# Multi-Contrast MRI Reconstruction with Structure-Guided Total Variation

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## Multi-Contrast MRI Reconstruction with Structure-Guided Total Variation*

Matthias J. Ehrhardt† and Marta M. Betcke‡

**Abstract.** Magnetic resonance imaging (MRI) is a versatile imaging technique that allows different contrasts depending on the acquisition parameters. Many clinical imaging studies acquire MRI data for more than one of these contrasts—such as for instance $T_1$ and $T_2$ weighted images—which makes the overall scanning procedure very time consuming. As all of these images show the same underlying anatomy one can try to omit unnecessary measurements by taking the similarity into account during reconstruction. We will discuss two modifications of total variation—based on i) location and ii) direction—that take structural a priori knowledge into account and reduce to total variation in the degenerate case when no structural knowledge is available. We solve the resulting convex minimization problem with the alternating direction method of multipliers that separates the forward operator from the prior. For both priors the corresponding proximal operator can be implemented as an extension of the fast gradient projection method on the dual problem for total variation. We tested the priors on six data sets that are based on phantoms and real MRI images. In all test cases exploiting the structural information from the other contrast yields better results than separate reconstruction with total variation in terms of standard metrics like peak signal-to-noise ratio and structural similarity index. Furthermore, we found that exploiting the two dimensional directional information results in images with well defined edges, superior to those reconstructed solely using a priori information about the edge location.

**Key words.** total variation, magnetic resonance imaging, MRI, a priori information, image reconstruction, regularization, structural similarity

**AMS subject classifications.** 47A52, 49M30, 65J22, 94A08

### 1. Introduction.

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# Magnetic Resonance Imaging (MRI)

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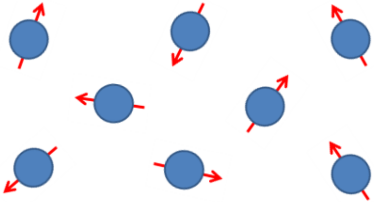

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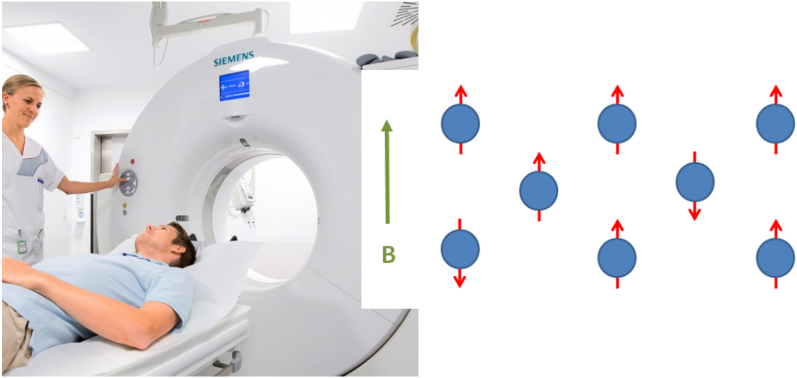


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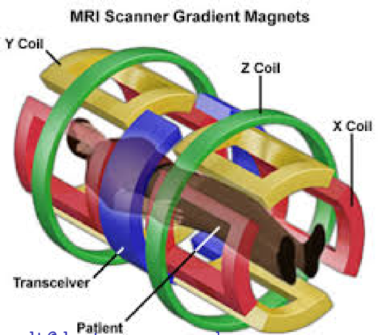


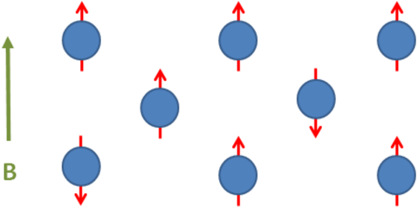


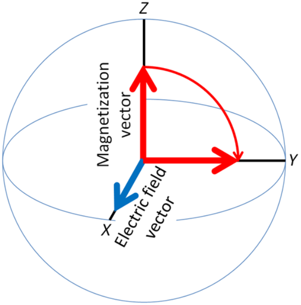


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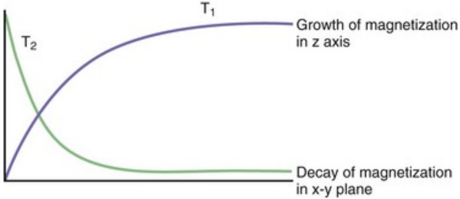
# Relaxation Parameters: $T_1$ and $T_2$




http://clinicalgate.com/neuroimaging-structural-imaging-magnetic-resonance-imaging-computed-tomography/

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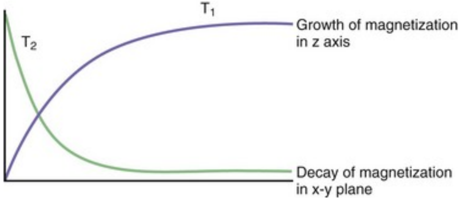


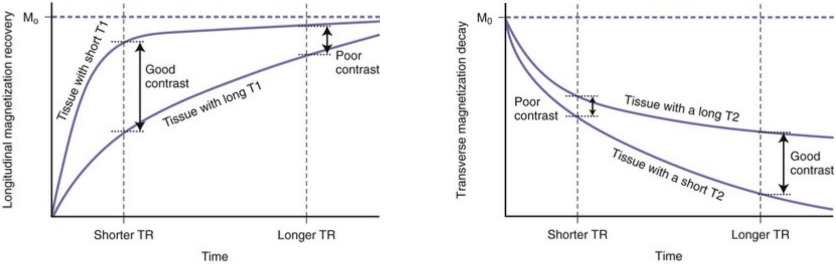


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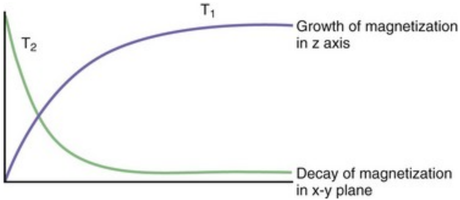


