5D Respiratory Motion Model Based Image Reconstruction Algorithm for 4D Cone-Beam Computed Tomography

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Numerical Results

Conclusion and future work

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4D Cone-Beam CT (4DCBCT)

- 4DCBCT provides respiratory phase-resolved volumetric images
- utilized in radiation therapy to provide 4D image guidance in lung and upper abdomen area.
- However, clinical application of 4DCBCT is currently limited due to the long scan time and low image quality

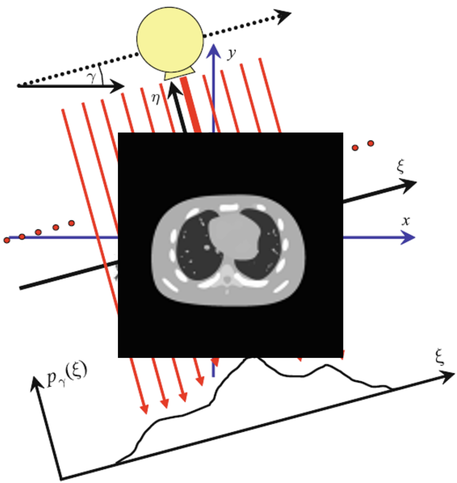

Figure: 4DCBCT

Challenges for 4DCBCT reconstruction

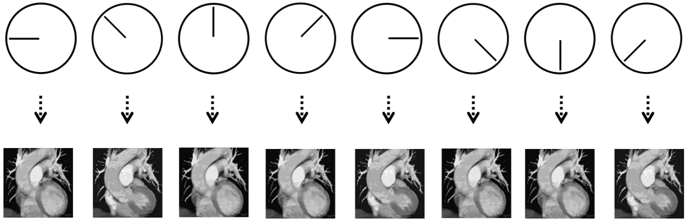

Figure: Projections: single projection per image

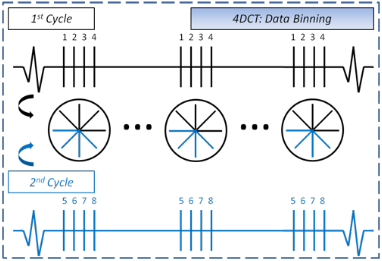

Figure: Rebinnig by cycle

- Few projections
- Binning errors
  - Inaccurate binning
  - Uneven binning

Some existing approaches

- No motion estimation: rebinning and reconstruction phase-by-phase (Jia et al., MICCAI, 2010; Gao et al., Medical Physics, 2012.);
- No motion estimation, no rebinning: Low rank matrix/rank sparsity (Cai et al., IEEE TMI, 2014);
- Time-dependent motion estimation (Christoffersen et al., IEEE TMI, 2013; Wang et al., Medical Physics, 2013; Yan et al., Medical Physics, 2014.).

Reconstruction by Rebinning

- Rebinning: Let $\{I_t(x_i, y_j), 1 \leq i, j \leq N, 1 \leq t \leq T\}$, y_t the binned projection data to the phase t, and A_t is the X-ray transform at these binned angles.

- The conventional phase-by-phase 4DCBCT methods

$$\min_{I_t} \|A_t I_t - y_t\|_2^2 + \mu |\nabla I_t|_1, 1 \leq t \leq T$$

- The state-of-art spatiotemporal-TV-based 4DCBCT

$$\min_{\{I_t\}} \sum_t \|A_t I_t - y_t\|_2^2 + \mu \sum_t |\nabla I_t|_1 + \lambda |\partial_t I_t|_1$$

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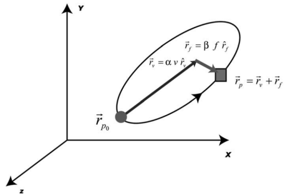
5D Respiratory Motion Model[3]

Developed to accurately estimate the respiratory motion from CT images.



$$\vec{X} = \vec{X}_0 + \vec{\alpha}(\vec{X}_0)v + \vec{\beta}(\vec{X}_0)f$$

- X_0 is the reference position
- X the predicted position
- $\vec{\alpha}(\vec{X}_0)$ and $\vec{\beta}(\vec{X}_0)$ motion vectors
- v the breathing amplitude, f the breathing rate.

