Statistics of Basketball Scoring and Lead Changes

in collaboration with Alan Gabel, Marina Kogan, Aaron Clauset

2015 SIAM Conference on Applications of Dynamical Systems

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dunk courtesy of Dr. J

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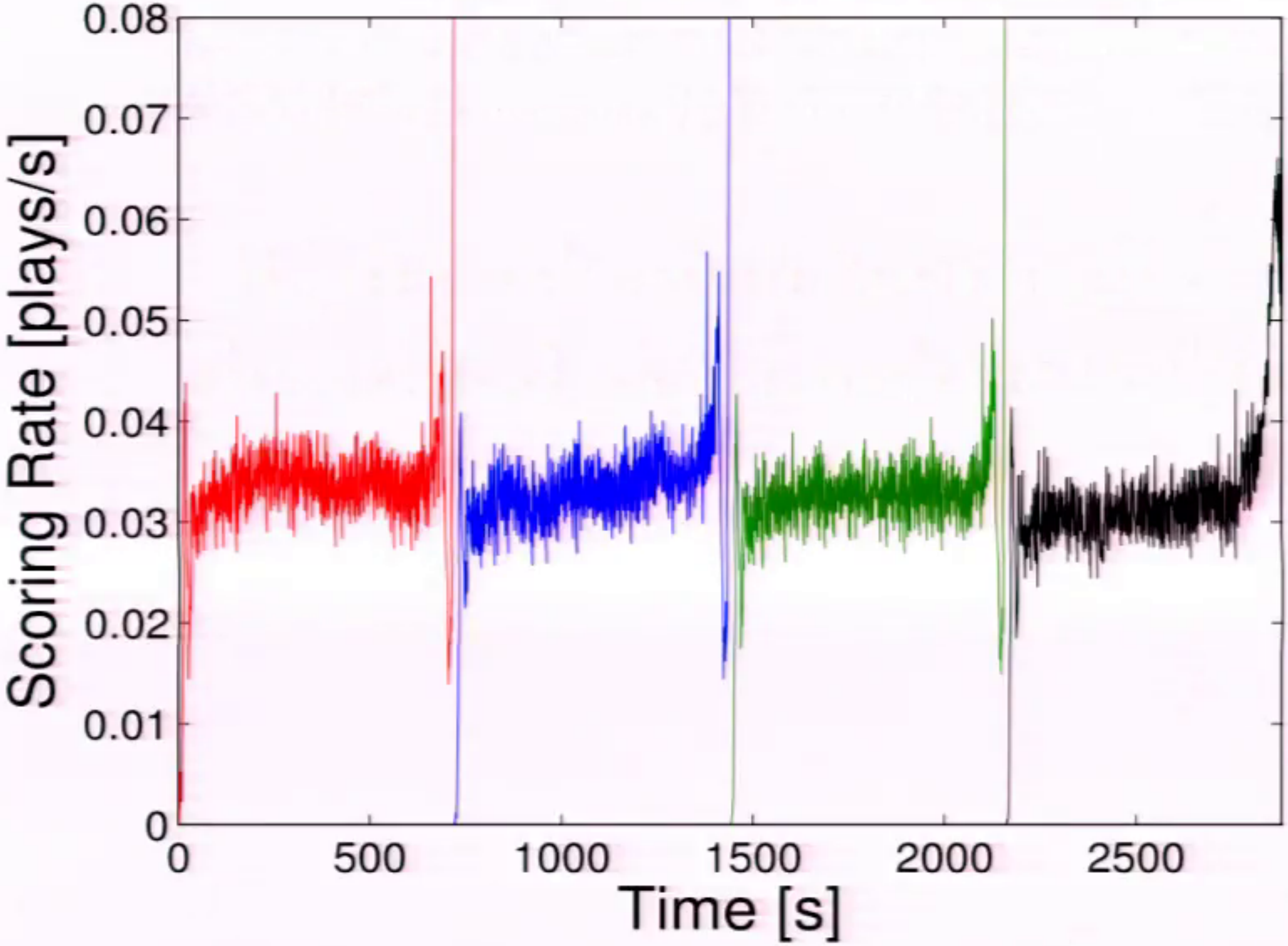
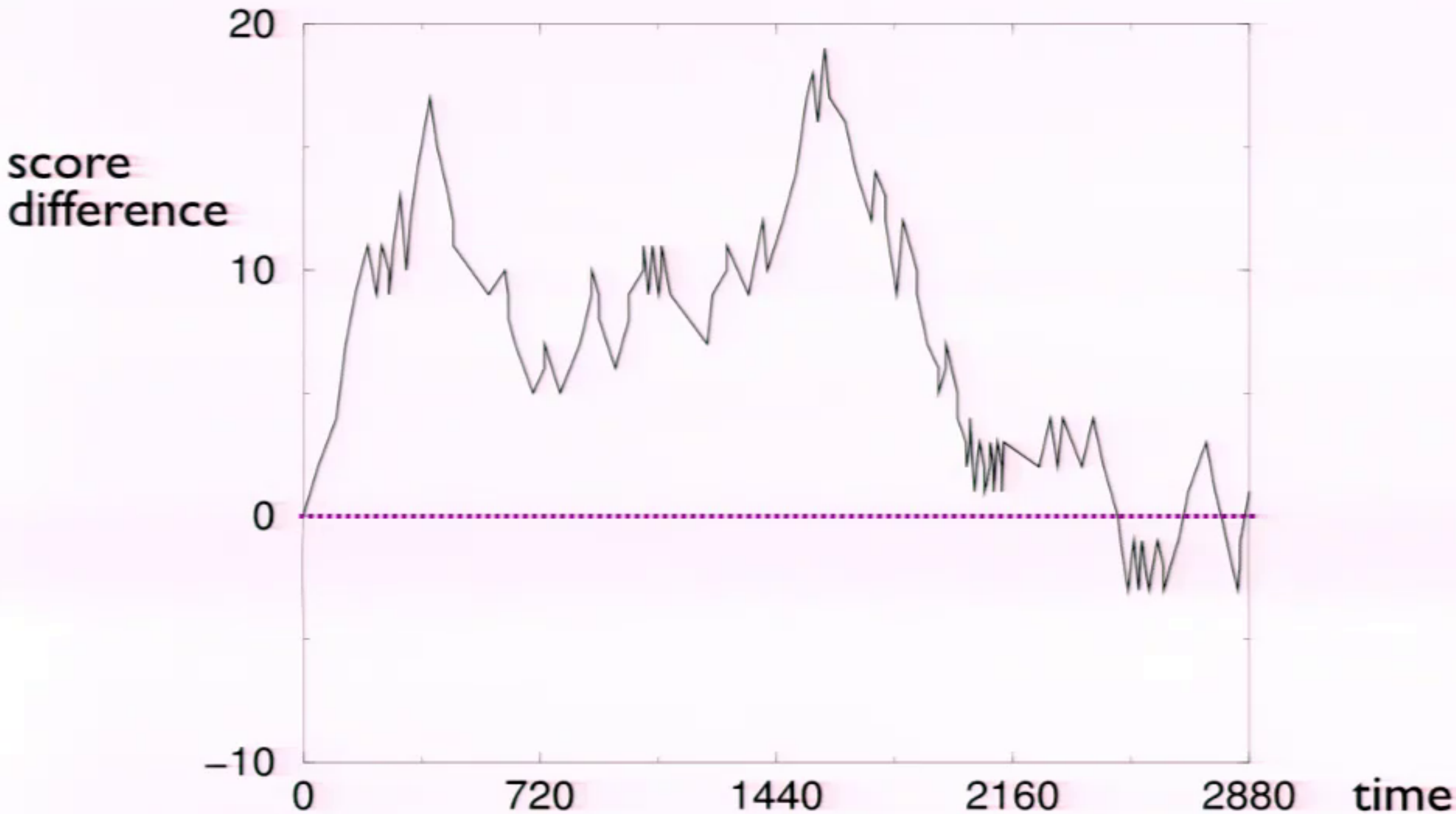
Can basketball scoring be described as a random walk?