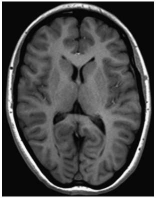

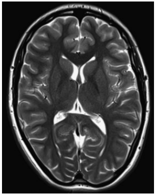

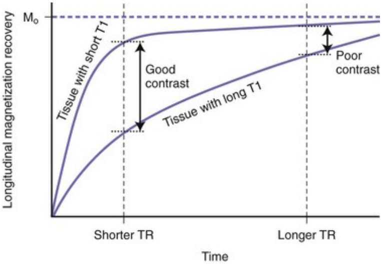


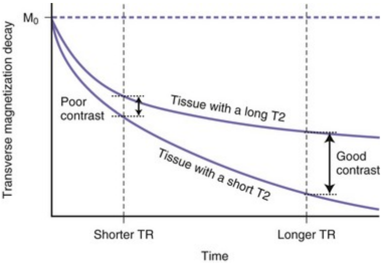


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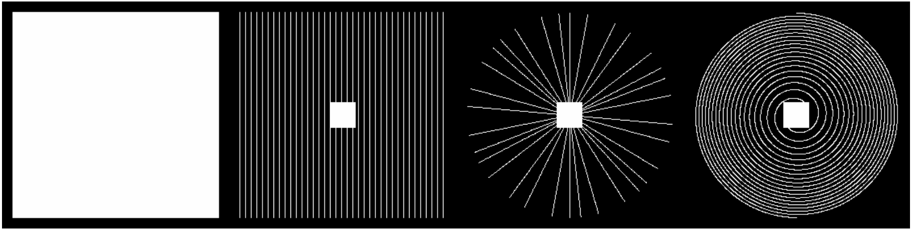
# Magnetic Resonance Imaging (MRI), models

Simple model:

$$b_i = \int_\Omega u(x) \exp(\langle x, k_i \rangle) dx + n_i$$

- Gaussian noise model, $n_i \sim \mathcal{N}(0, \sigma)$



# Speed of MRI

- The measuerements $b_i \in \mathbb{C}, i = 1, \ldots, M$ are **sequentially** acquired.

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  - dynamic imaging
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  - time is money: higher patient throughput, cheaper scans

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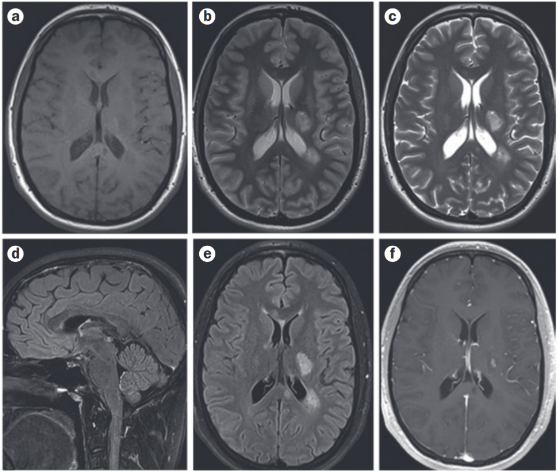
- Scanning not arbitrary fast. **Fast scans** needed
  - dynamic imaging
  - patient comfort
  - time is money: higher patient throughput, cheaper scans
- Speed-up by acquiring for less data
  - **Sparse MRI**: "Omit" unnecessary measurements and replace by **a-priori information** about the object (Lustig et al., 2007)
  - **Parallel MRI**: sample some data in parallel

# Multi-Contrast MRI




pre-contrast

a) $T_1$-weighted

b,c) dual-echo $T_2$

post-contrast

d) 2D $T_2$ FLAIR

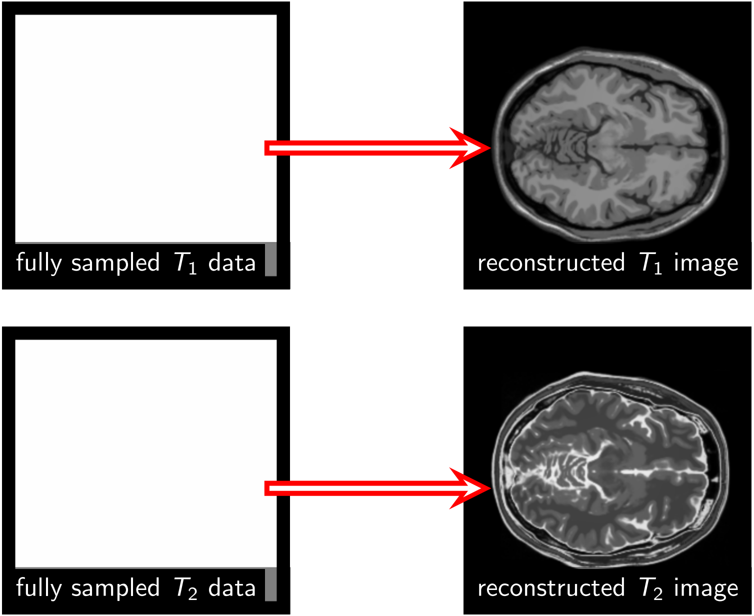
e) 2D $T_2$ FLAIR

f) $T_1$-weighted

- Standardized brain MRI protocol for multiple sclerosis
- 6 scans, total duration: 30 min

Rovira et al., Nature Reviews Neurology, 2015

# Multi-Contrast MRI Reconstruction

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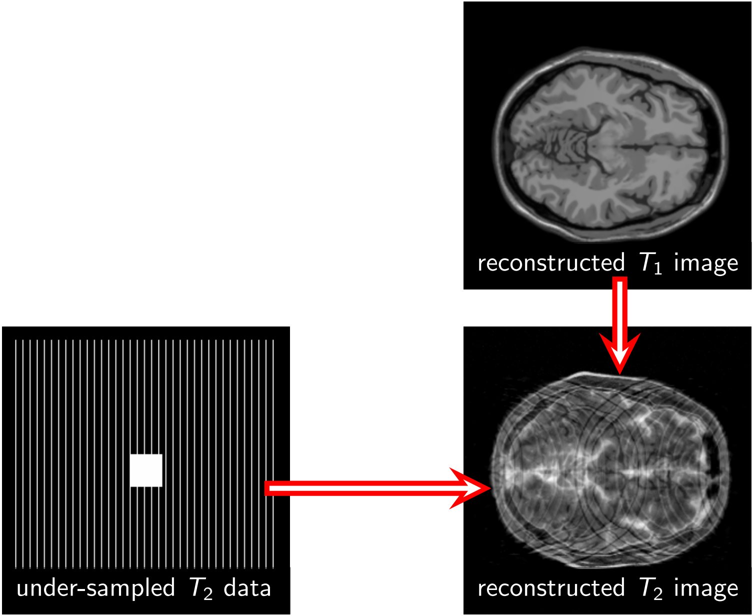