[3] Low et al., International Journal of Radiation Oncology Biology Physics, 2005.

5D Model based reconstruction model

- Let $\{I_t\}$ be the image sequence to be reconstructed from the observed projection data $\{y_t\}$ (without binning).
- The object at $\vec{X}$ in I_0 deforms to a new location $\vec{X}_t$ in an arbitrary image phase I_t through the 5D motion model, i.e.

$$I_0(\vec{X}) = I_t(\vec{X}_t)$$
$$\vec{X}_t = \vec{X} + v_t\vec{M}_1 + f_t\vec{M}_2, \text{ for } 1 \leq t \leq T$$

  where the displacement vectors (M_1, M_2) are time-independent and the breathing amplitude (f_t) and the breathing rate (v_t) are time-dependent.
- In practice, we assume that the breathing amplitude f_t and rate v_t can be experimentally measured in advance (for regular breathing).

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5D Model based reconstruction model

- In 2D, let $\vec{M}_1 = (M_{1x}, M_{1y})$ and $\vec{M}_2 = (M_{2x}, M_{2y})$. $M = (M_{1x}, M_{1y}, M_{2x}, M_{2y})$.

$$I_0(\vec{X}) = I_t(\vec{X} + L_t(M))$$

with $L_t(M) = v_t\vec{M}_1 + f_t\vec{M}_2$.

- Linearization

$$I_0(\vec{X}) \approx I_t(\vec{X}) + \nabla^T I_t \cdot L_t(M)$$

- Finally, we attempt to solve

$$\min_{I_0, M} \sum_t \|A_t(I_0 - \nabla^T I_t \cdot L_t(M)) - y_t\|_2^2 + \mu|\nabla I_0|_1 + \lambda|\nabla M|_1.$$

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Numerical method

- Given I_t^K, we apply the proximal alternating minimization (PAM)[4] to solve

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$$\begin{cases} I_0^{K+1} = \underset{I_0}{\operatorname{argmin}} & \sum_t \|A_t(I_0 - \nabla^T I_t^K \cdot L_t(M^K)) - y_t\|_2^2 \\ & +\mu|\nabla I_0|_1 + \frac{1}{2\sigma}\|I_0 - I_0^K\|_2^2 \\ M^{K+1} = \underset{M}{\operatorname{argmin}} & \sum_t \|A_t(I_0^{K+1} - \nabla^T I_t^K \cdot L_t(M)) - y_t\|_2^2 \\ & +\lambda|\nabla M|_1 + \frac{1}{2\eta}\|M - M^K\|_2^2. \end{cases}$$

- Given (I_0^{K+1}, M^{K+1}), update I_t^{K+1} according to the deformation model

$$I_t^{K+1}(\vec{X} + L_t(M^{K+1})) = I_0^{K+1}(\vec{X})$$

Note that interpolation is needed to compute the cartesian coordinate of I_t from the deformation of I_0.

[4] Attouch-Bolte-Redont-Soubeyran,2010

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$$\min_{I_0,M}\sum_t \|A_t(I_0-\nabla^T I_t^K\cdot L_t(M))-y_t\|_2^2+\mu|\nabla I_0|_1+\lambda|\nabla M|_1.$$

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[4] Attouch-Bolte-Redont-Soubeyran,2010

Convergence Analysis

- Consider approximated model:

$$\min_{0\le I_0\le\alpha,|M|_\infty\le\beta} P(I_0,M) := \|A(I_0-\nabla^T I_0\cdot L_t(M))-y_t\|_2^2+\mu|\nabla I_0|_1+\lambda|\nabla M|_1$$