(Almost) Constant Scoring Rate

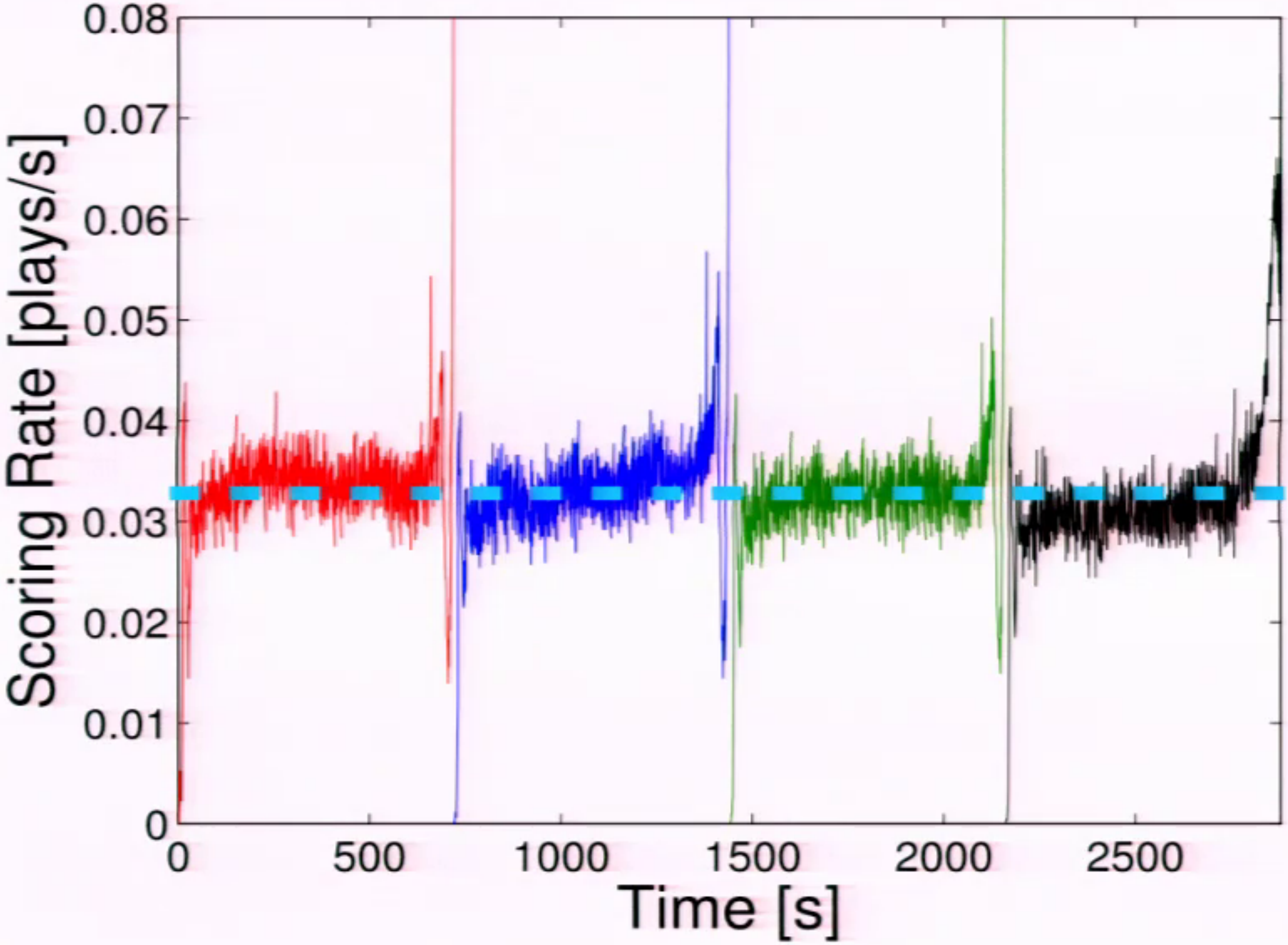


Score Evolution in a Typical NBA Basketball Game

Chicago-Denver 11/26/2010

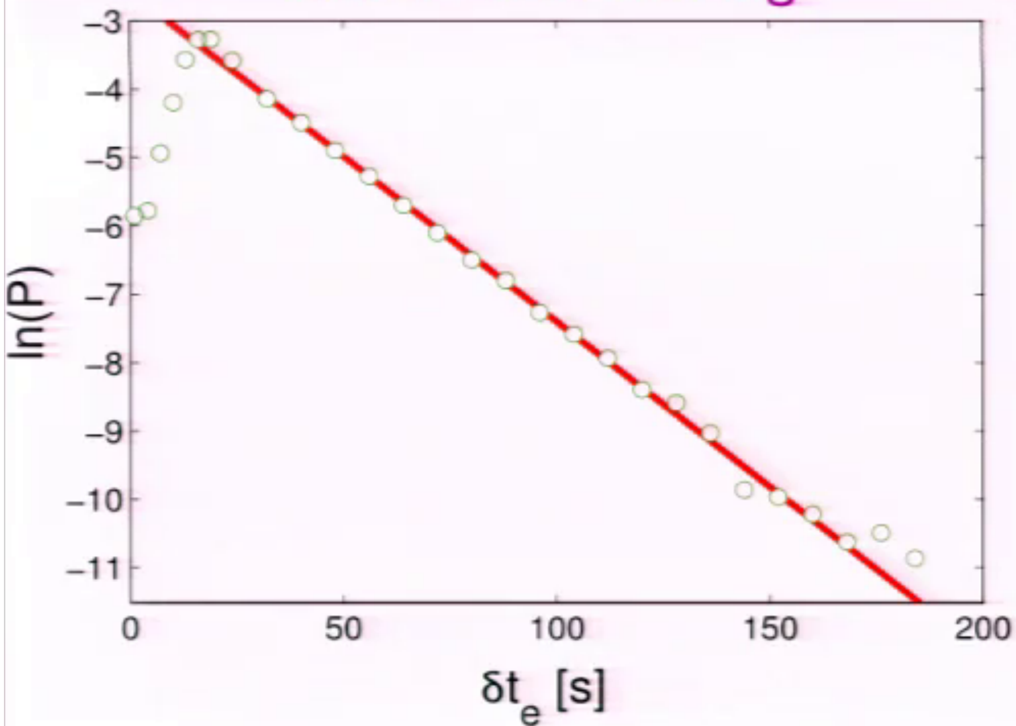


(Almost) Constant Scoring Rate



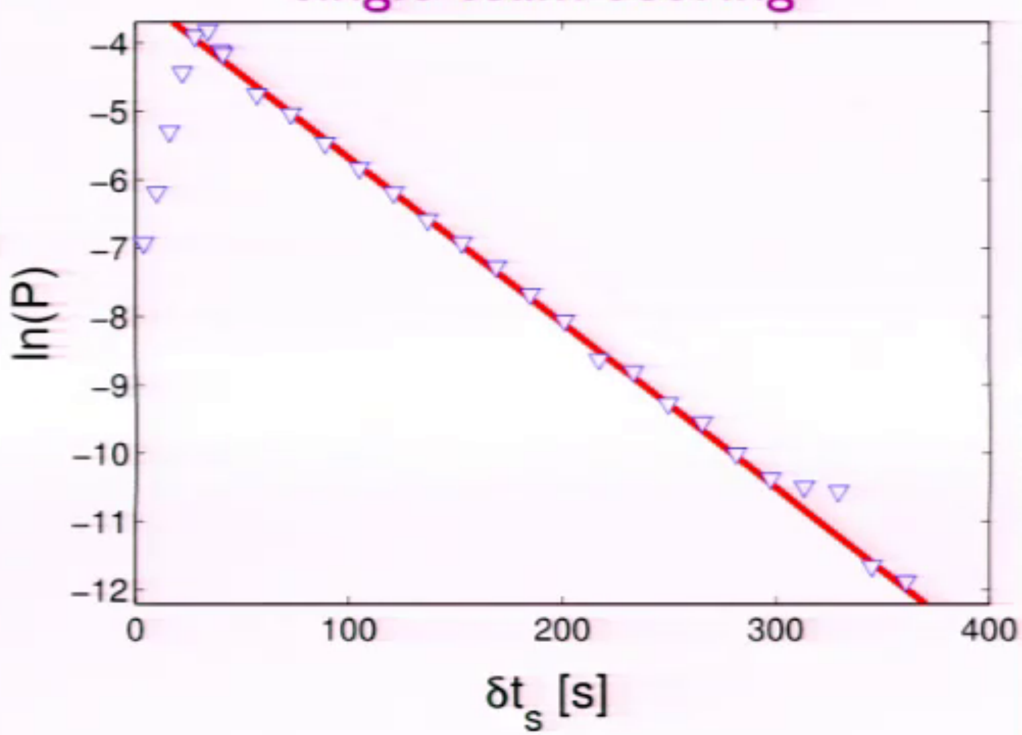
Random Time Intervals Between Scores

either team scoring



$$P(\delta t) \sim e^{-\lambda \delta t}$$

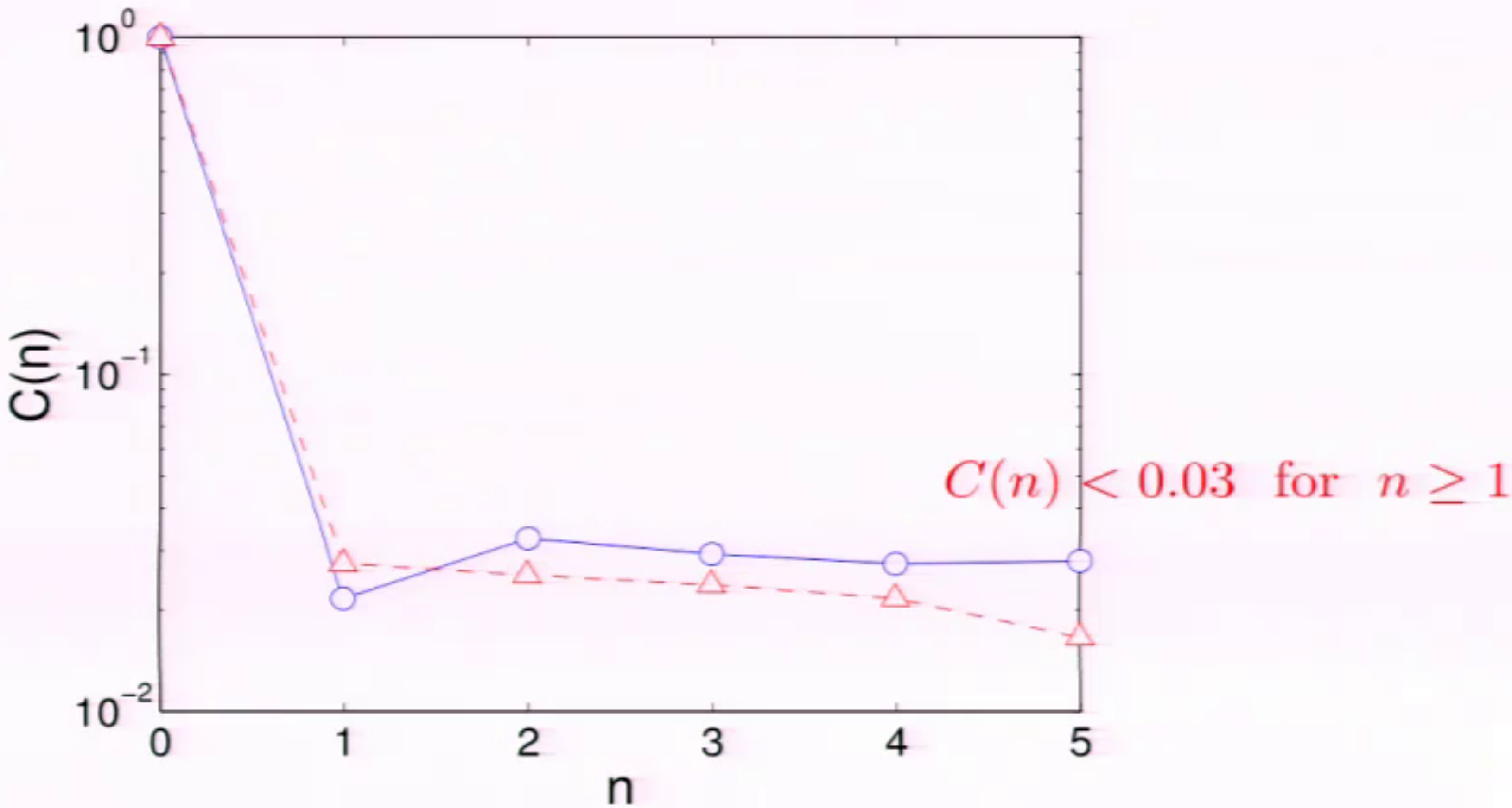
single team scoring