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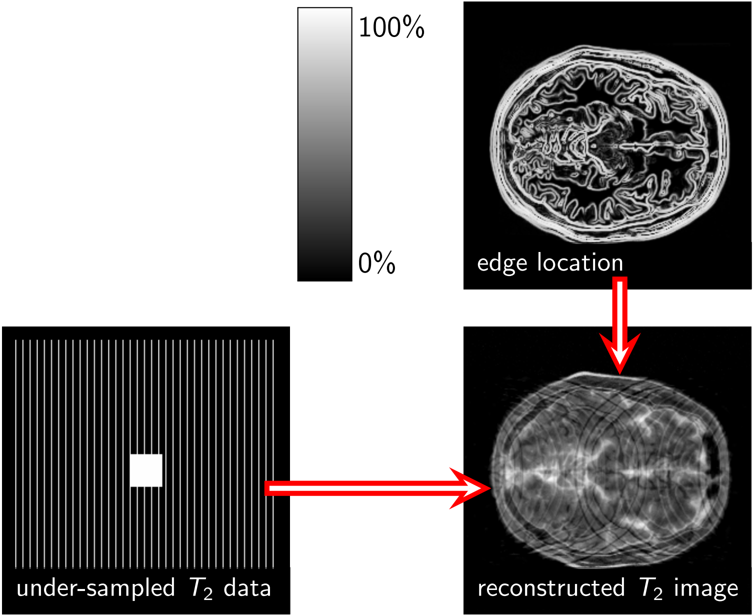
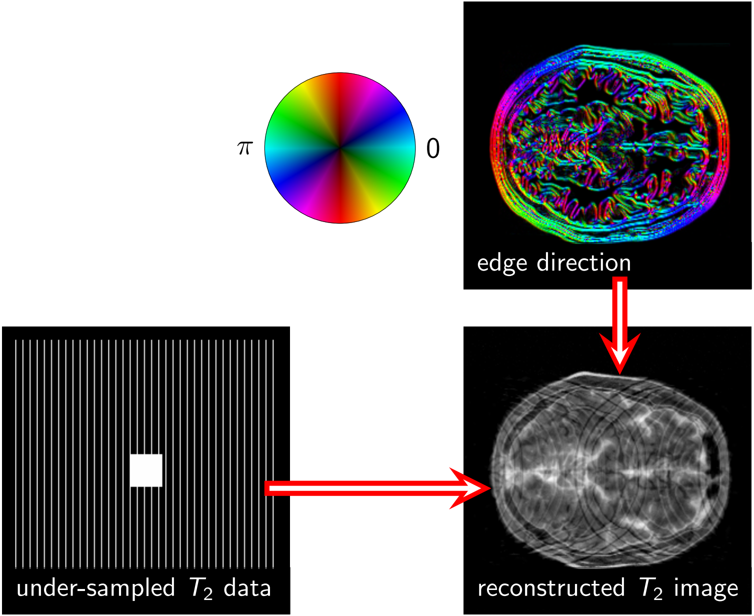


# Multi-Contrast MRI Reconstruction

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1) Mathematical Model

2) Algorithms

3) Numerical Results

# Mathematical Model

# MRI Reconstruction

## Variational Approach

$$u^* \in \arg\min_{u \geq 0} \left\{ \frac{1}{2} |Au - b|^2 + \alpha \mathcal{J}(u) \right\}$$

$$A : \mathbb{R}^N \to \mathbb{C}^M, \quad A = \mathrm{Re}^* \circ F \circ S$$

- 2D discrete Fourier transform $F$
- sampling operator $S$

# Modelling A-Priori Information

## Total Variation

$$\mathcal{J}(u) = \mathrm{TV}(u) := \sum_i |\nabla u_i|$$

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## Weighted Total Variation

$$\mathcal{J}(u) = \mathrm{wTV}(u) := \sum_i w_i |\nabla u_i|, \quad 0 \leq w_i \leq 1$$

$$w_i = \frac{\eta}{|\nabla v_i|_\eta}, \quad |\nabla v_i|_\eta^2 = |\nabla v_i|^2 + \eta^2, \quad \eta > 0$$

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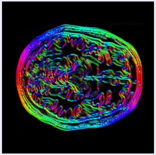
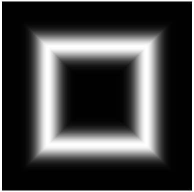
## Directional Total Variation

$$\mathcal{J}(u) = \mathrm{dTV}(u) := \sum_i |D_i \nabla u_i|$$

$$D_i = I - \xi_i \xi_i^T, \quad \xi_i = \nabla v_i / |\nabla v_i|_\eta$$



# Parallel Level Set Prior



Ehrhardt et al. 2016 (to appear in IEEE TMI)

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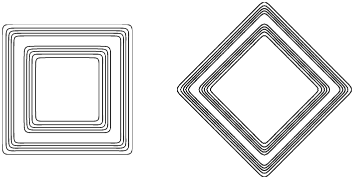

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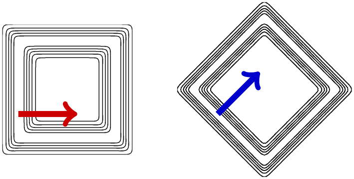

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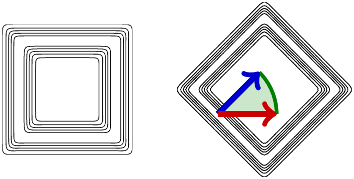

$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Ehrhardt et al. 2016 (to appear in IEEE TMI)

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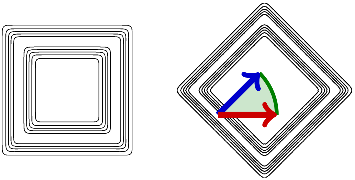

$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

## Measure Similar Structures

$$\mathcal{S}(u) := \sum_i \left( |\nabla u_i|^2 - \langle \nabla u_i, \xi_i \rangle^2 \right)^{1/2}$$

- $\xi_i \propto \nabla v_i / |\nabla v_i|_\eta, \quad |\nabla v_i|_\eta := \sqrt{|\nabla v_i|^2 + \eta^2}, \quad \eta > 0$

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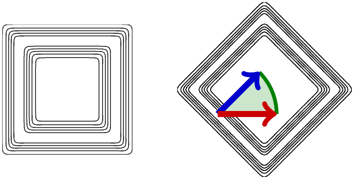

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- $0 \leq \mathcal{S}(u) \leq \mathrm{TV}(u)$
- $\mathcal{S}(u) = 0 \Leftrightarrow u \sim v \ (\nabla u \parallel \nabla v)$

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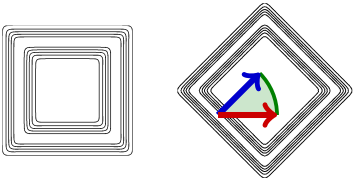

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- $\mathcal{S}(u) = 0 \Leftrightarrow u \sim v \ (\nabla u \parallel \nabla v)$