- Analysis based on Kurdyka-Łojasiewicz (K-Ł) Inequality (Attouch-Bolte-Redont-Soubeyran, Mathematics of Operations Research, 2010).

$$\begin{cases} f(I_0) & = \mu|\nabla I_0|_1+\mathcal{C}_{0\le I_0\le\alpha}(I_0) \\ Q(I_0,M) & = \sum_t \|A_t(I_0-\nabla^T I_0\cdot L_t(M))-y_t\|_2^2, \\ g(M) & = \lambda|\nabla M|_1+\mathcal{C}_{-\beta\le M\le\beta}(M) \end{cases}$$

- It can be shown that
  - $P(I_0,M)$ is a K-Ł function.
  - The sequence $Z^K=(I_0^K,M^K)$ is subsequence-convergent.
  - $\nabla Q(I_0,M)$ has a Lipschitz constant on any bounded set.
  - The sequence $Z^K=(I_0^K,M^K)$ converges to the critical point of $P(I_0,M)$.

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Simulation

- Methods in comparison: FBP, spatial TV (phase-by-phase), spatialtemporal-TV
- 570 projections are evenly distributed between 0 and 2π and the image size is 500×500 (visualization size 500×300)
- For the proposed method, every I_t has a single projection data y_t.
- The motion are simulated via 5D model with measured data.

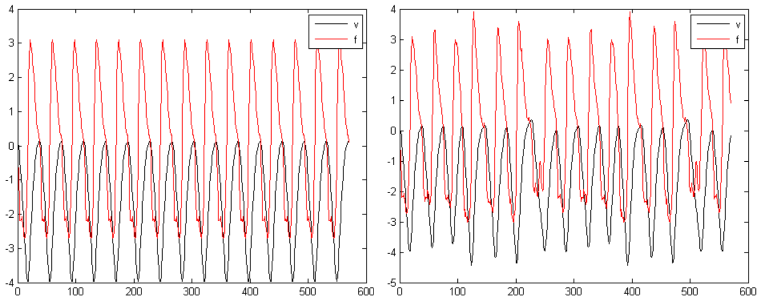

Figure: Breathing amplitude v and breathing rate f. (Left) Periodic breathing; (Right) non-periodic breathing.

Rebinning for the methods in comparison

- 570 projections are binned to 10 phases.

Phase / Period	1st	2nd	3rd	4th	5th`	6th	7th	8th	9th	10th
1	1-3	4-7	8-11	12-15	16-19	20-23	24-27	28-31	32-35	36-38
2	39-41	42-45	……							
3	77-79									
……	……									

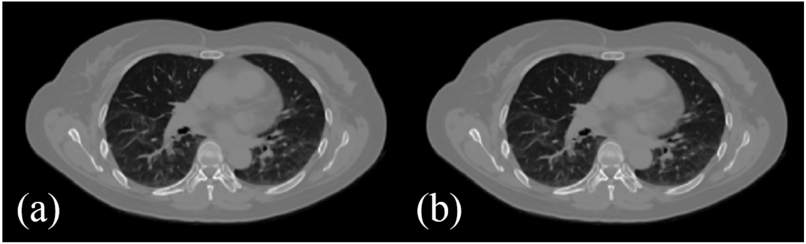

Figure: Ground truth(Phase 3). (a) Periodic breathing; (b) non-periodic breathing.