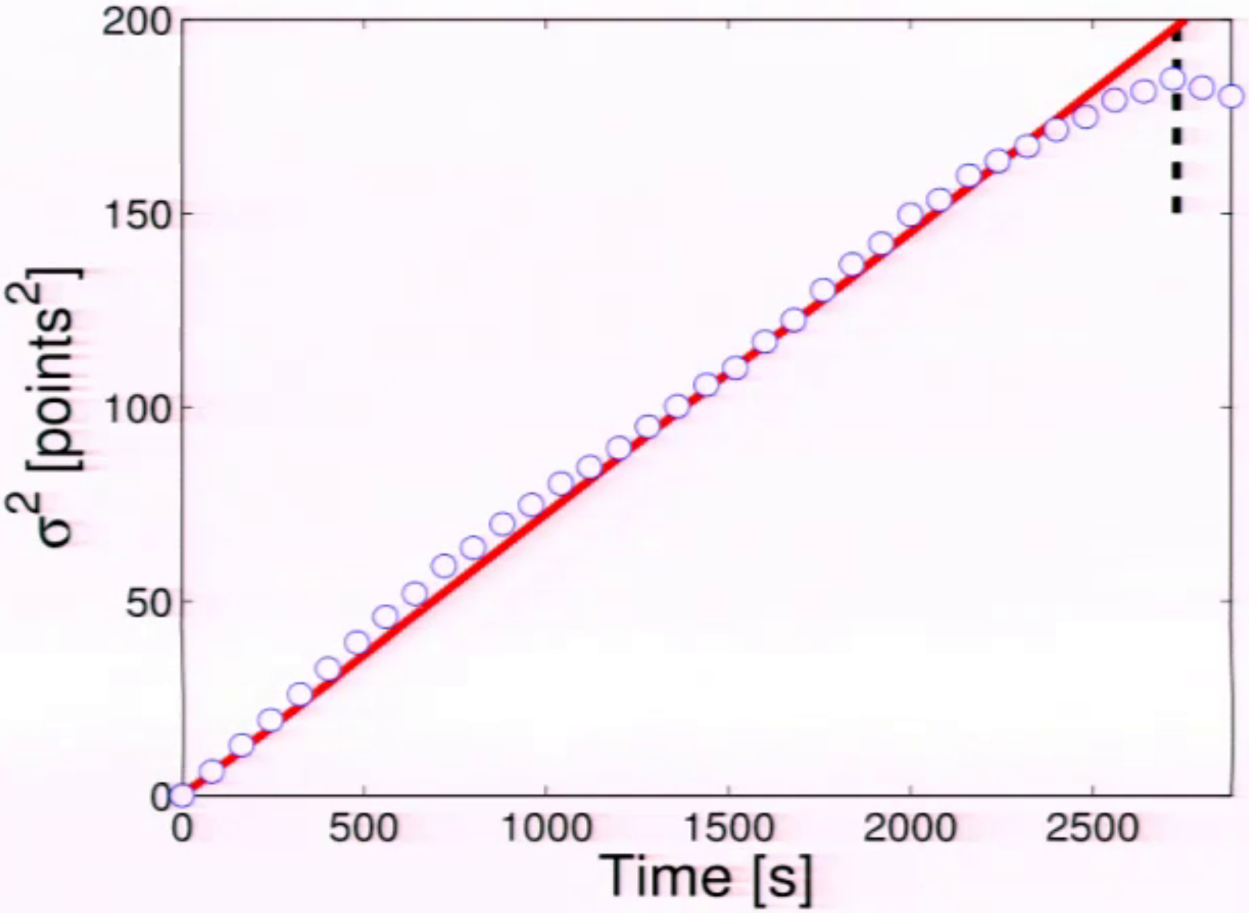
No "Hot Hand" Gilovich, Vallone & Tversky (1985)

Correlation Between Successive Scores

$$C(n) \equiv \frac{\sum_k (t_k - \langle t \rangle)(t_{k+n} - \langle t \rangle)}{\sum_k (t_k - \langle t \rangle)^2}$$



Linear Growth in the Score Difference Variance



Statistics of Lead Changes

- How many lead changes occur?
- How long does one team lead?
- When does the last lead change occur?
- When is a lead "safe"?