## Directional Total Variation = Parallel Level Set Prior

$$\mathrm{dTV}(u) = \mathcal{S}(u)$$

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# Algorithms

# Reformulate the Optimization Problem

$$\min_{u \geq 0} \left\{ \frac{1}{2} |Au - b|^2 + \alpha \mathcal{J}(u) \right\}$$

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# Reformulate the Optimization Problem

$$\min_{u \geq 0} \left\{ \frac{1}{2} |Au - b|^2 + \alpha \mathcal{J}(u) \right\}$$

$$\min_{u} \left\{ \frac{1}{2} |SF \operatorname{Re}^* u - b|^2 + \alpha \mathcal{J}(u) + \chi_{\geq 0}(u) \right\}$$

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$$\min_{u,x,z} \left\{ \frac{1}{2} |Sx - b|^2 + \alpha \mathcal{J}(u) + \chi_{\geq 0}(u) \right\}, \quad \text{s.t. } x = F \operatorname{Re}^* z, u = z$$

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Lagrangian Formulation, $\rho > 0$

$$\begin{aligned}\mathcal{L}(u, x, z) = &\frac{1}{2} |Sx - b|^2 + \alpha \mathcal{J}(u) + \chi_{\geq 0}(u) \\ &+ \frac{\rho}{2} \left[ |\nu - (x - F \operatorname{Re}^* z)|^2 + |\mu - (u - z)|^2 \right]\end{aligned}$$

# Solve Optimization Problem with ADMM

$$\begin{aligned}\mathcal{L}(u, x, z) = &\frac{1}{2}|Sx - b|^2 + \alpha \mathcal{J}(u) + \chi_{\geq 0}(u) \\ &+ \frac{\rho}{2}\Big[|\nu - (x - F\,\mathrm{Re}^*\, z)|^2 + |\mu - (u - z)|^2\Big]\end{aligned}$$

**Alternating Direction Method of Multipliers (ADMM):** Iterate

Update first block

$$u^k, x^k = \arg\min_{u,x} \mathcal{L}(u, x, z^{k-1})$$

Update second block

$$z^k = \arg\min_{z} \mathcal{L}(u^k, x^k, z)$$

Update Lagrange multipliers

# Solve Optimization Problem with ADMM

$$
\begin{aligned}
\mathcal{L}(u, x, z) =& \frac{1}{2}|Sx - b|^2 + \alpha \mathcal{J}(u) + \chi_{\geq 0}(u) \\
&+ \frac{\rho}{2}\Big[|\nu - (x - F\,\mathrm{Re}^*\, z)|^2 + |\mu - (u - z)|^2\Big]
\end{aligned}
$$

**Alternating Direction Method of Multipliers (ADMM):** Iterate
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$$x^k = \arg\min_x \mathcal{L}(\sim, x, z^{k-1})$$

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**Alternating Direction Method of Multipliers (ADMM):** Iterate
Update first block

$$u^k = \mathrm{prox}_{\alpha/\rho\mathcal{J}+\chi_{\geq 0}}(z^{k-1} - \mu^{k-1})$$

$$x^k = \arg\min_x \mathcal{L}(\sim, x, z^{k-1})$$

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**Alternating Direction Method of Multipliers (ADMM):** Iterate

Update first block

$$u^k = \mathrm{prox}_{\alpha/\rho\mathcal{J}+\chi_{\geq 0}}(z^{k-1} - \mu^{k-1})$$

$$x^k = (S^*S + \rho I)^{-1}[S^*b + \rho(Fz^{k-1} - \nu^{k-1})]$$

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$$z^k = \arg\min_z \mathcal{L}(u^k, x^k, z)$$

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$$x^k = (S^* S + \rho I)^{-1}[S^* b + \rho(F z^{k-1} - \nu^{k-1})]$$

Update second block

$$z^k = \frac{1}{2}[\mathrm{Re}\, F^{-1}(x^k + \nu^{k-1}) + u^k + \mu^{k-1}]$$

Update Lagrange multipliers

# Proximal Operator of Structural-Guided Total Variation

$$
\begin{aligned}
u^k &= \operatorname{prox}_{\alpha/\rho\mathcal{J}+\chi_{\geq 0}}(\cdot) \\
&= \arg\min_{u}\left\{\frac{1}{2}|u-(\cdot)|^2+\alpha/\rho|D\nabla u|_1+\chi_{\geq 0}(u)\right\}
\end{aligned}
$$

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# Proximal Operator of Structural-Guided Total Variation

$$
\begin{aligned}
u^k &= \operatorname{prox}_{\alpha/\rho \mathcal{J} + \chi_{\geq 0}}(\cdot) \\
&= \arg\min_u \left\{ \frac{1}{2} |u - (\cdot)|^2 + \alpha/\rho |D\nabla u|_1 + \chi_{\geq 0}(u) \right\} \\
&= \arg\min_u \left\{ \frac{1}{2} |u - (\cdot)|^2 + \alpha/\rho |\tilde{\nabla} u|_1 + \chi_{\geq 0}(u) \right\}
\end{aligned}
$$

Same algorithms as for “normal” total variation. Here: FISTA

$$
\|\tilde{\nabla}\| = \|D\nabla\| \leq \|D\|\|\nabla\| \leq \|\nabla\| \tag{1}
$$

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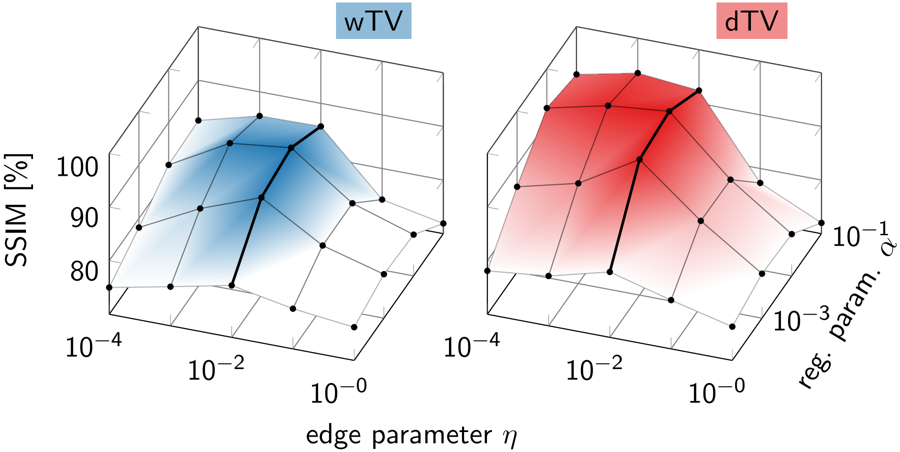
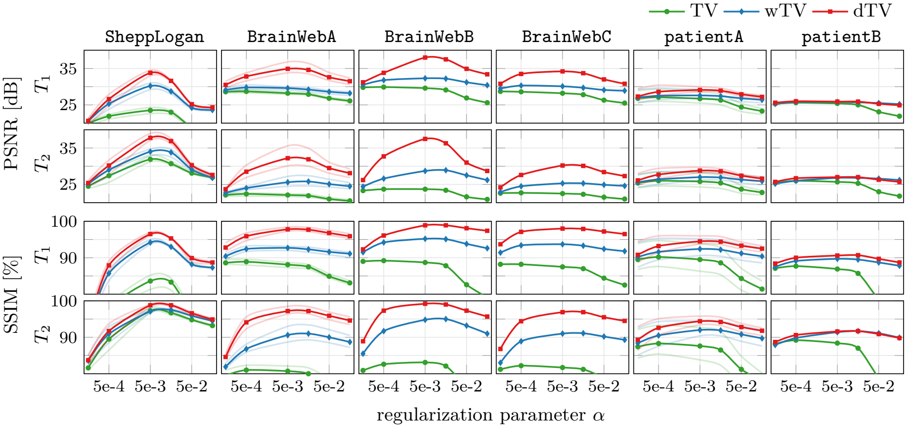
## Numerical Results

