{OUT}
subnetwork, 5
$I^5_{i,IN}$
subnetwork, 53
$I^{53}_{i,IN}$

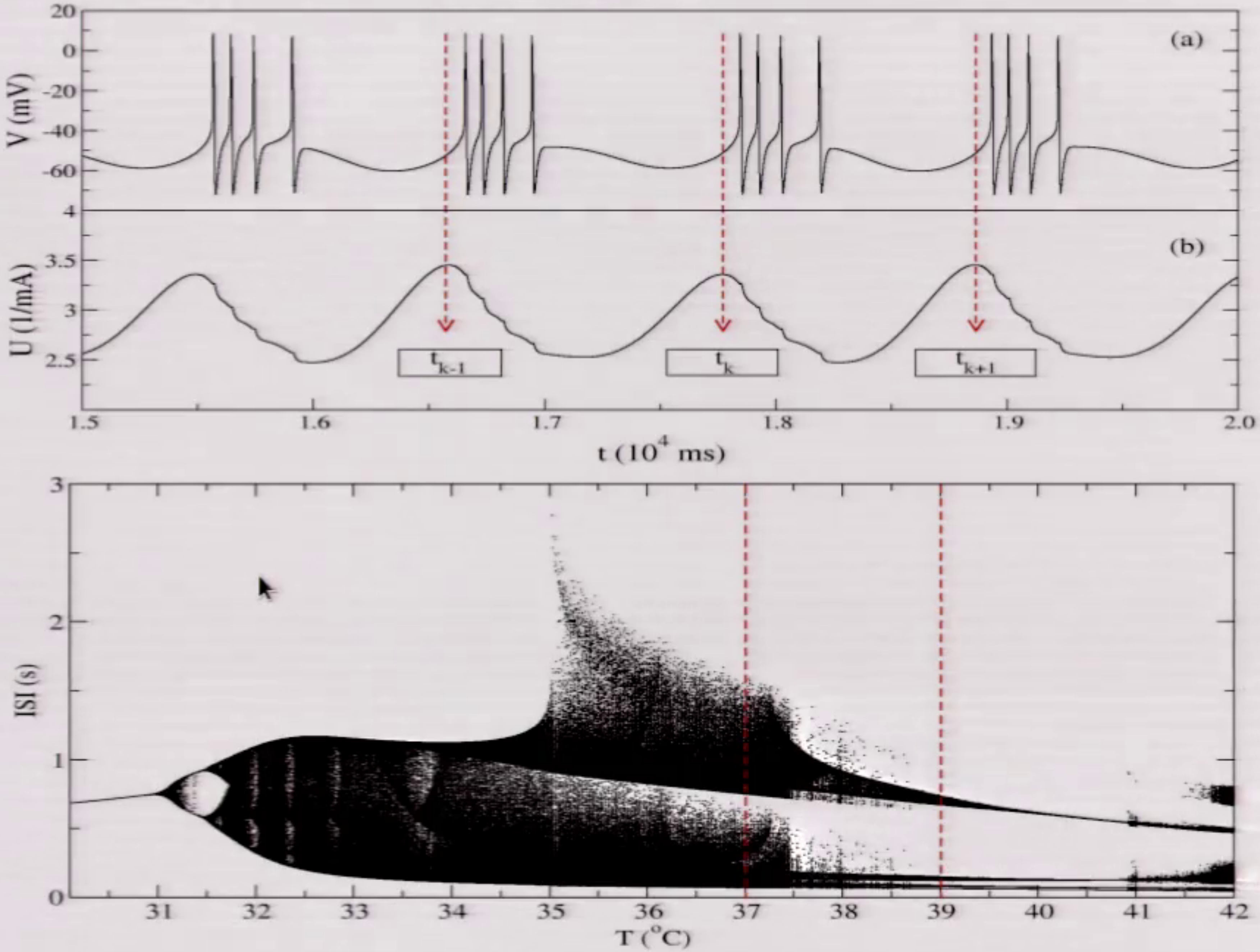


E. L. Lameu, C. A. S. Batista, A. M. Batista, K. Iarosz, R. L. Viana, S. R. Lopes, and J. Kurths, Suppression of bursting synchronization in clustered scale-free (``rich-club'') neuronal networks, Chaos, Vol. 22 (2012) Article 043149

Each subnetwork has 256 nodes, which are evolved with Huber-Braun set of equations, coupled with different Small-World (NW) matrices.