Number of Lead Changes

evenly matched teams: $m \sim \sqrt{N}$

$$\int^{N} dN' \, P(0, N') \sim \sqrt{N}$$

evenly matched teams;
antipersistence p: $m \sim \sqrt{N(1-p)/p}$

Antipersistence: After team A scores

team A scores next with probability p=0.35
team B scores next with probability 1-p=0.65

Distribution of Number of Lead Changes

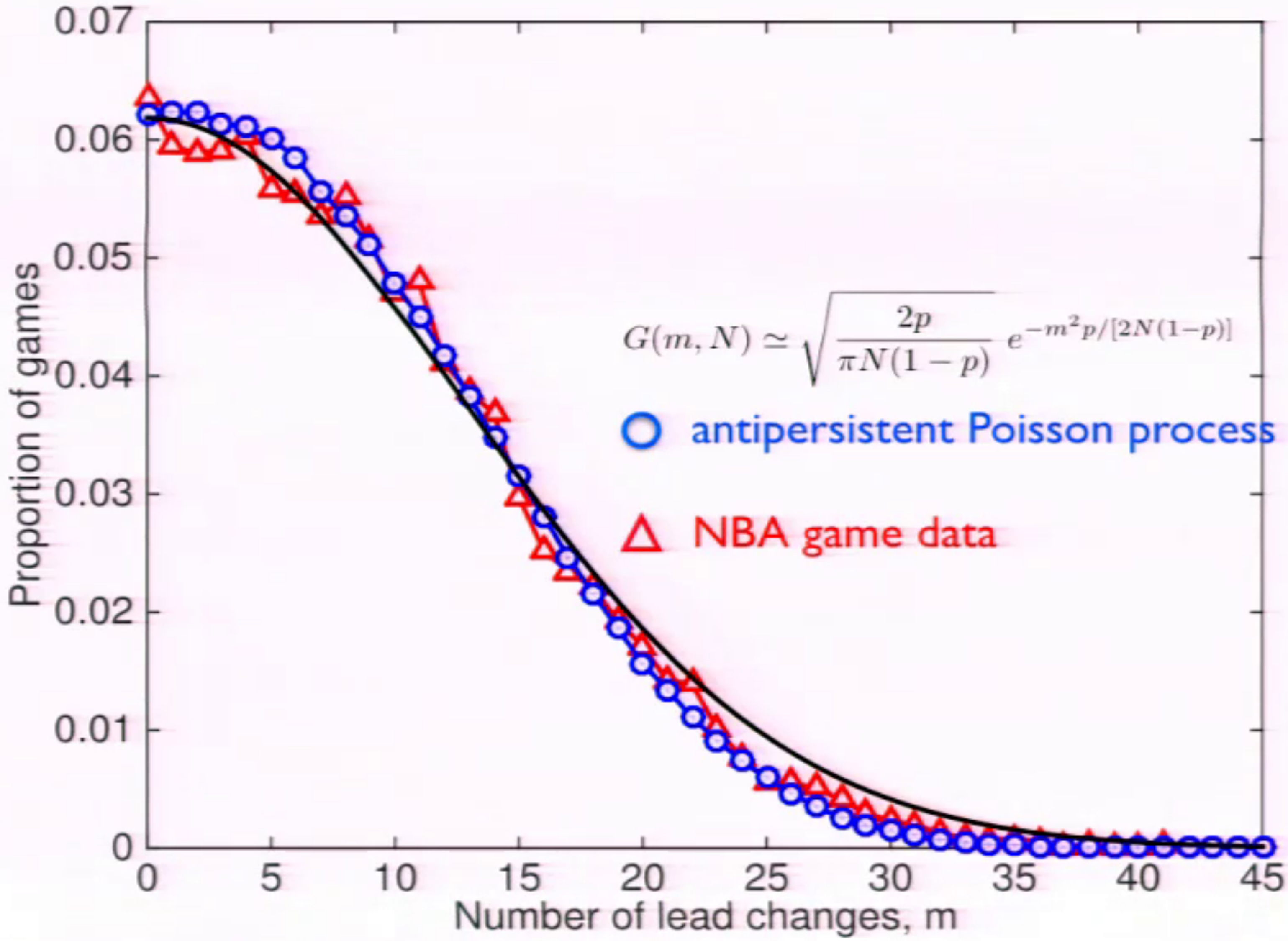
no antipersistence:

$$G(m,N) \simeq \sqrt{\frac{2}{\pi N}}\, e^{-m^2/2N}$$

with antipersistence:

$$G(m,N) \simeq \sqrt{\frac{2p}{\pi N(1-p)}}\, e^{-m^2 p/[2N(1-p)]}$$

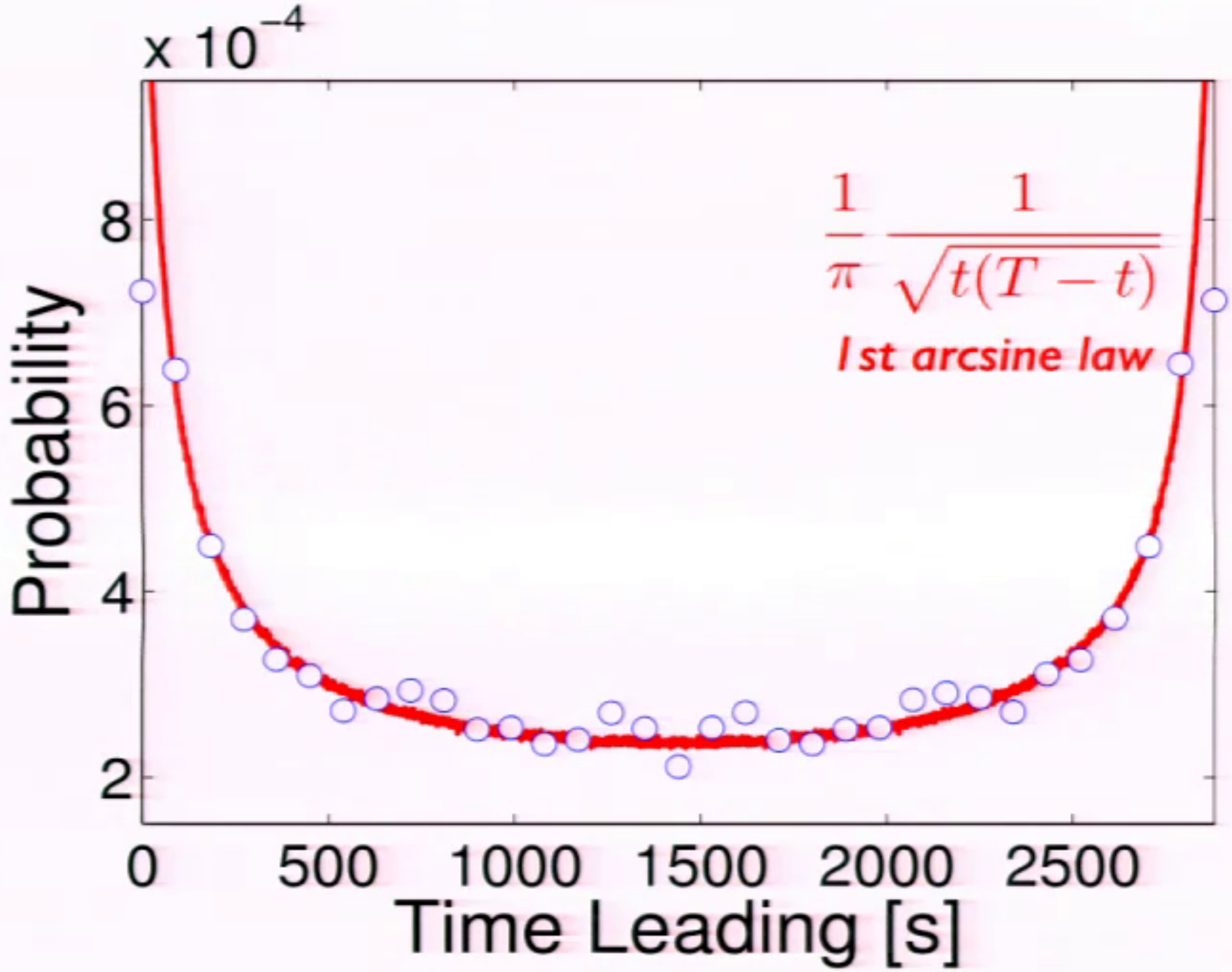
Distribution of Number of Lead Changes



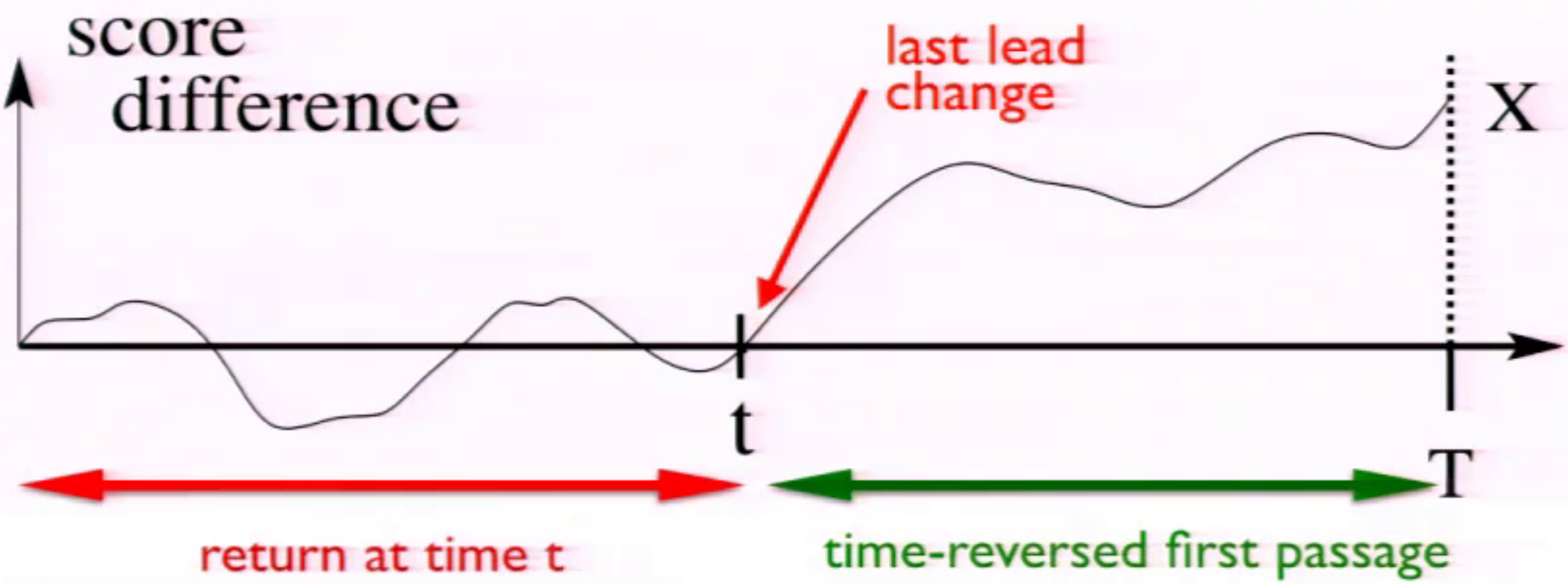
How Long Does One Team Lead?

Chicago-Denver 11/26/2010

How Long Does One Team Lead?



When Does the Last Lead Change Occur?



$$\frac{1}{\sqrt{4\pi D t}}$$

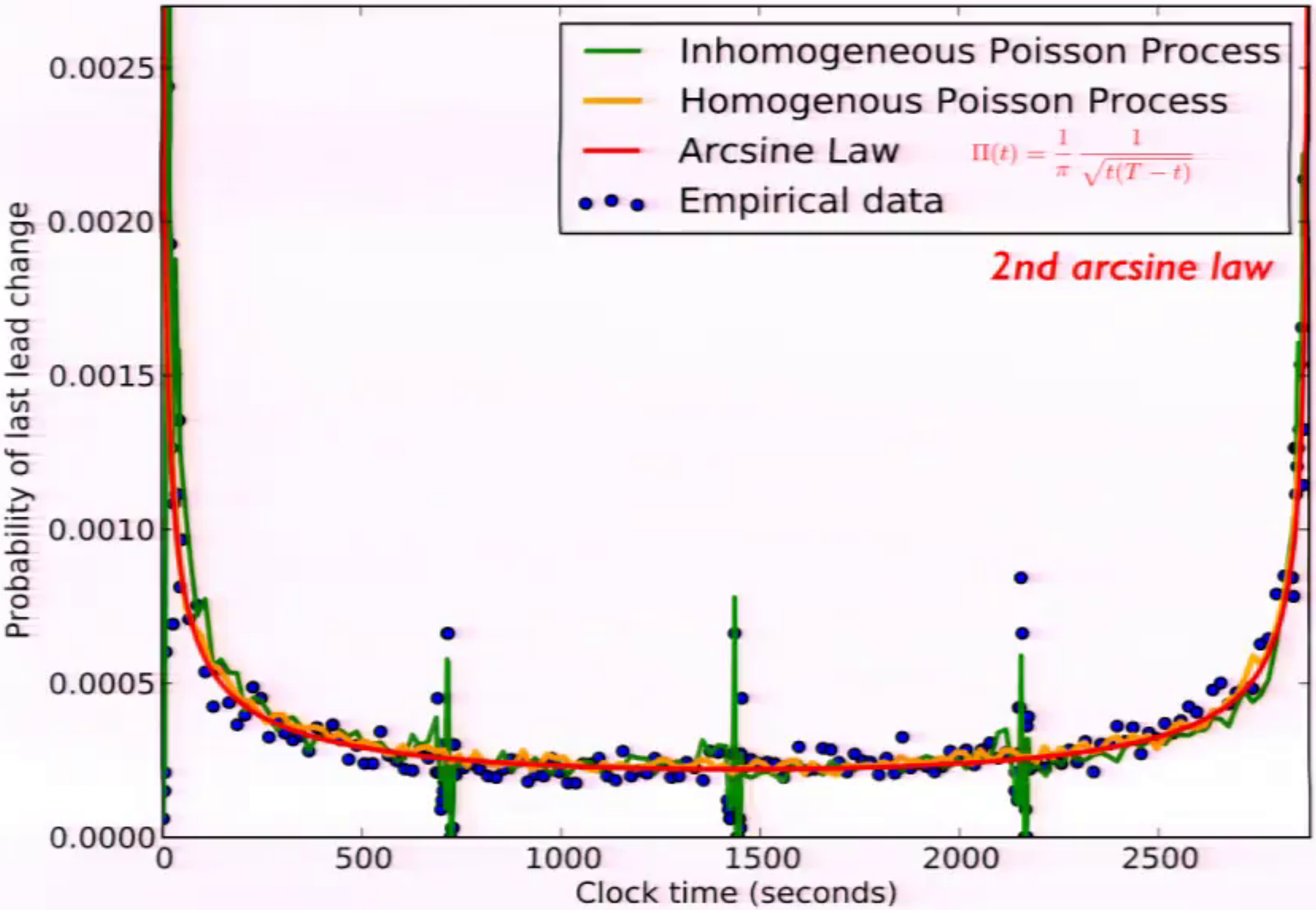
$$\frac{x}{\sqrt{4\pi D (T-t)^3}}\, e^{-x^2/[4D(T-t)]}$$

When Does the Last Lead Change Occur?