# Parameter Estimation (example for $T_2$)

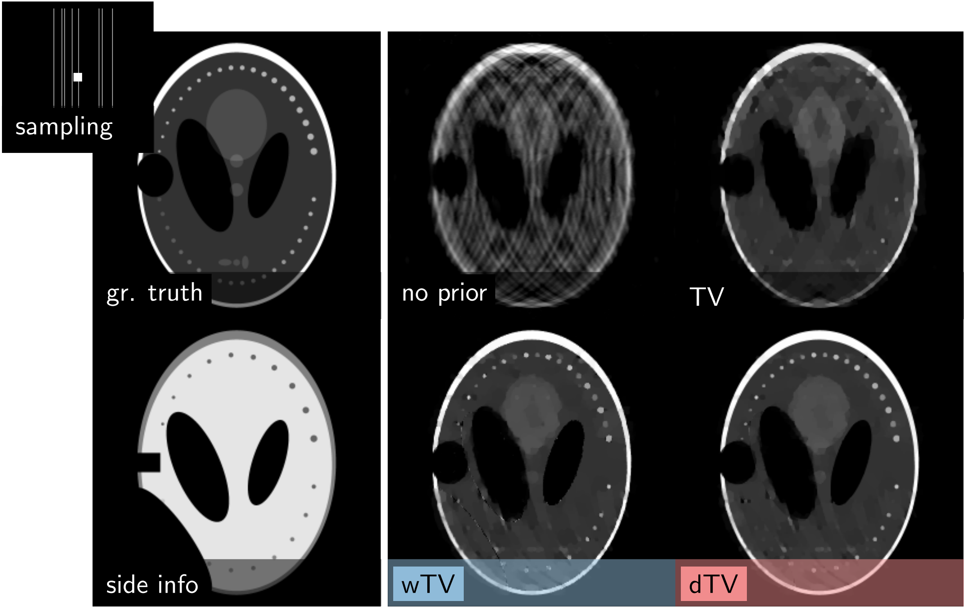



- Best results for $\eta = 10^{-2}$. Trade-off: regularization v structure
- For large $\eta$, both methods perform as TV (not shown)

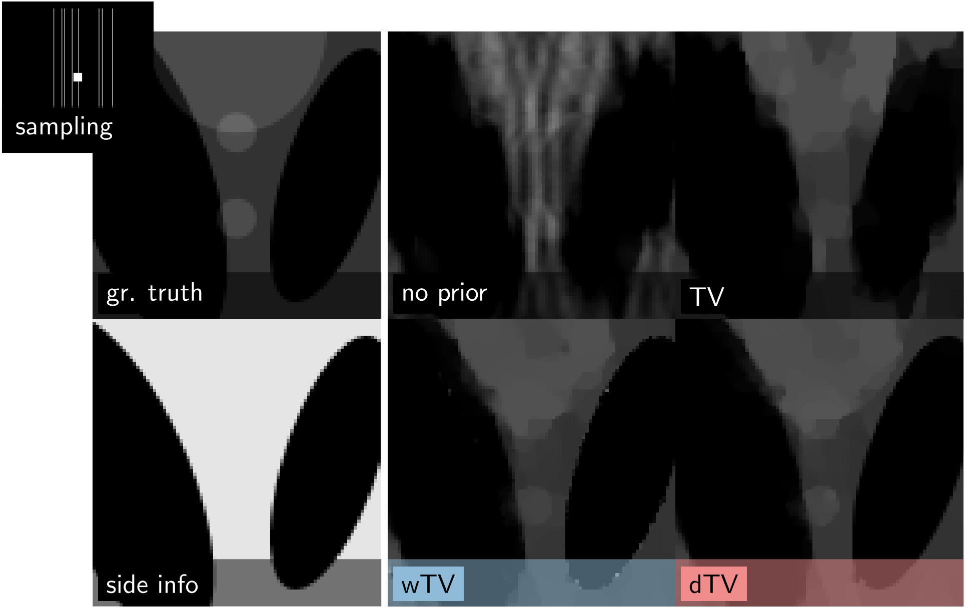
# Results with varying Regularization Parameter



# Results



# Results

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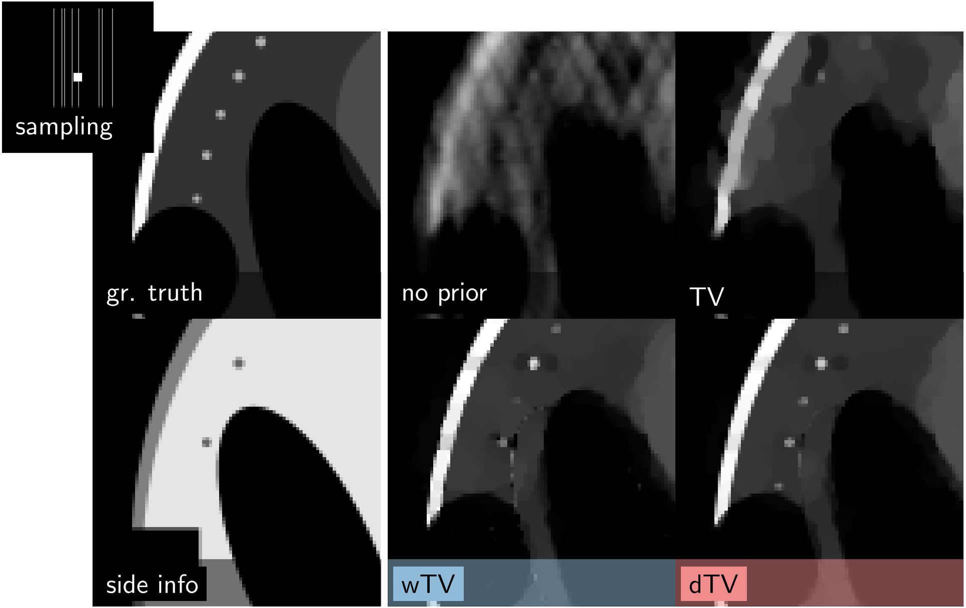
# Results