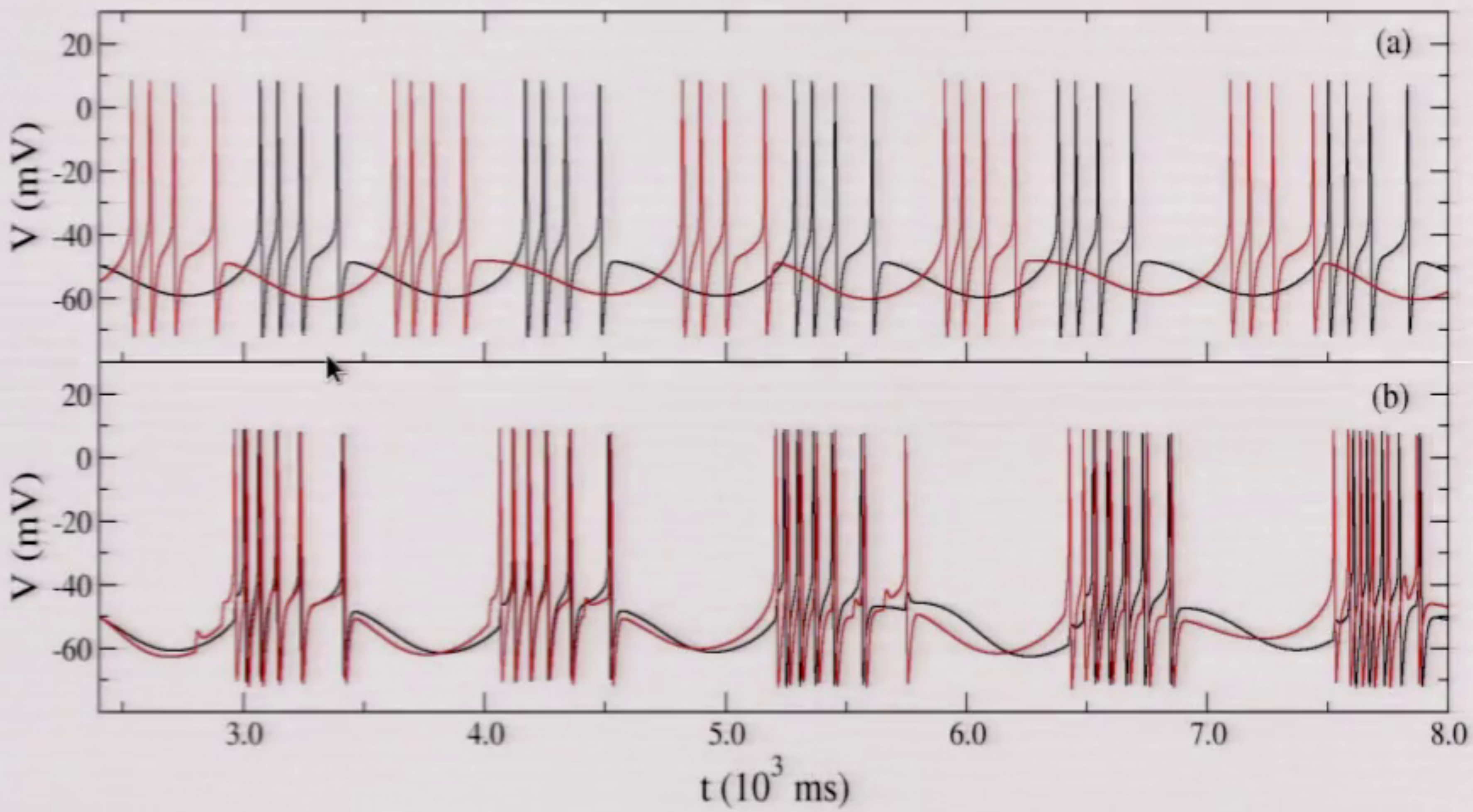
This system exhibits lots of different dynamical behaviors, but here we consider a bursting regime.