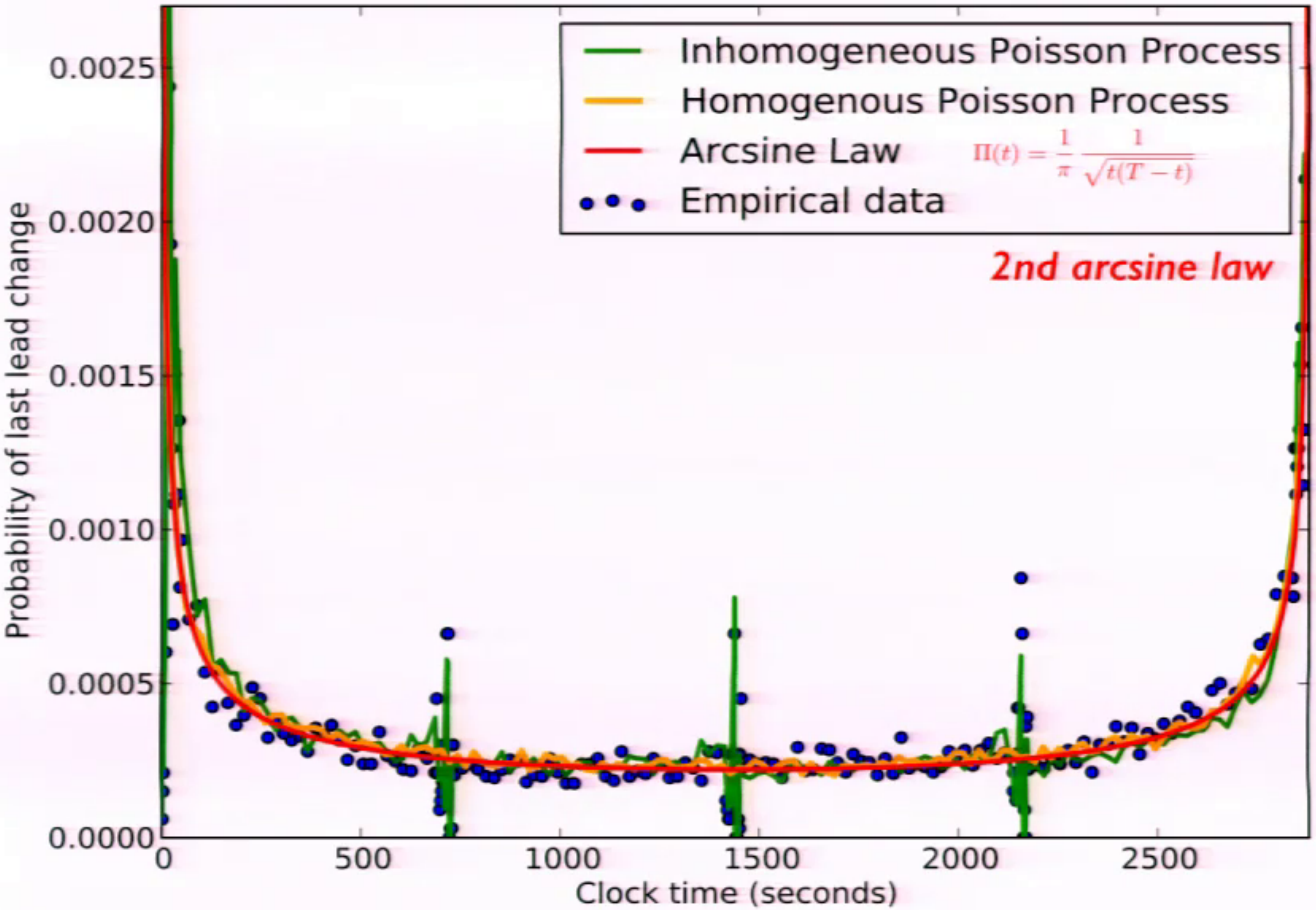
$\Pi(t)$ = Probability that last lead change occurs at time t

$$\begin{aligned}
\Pi(t) &= 2\int_0^\infty dx\, P(x{=}0,t)\, F(x,T{-}t) \\
&= 2\int_0^\infty \frac{dx}{\sqrt{4\pi Dt}}\, \frac{x}{\sqrt{4\pi D(T{-}t)^3}}\, e^{-x^2/[4D(T-t)]} \\
&= \frac{1}{\pi}\, \frac{1}{\sqrt{t(T-t)}}
\end{aligned}$$