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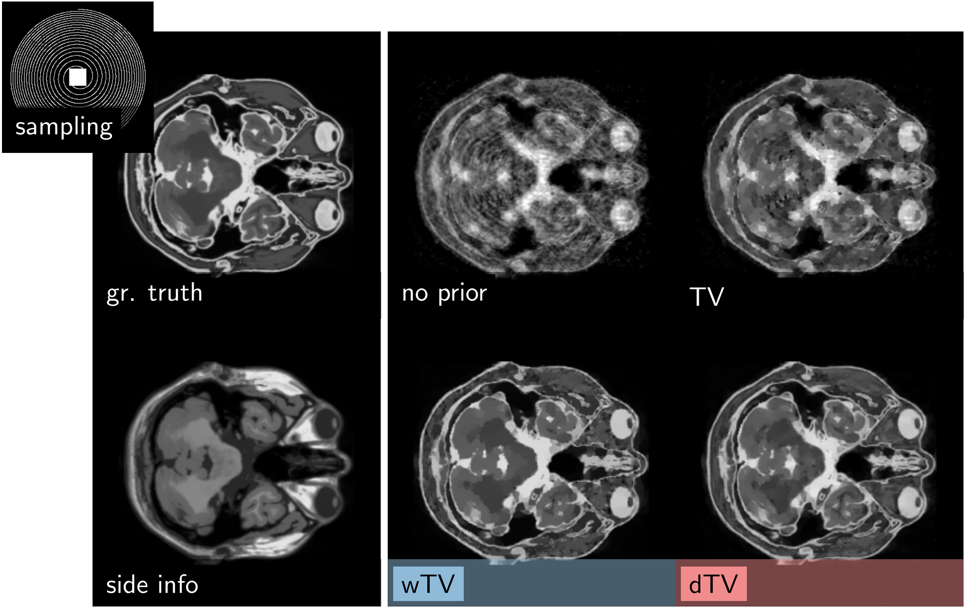

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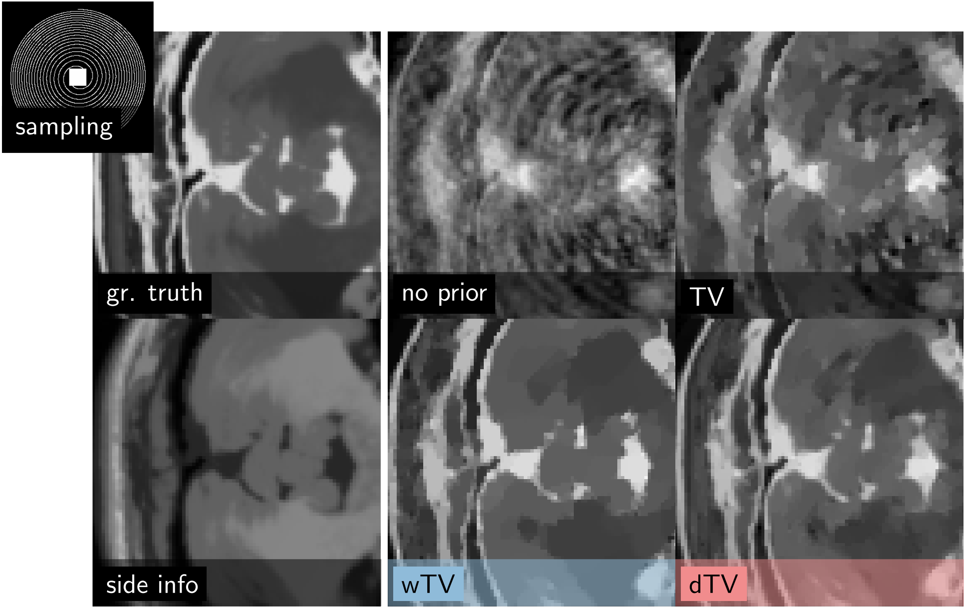


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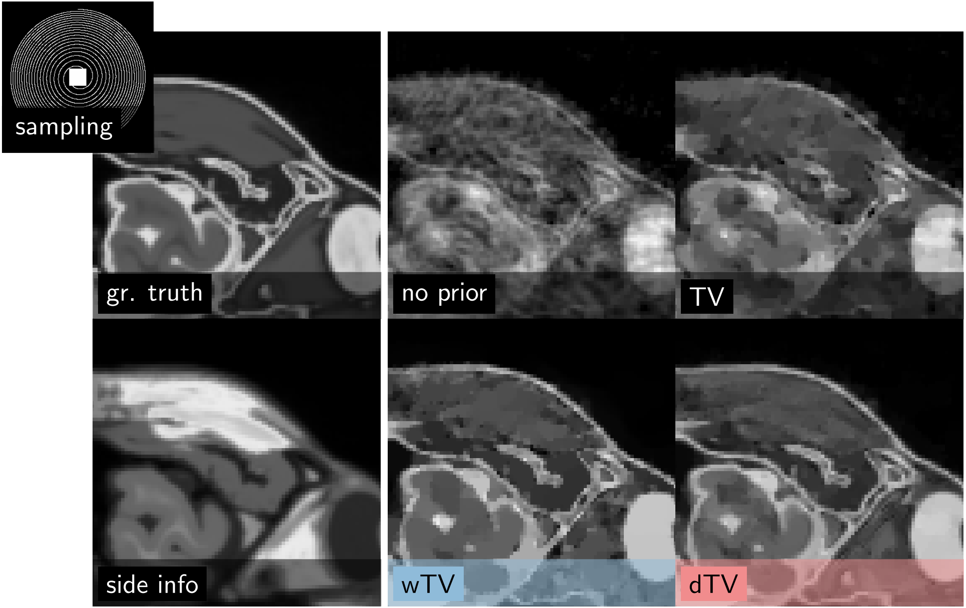
# Results