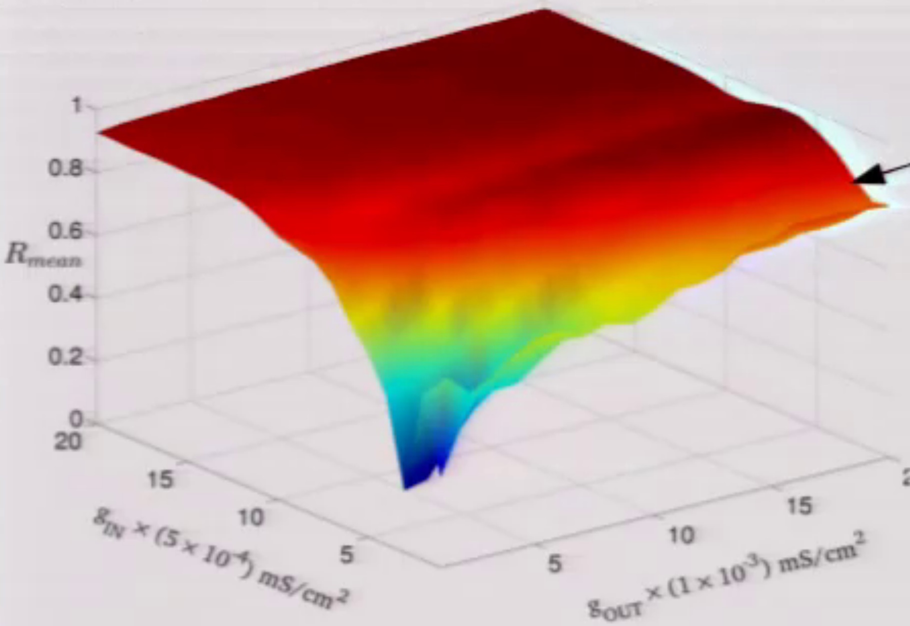
With this phases defined, we are able to use the Kuramoto order parameter, and analyze the burst phase synchronization.

$$\varphi(t) = 2\pi k + 2\pi \frac{t - t_k}{t_{k+1} - t_k}, \qquad (t_k < t < t_{k+1})$$

$$R_j(t) = \left| \frac{1}{N} \sum_{i=1}^{N} e^{i\varphi_{ij}} \right|$$

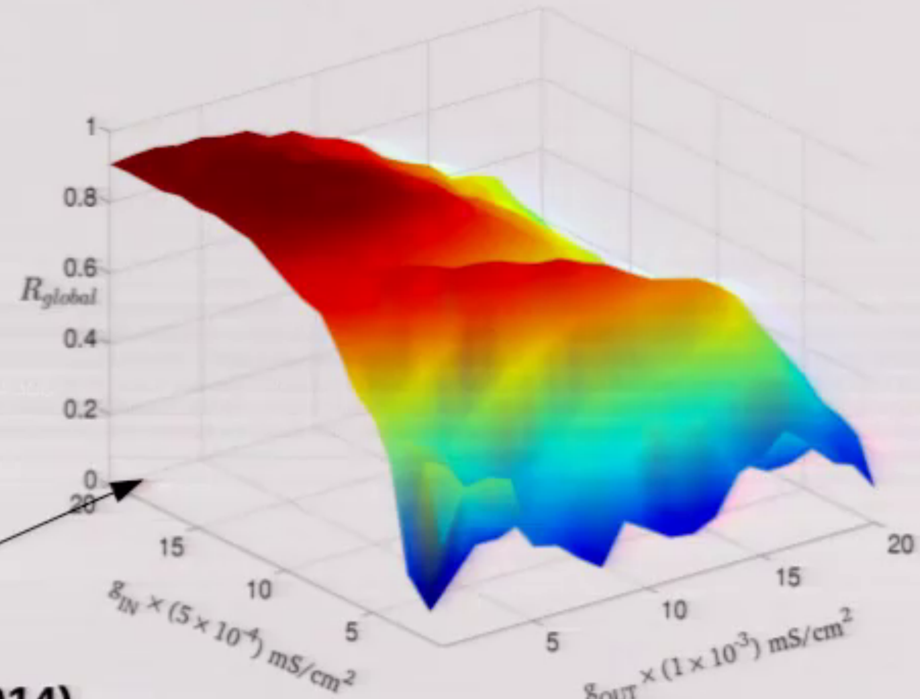


Results and Discussion



The mean order parameter of all sub networks! (phase sync. of sub networks)

The global order parameter of all neurons! (phase sync. all system)