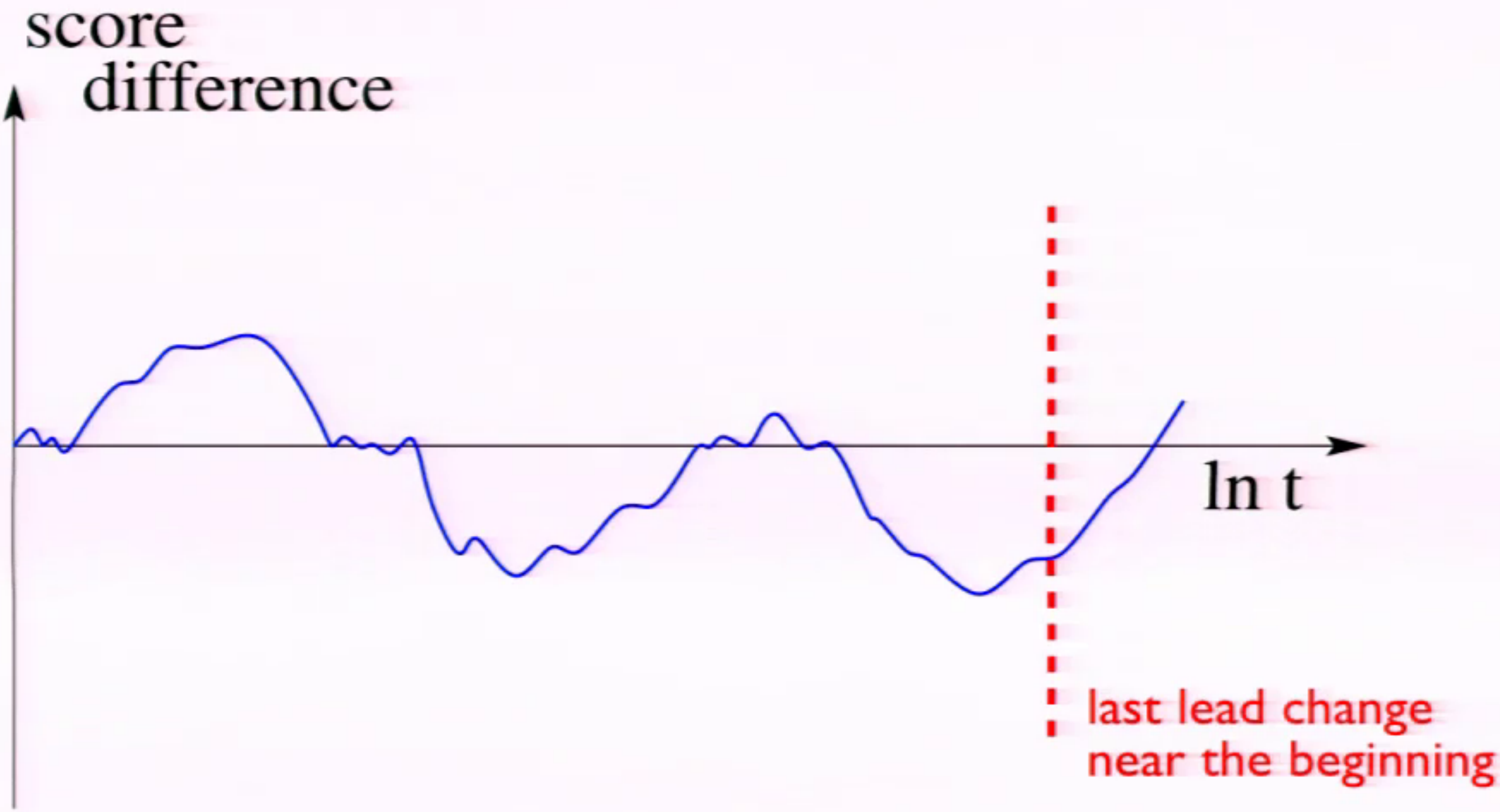
Probability When Last Lead Occurs



Probability When Last Lead Occurs

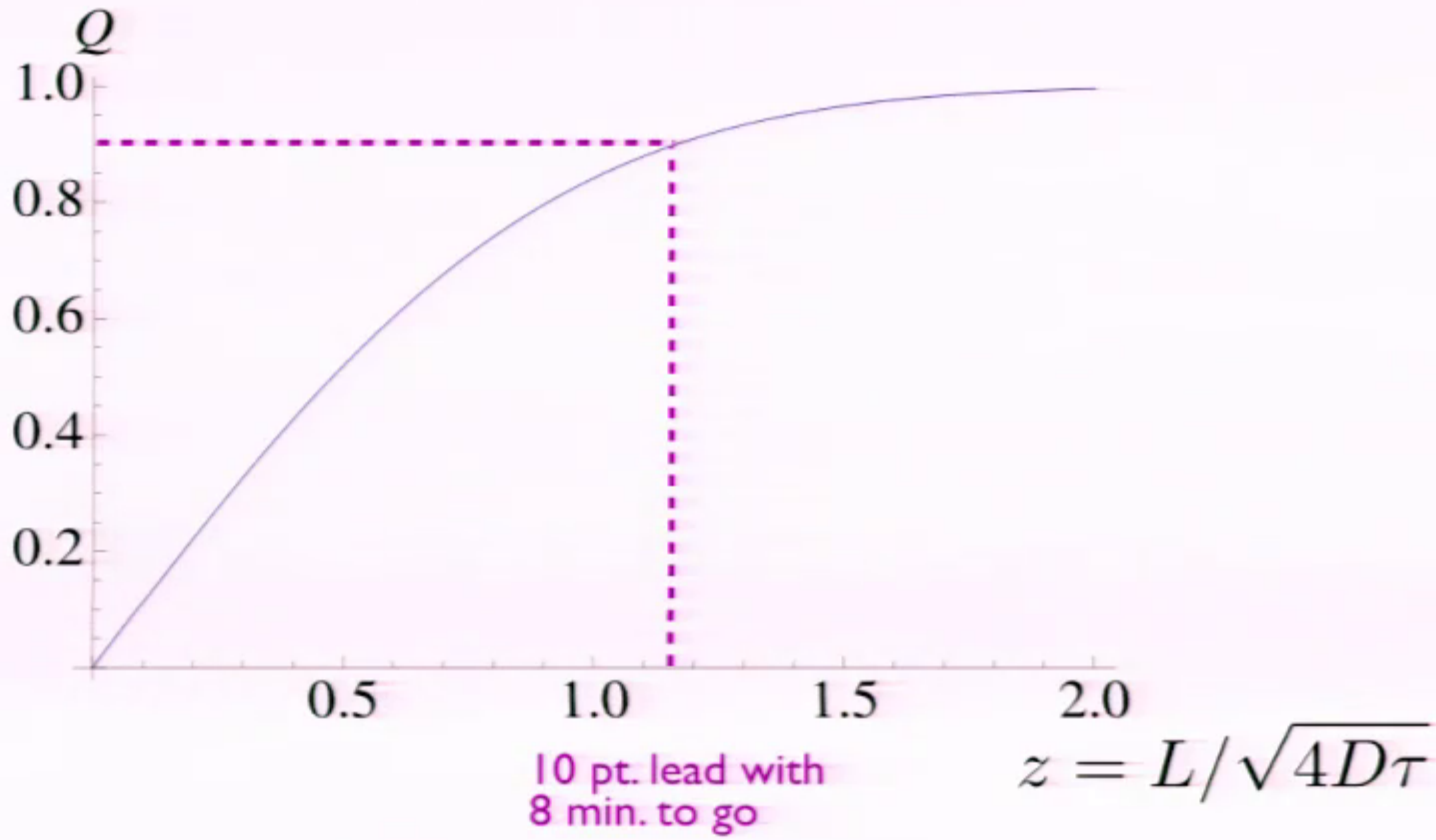


Why?

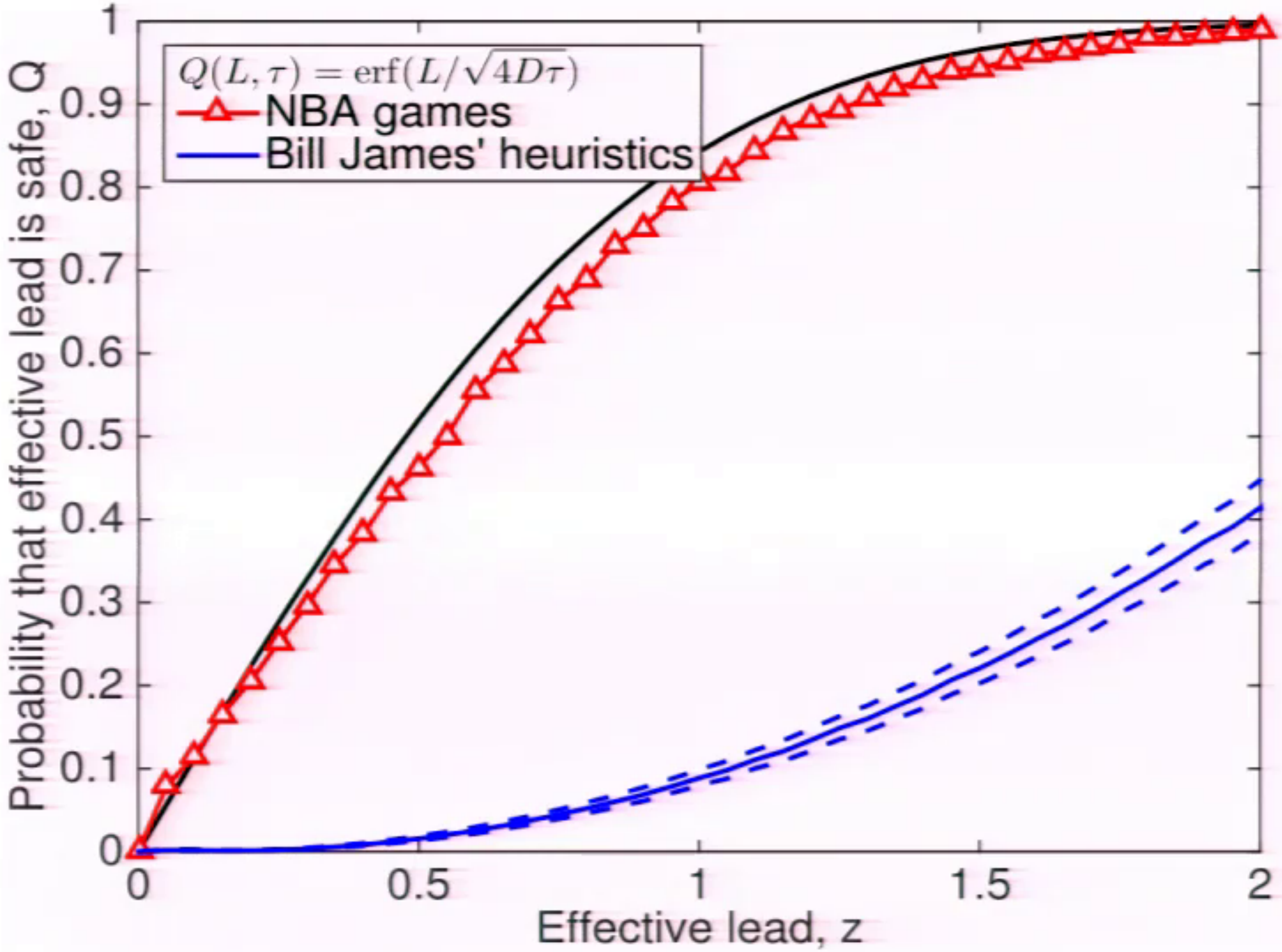


When Is a Lead Safe?

$$Q(L,\tau) = \mathrm{erf}(L/\sqrt{4D\tau})$$



When Is a Lead Safe?