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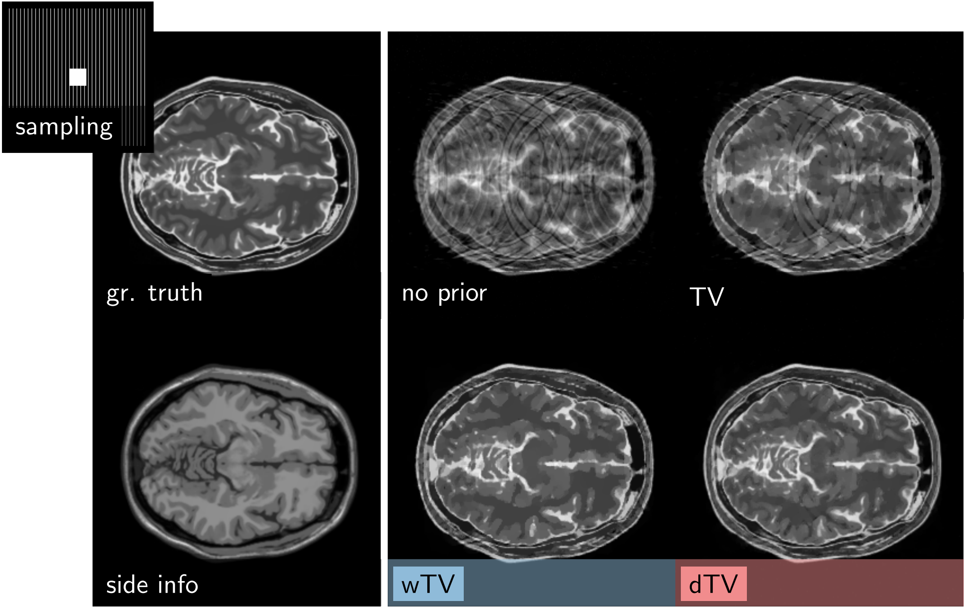


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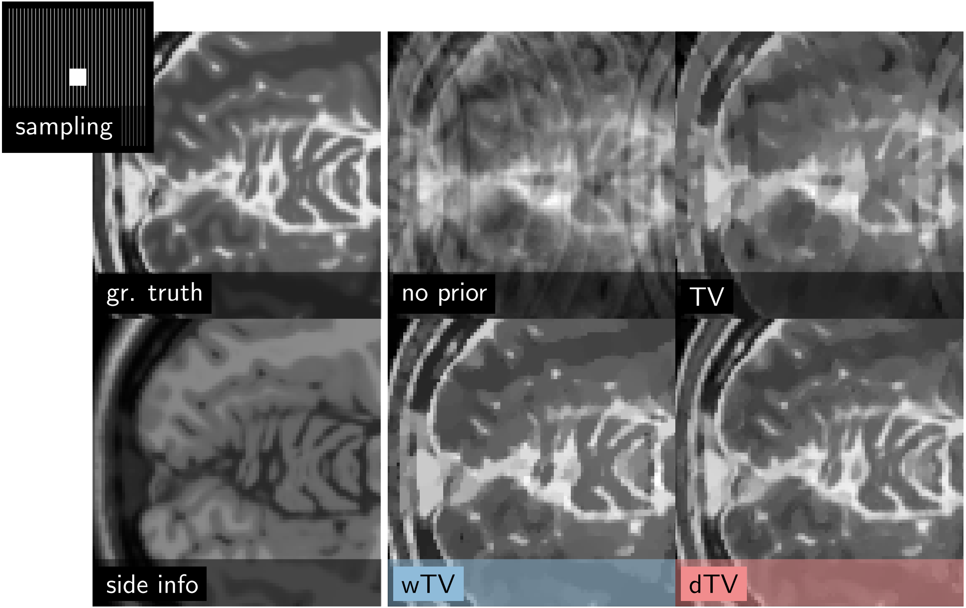


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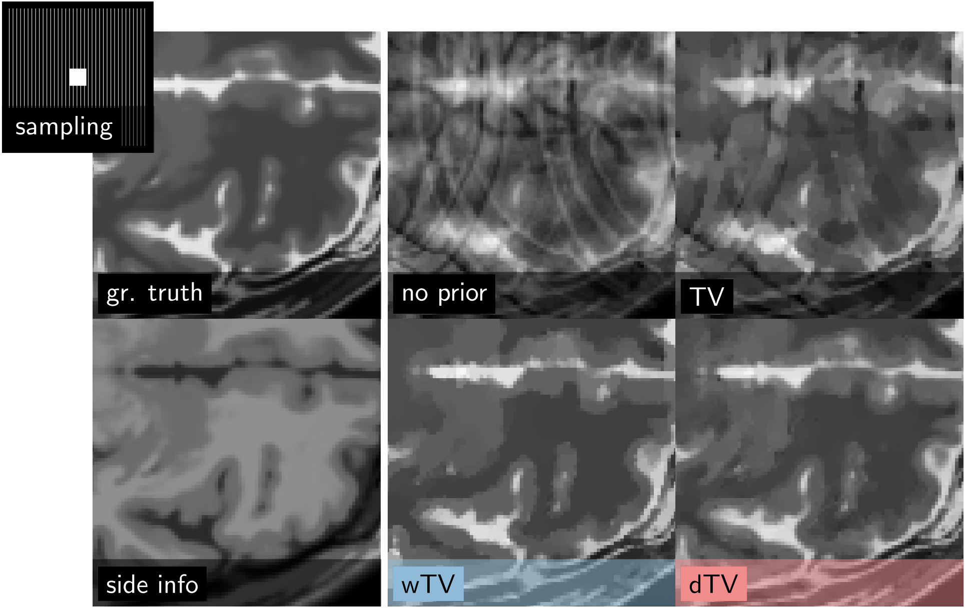

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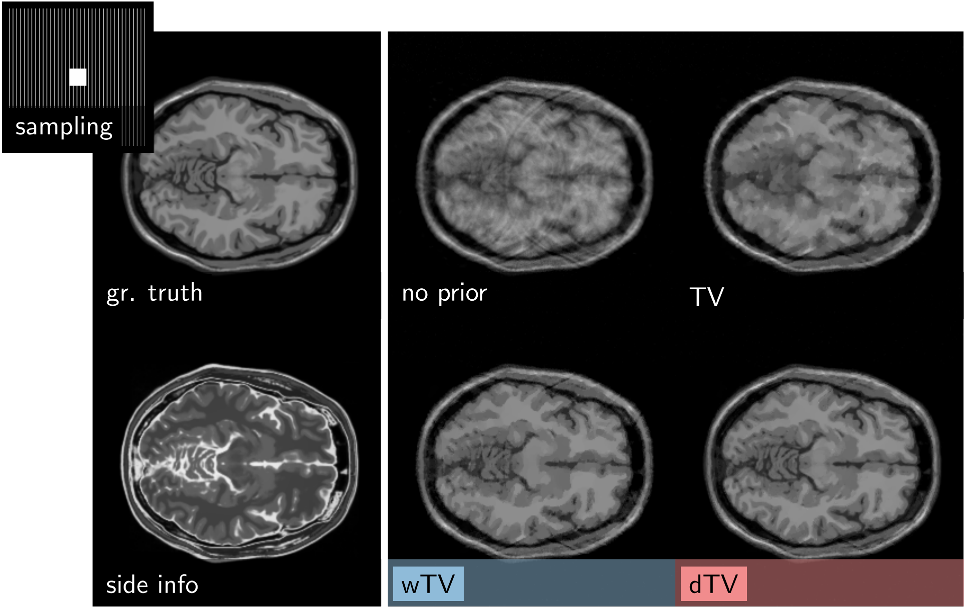