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Synchronization of bursting Hodgkin-Huxley-type neurons in clustered networks
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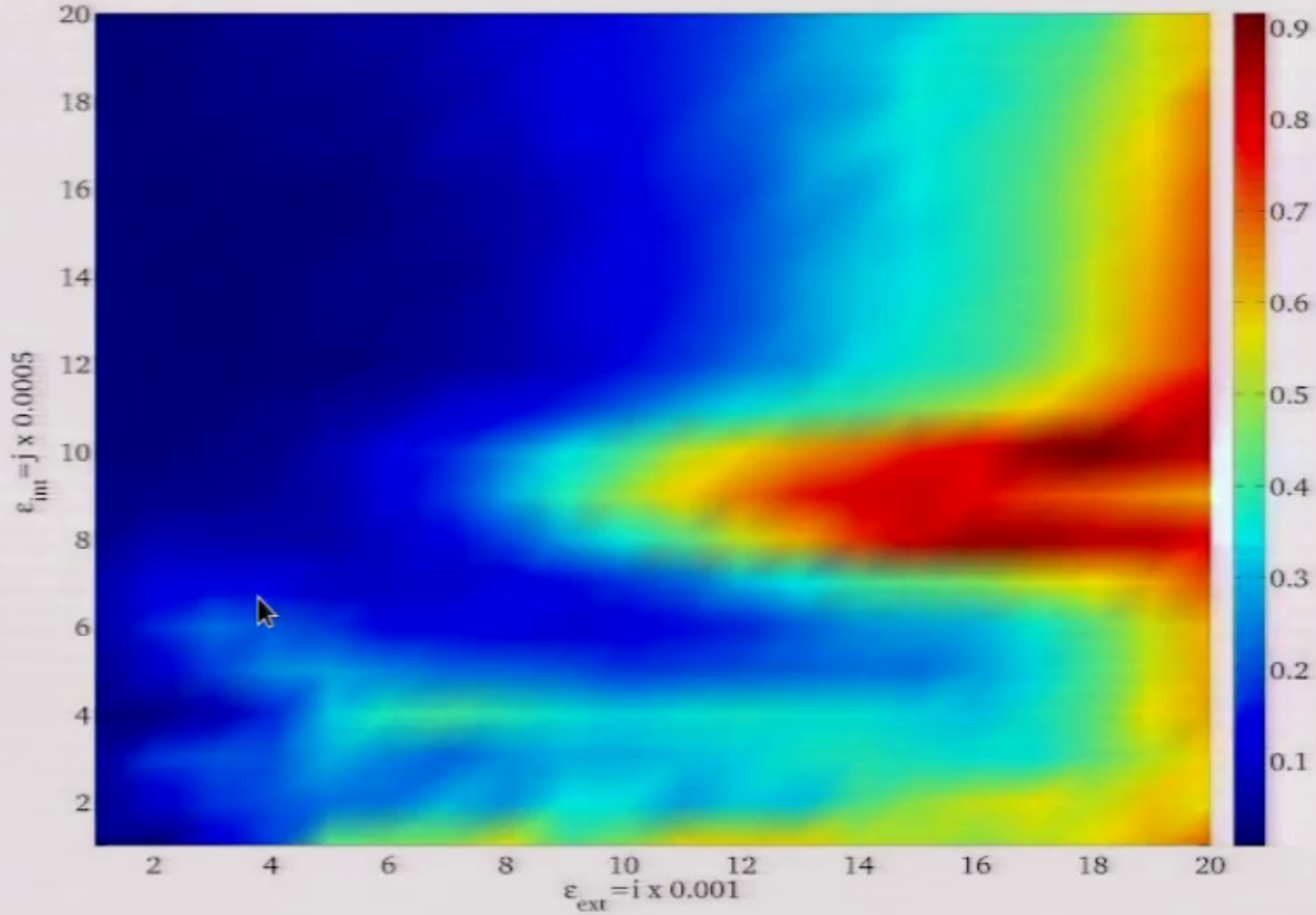
Subtraction from previous images!