Reconstruction results for periodic breathing

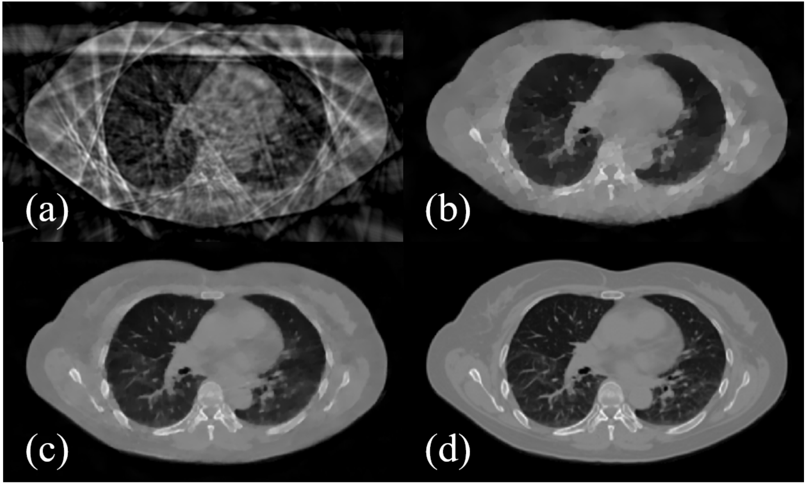

Figure: Reconstruction results for periodic breathing (Phase 3). (a) FBP; (b) spatial TV; (c) spatiotemporal TV; (d) 5D Method.

Reconstruction errors for periodic breathing

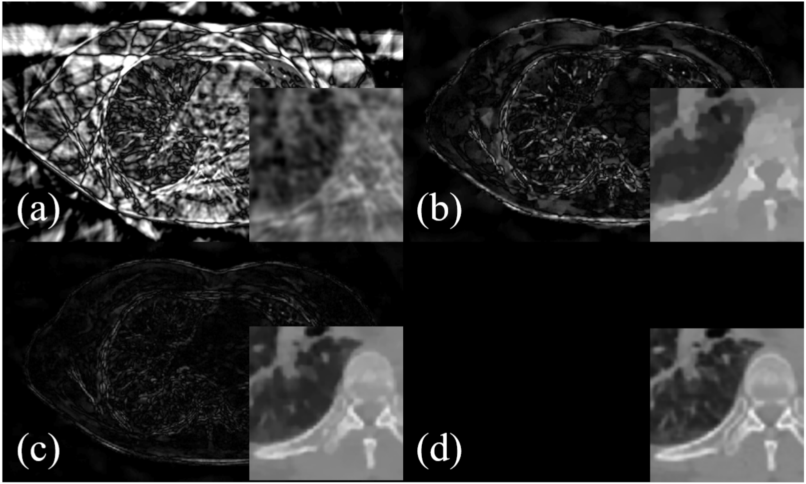

Figure: Reconstruction errors and zoom-in details for periodic breathing (Phase 3). (a) FBP; (b) spatial TV; (c) spatiotemporal TV; (d) 5D Method.

Reconstruction results for non-periodic breathing

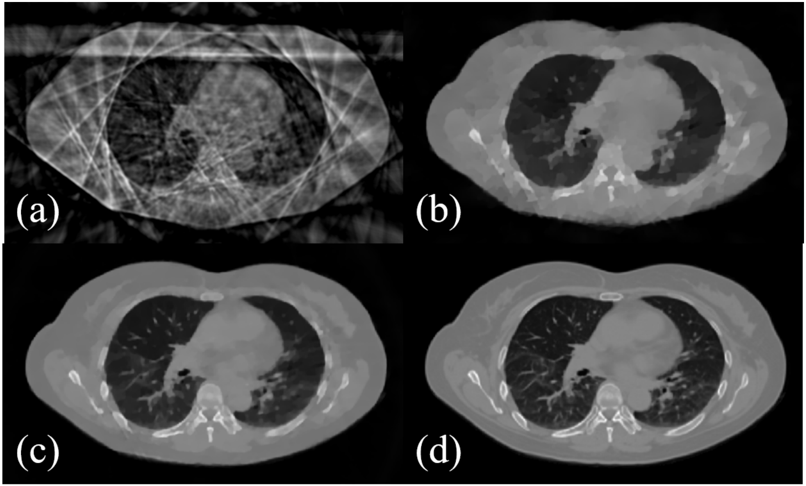

Figure: Reconstruction results for non-periodic breathing (Phase 3). (a) FBP; (b) spatial TV; (c) spatiotemporal TV; (d) 5D Method.