Bill James
Inventor of *Sabermetrics*

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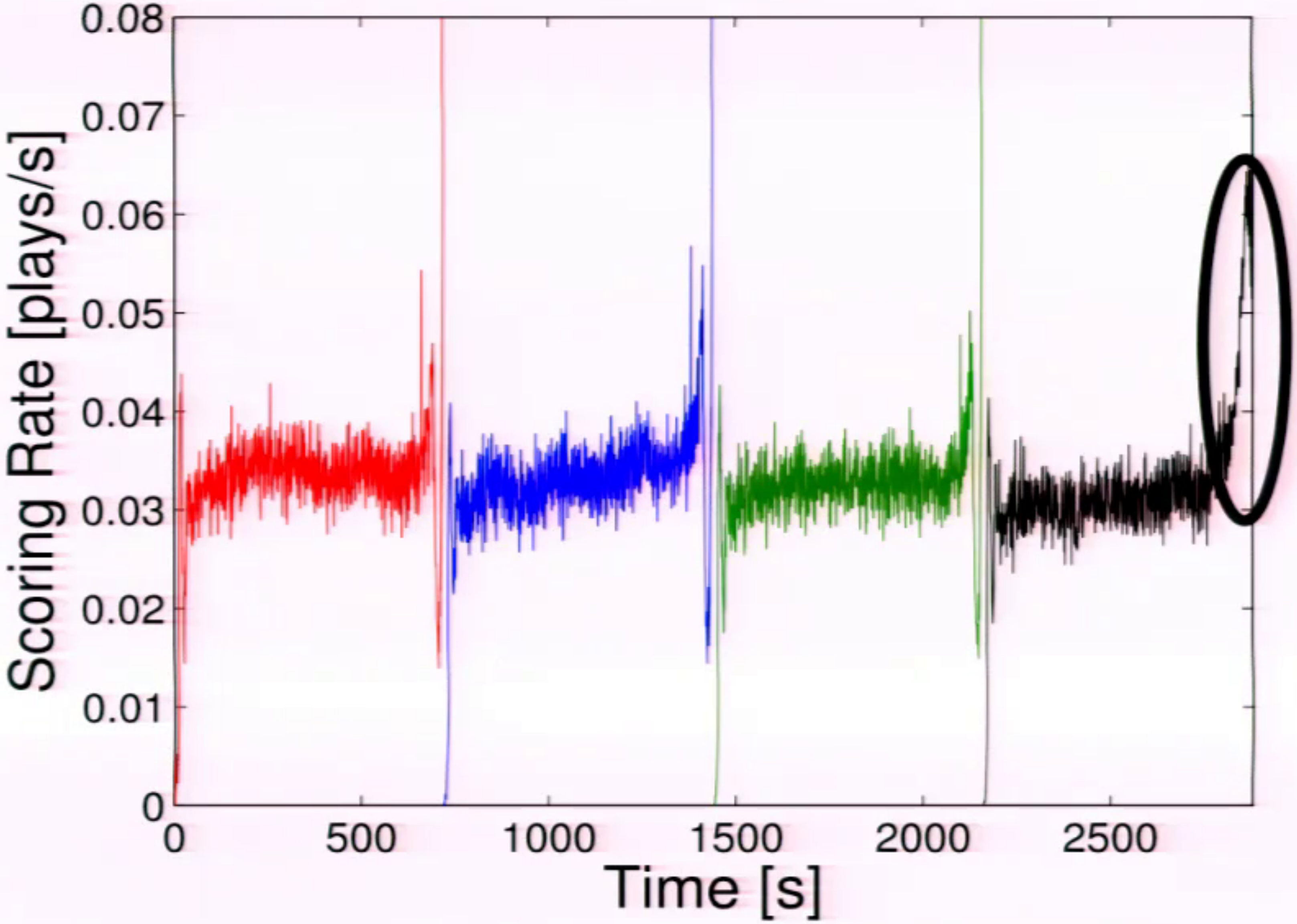
Can basketball scoring be described as a random walk?

Yes!

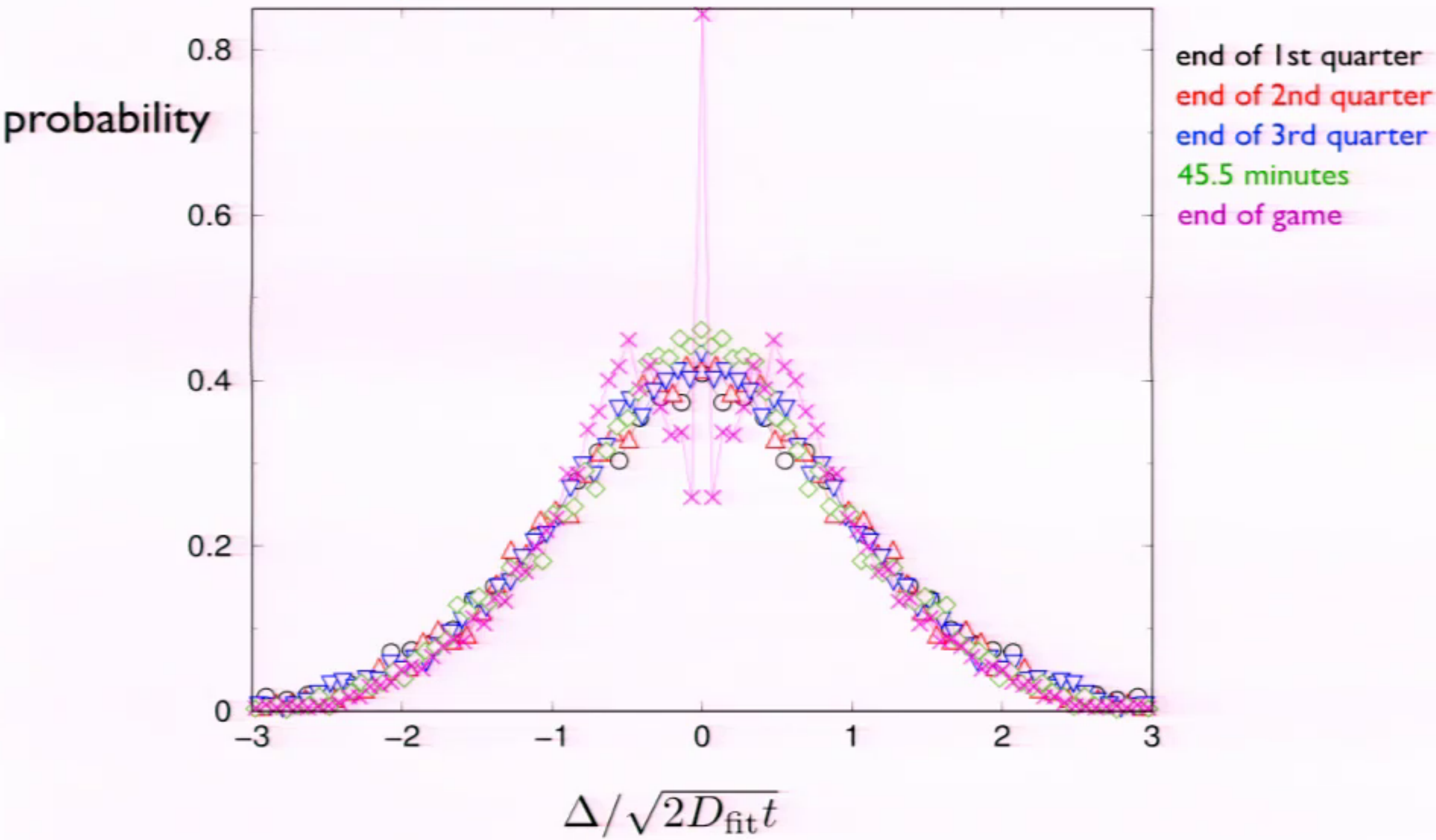
Some Buts……….

0. ***Last 2.5 minutes:*** anomalous.



Scoring Rate [plays/s]
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
500
1000
1500
2000
2500
Time [s]

Almost Gaussian Score Difference Distribution
except in the last 2.5 minutes



Some Buts..........

0. ***Last 2.5 minutes:*** anomalous.

1. ***Bias:*** Teams typically not that unevenly matched.

2. ***Antipersistence:*** After team A scores, same team scores next with probability 0.35.

3. ***Return to zero:*** winning team coasts, losing team desperate.