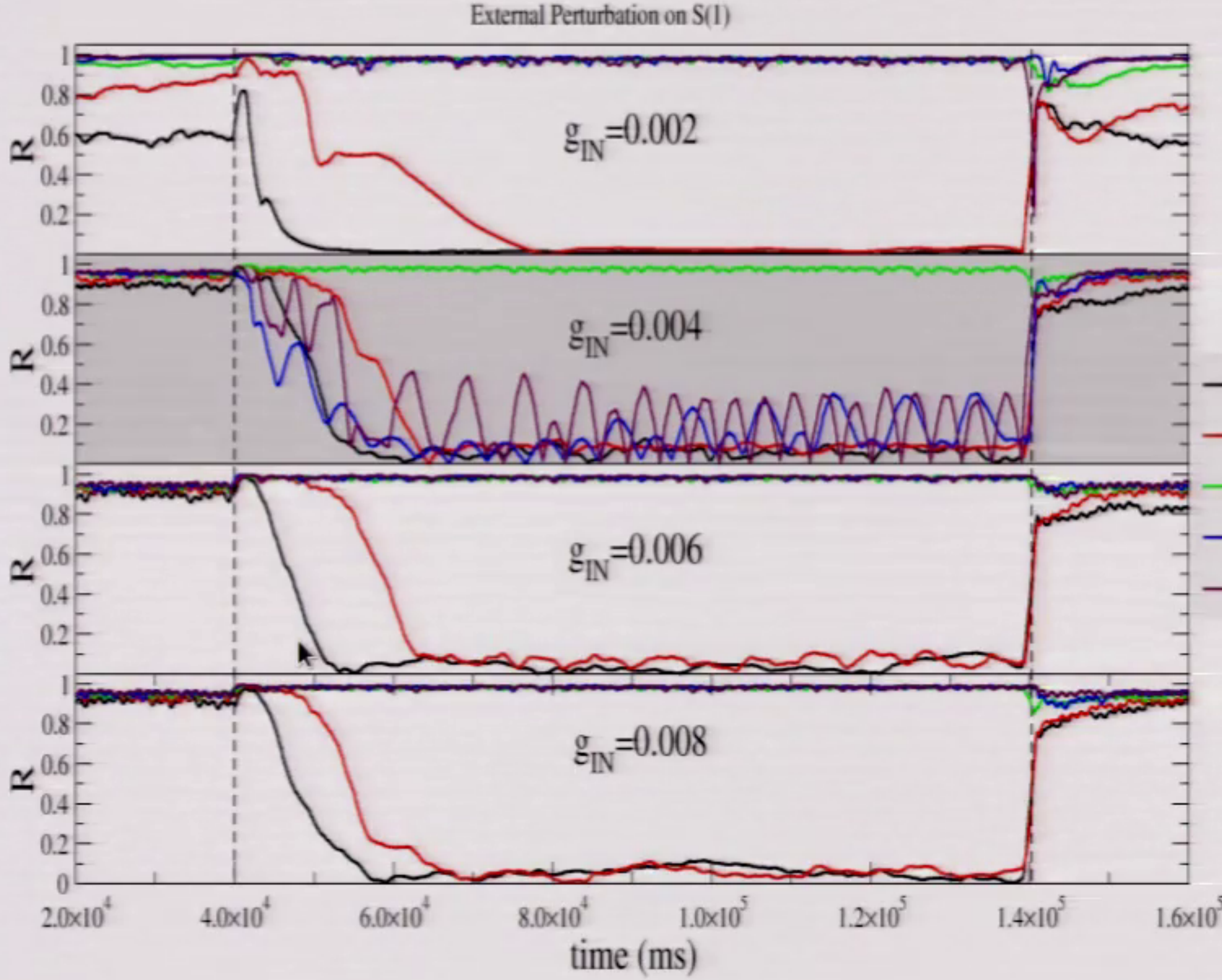


To understand what is happening in the malleable region we consider

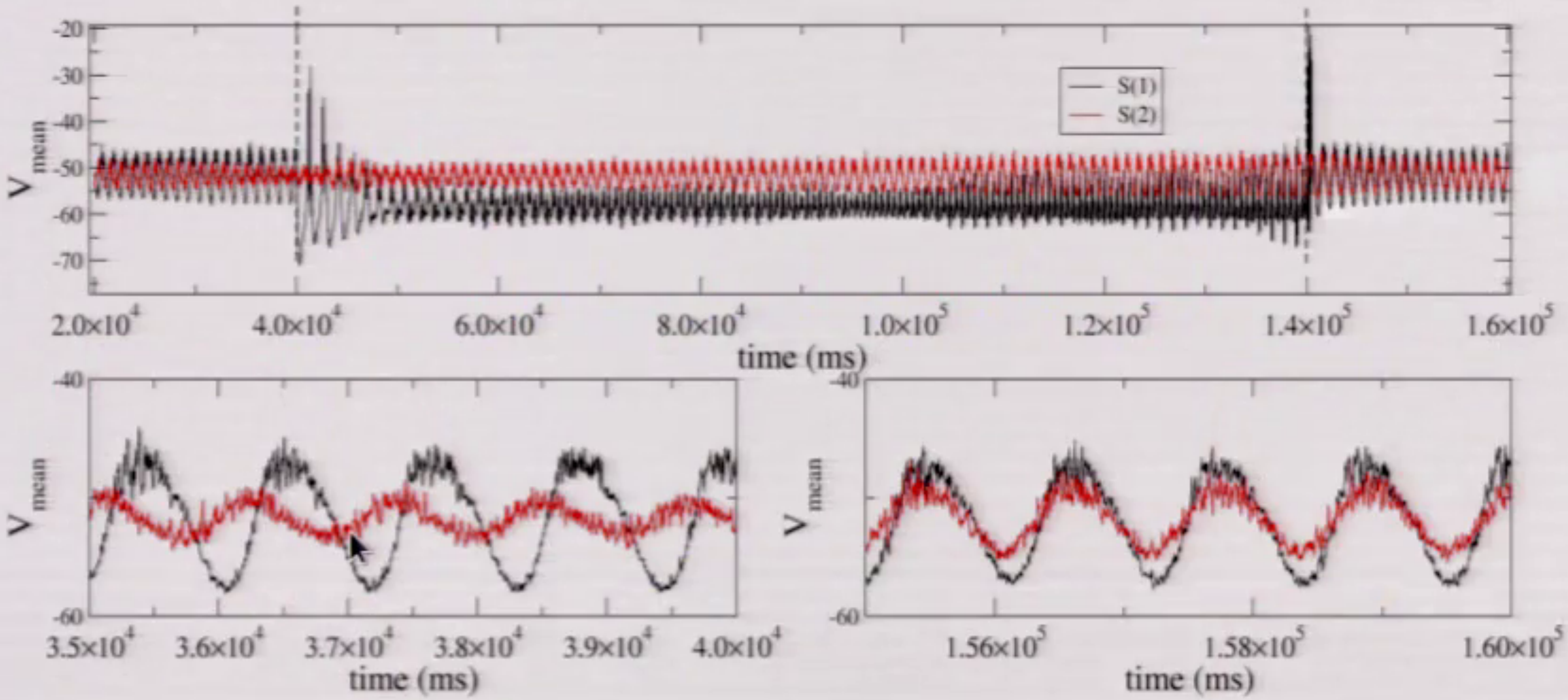
- **Just two sub network coupled;**

- We insert an external feedback (amplitude 1.5% from the original mean signal) in the sub network 1 (to emulate the effect of all other sub networks.

S(1)