Reconstruction errors for non-periodic breathing (Phase 3)

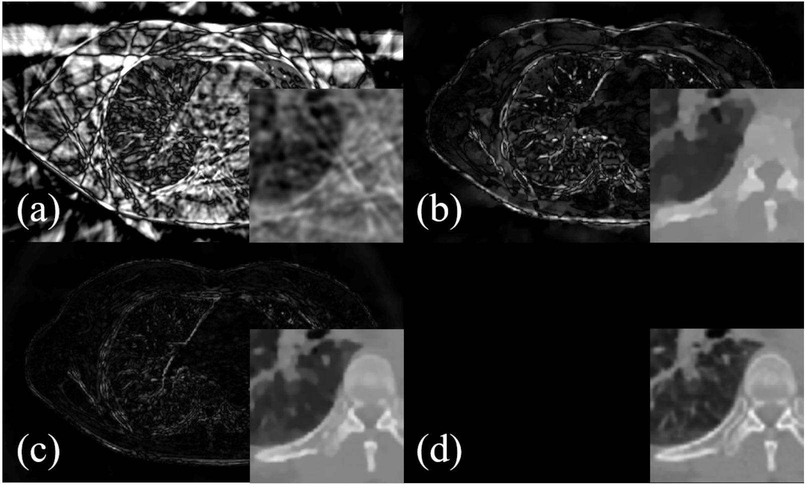

Figure: Reconstruction errors and zoom-in details for non-periodic breathing (Phase 3). (a) FBP; (b) spatial TV; (c) spatiotemporal TV; (d) 5D Method.

Reconstructed motion vector

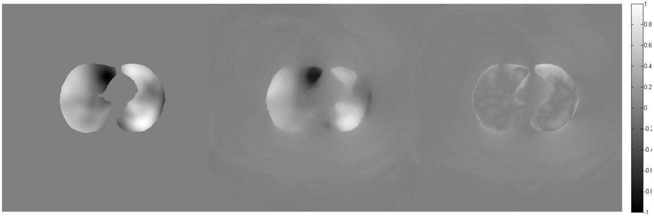

Figure: M_{1x} for periodic breathing. (Left) Ground truth; (Middle) Reconstructed; (Right) The error.

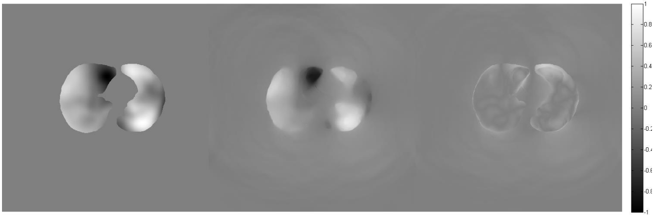

Figure: M_{1x} for non-periodic breathing. (Left) Ground truth; (Middle) Reconstructed; (Right) The error.

Quantitative reconstruction errors

Table: Relative errors between reconstructed images and ground truth (unit in %)

Method	FBP	spatial TV	spatiotemporal TV	5D Method
Periodic	27.63	3.75	2.91	0.44
Non-periodic	28.14	3.86	2.96	0.45

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- Conclusions
  - developed a new 4DCBCT image reconstruction method incorporating the breath motion.
  - improved image reconstruction from standard and state-of-art methods for both periodic and non-periodic breathing.
- Future work
  - validation on 4D image (ongoing) and real data.
  - convergence analysis on the original model.

Thank you

Thank you!

Jiulong Liu, Xue Zhang, Xiaoqun Zhang, Hongkai Zhao, Yu Gao, David Thomase, Daniel A Lowe, and Hao Gao.
5D respiratory motion model based image reconstruction algorithm for 4D cone-beam computed tomography.
Inverse Problems, Volume 31, Number 11, 2015