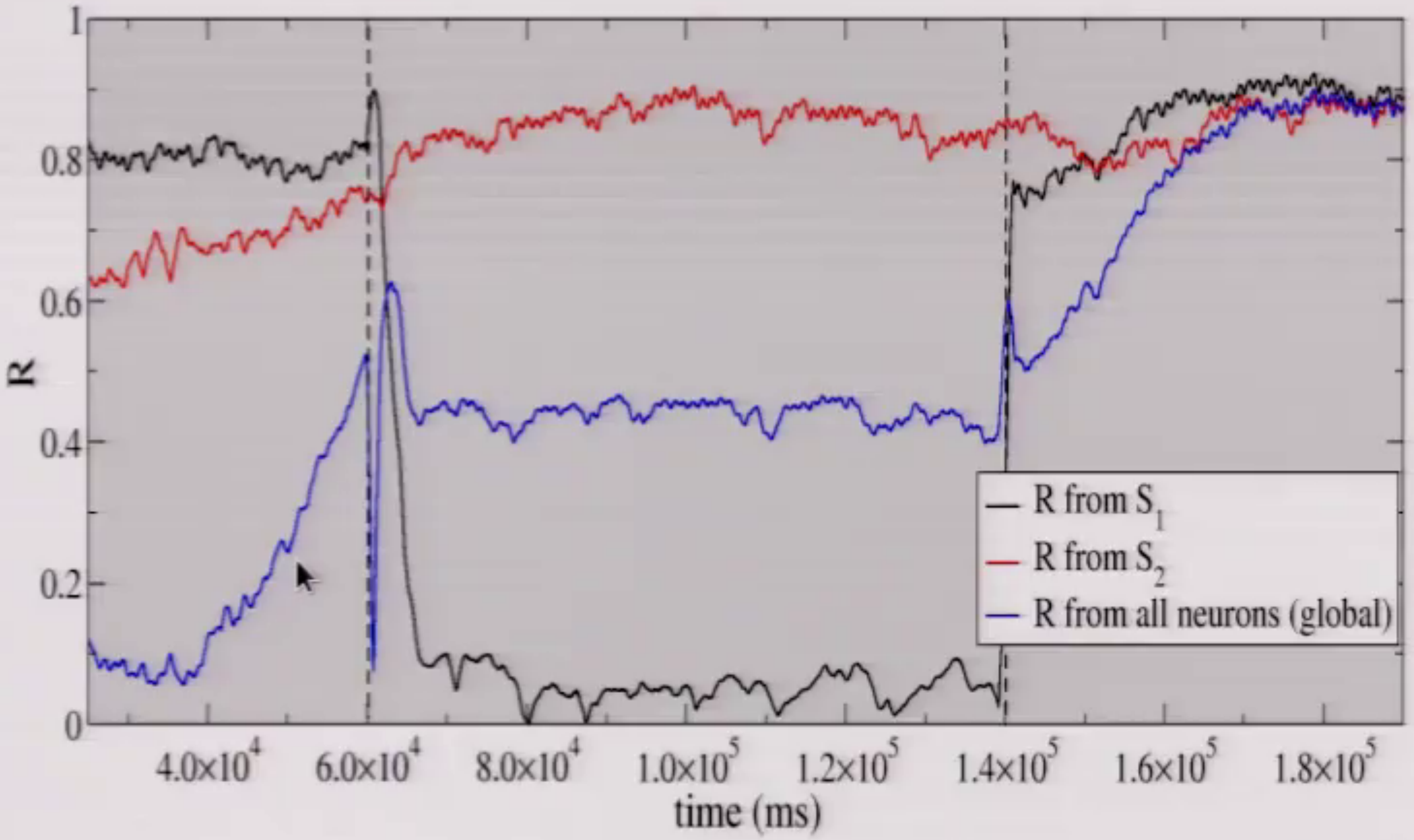


External Perturbation on S(1)
g_{IN}=0.002
g_{IN}=0.004
g_{IN}=0.006
g_{IN}=0.008
R
1
0.8
0.6
0.4
0.2
0
g_{OUT}=0.001
g_{OUT}=0.002
g_{OUT}=0.003
g_{OUT}=0.004
g_{OUT}=0.005
2.0×10^4
4.0×10^4
6.0×10^4
8.0×10^4
1.0×10^5
1.2×10^5
1.4×10^5
1.6×10^5
time (ms)



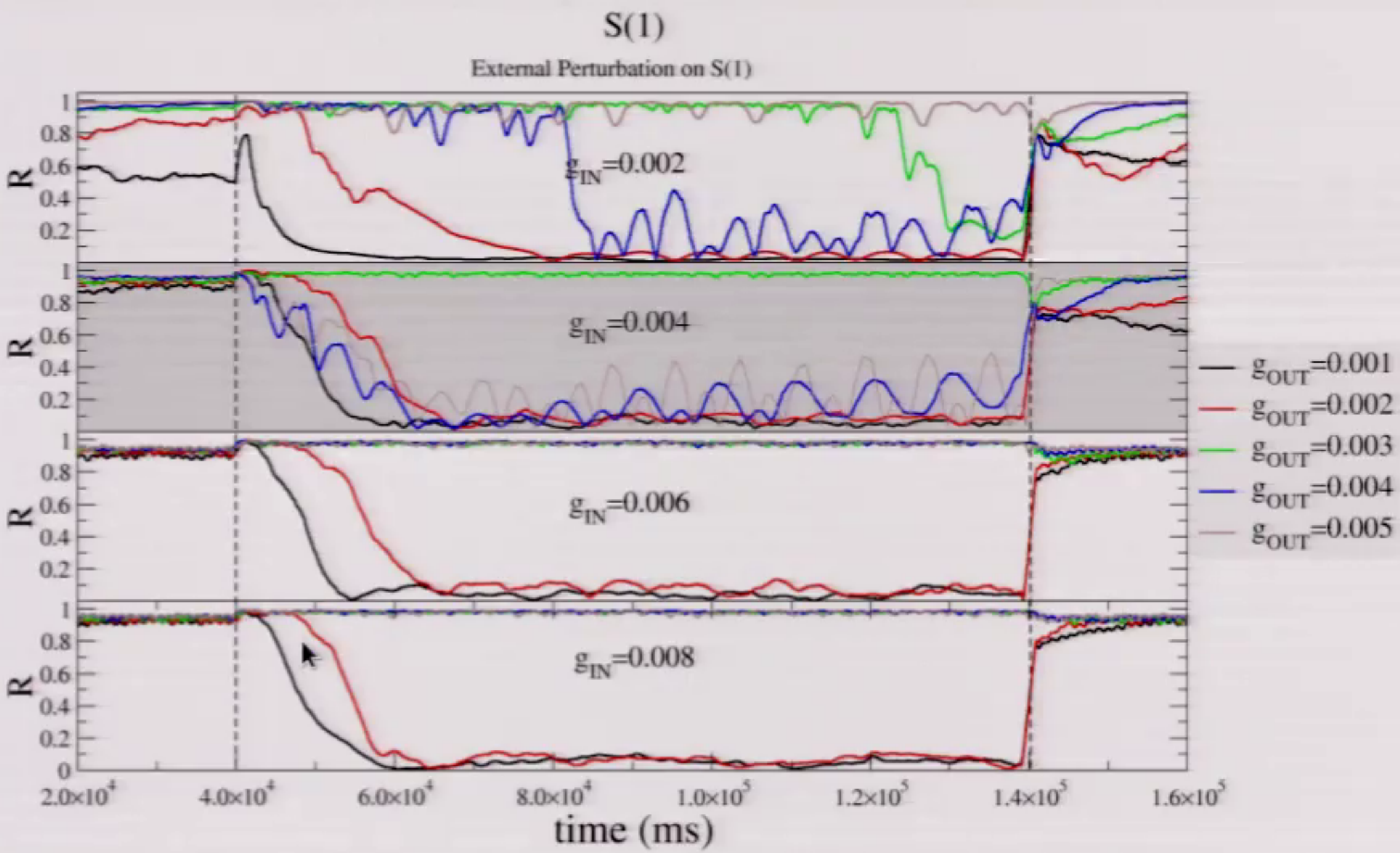
S(1)
S(2)
V_{mean}
time (ms)
2.0×10^4
4.0×10^4
6.0×10^4
8.0×10^4
1.0×10^5
1.2×10^5
1.4×10^5
1.6×10^5
3.5×10^4
3.6×10^4
3.7×10^4
3.8×10^4
3.9×10^4
4.0×10^4
1.56×10^5
1.58×10^5
1.60×10^5

g_out = 0.001
g_in = 0.004



- **Eight sub network coupled;**

- We insert an external feedback (amplitude 1.5% from the original mean signal) in the sub network 1 (to emulate the effect of all other sub networks).


S(1)
External Perturbation on S(1)
R
g_{IN}=0.002
g_{IN}=0.004
g_{IN}=0.006
g_{IN}=0.008
g_{OUT}=0.001
g_{OUT}=0.002
g_{OUT}=0.003
g_{OUT}=0.004
g_{OUT}=0.005
0
0.2
0.4
0.6
0.8
1
2.0×10^4
4.0×10^4
6.0×10^4
8.0×10^4
1.0×10^5
1.2×10^5
1.4×10^5
1.6×10^5
time (ms)

Conclusion

- Network of network is an open field of research.
- Many interesting features of those systems are not yet well understood.
- In particular, conditions of synchronization, or in general, conditions of how a subnetwork reacts to a coupling are not well known.
- Here we show examples of how internal and external couplings play an important role in the “malleability” of a particular coupling.
- We have shown that some particular values of internal and external couplings can be used to optimize the effects of the couplings.