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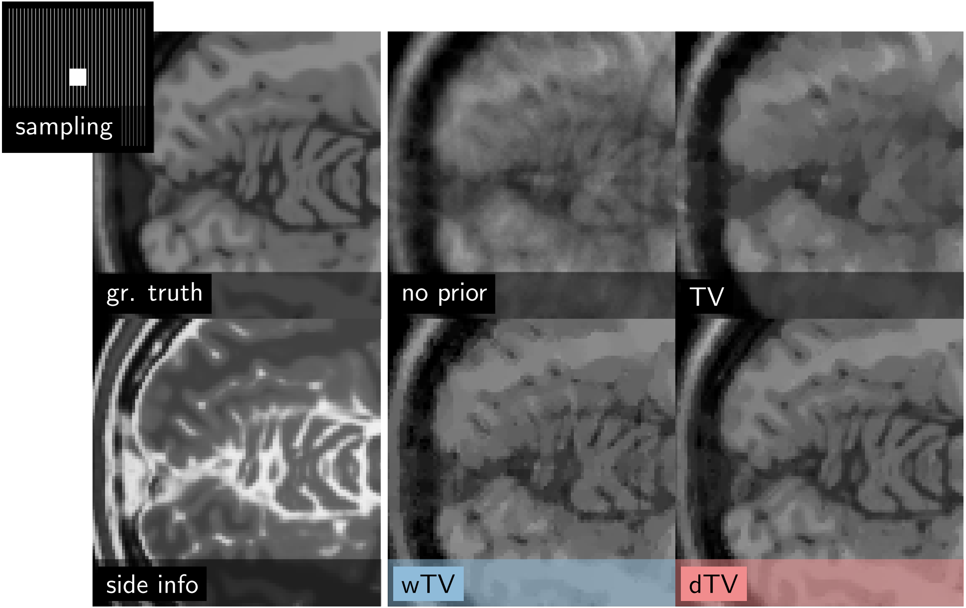


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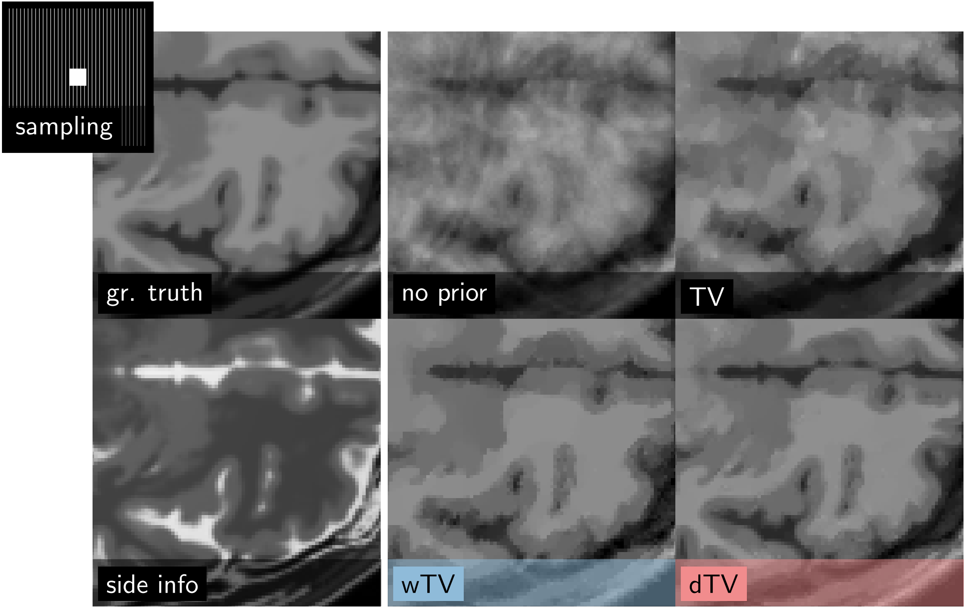
# Results

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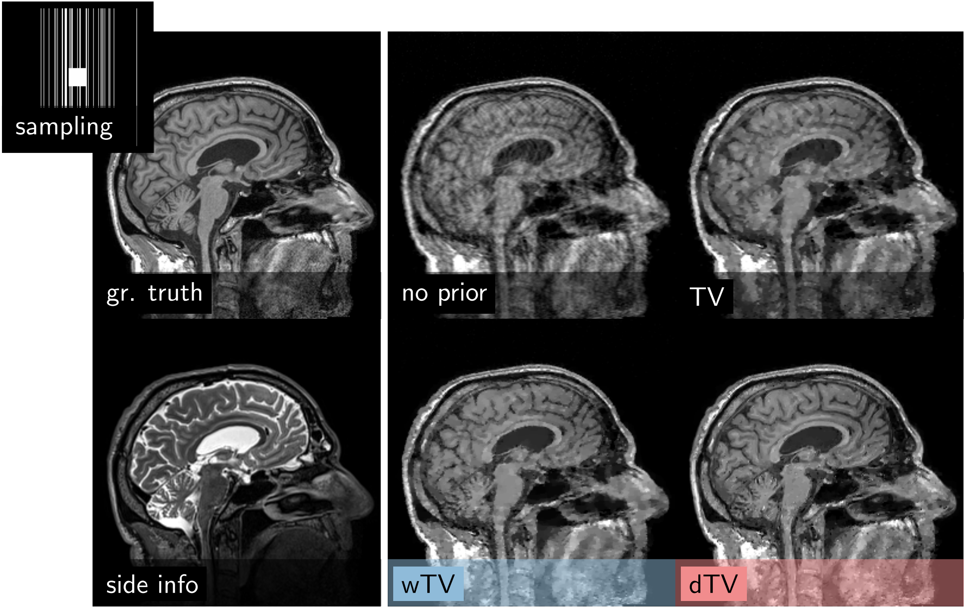


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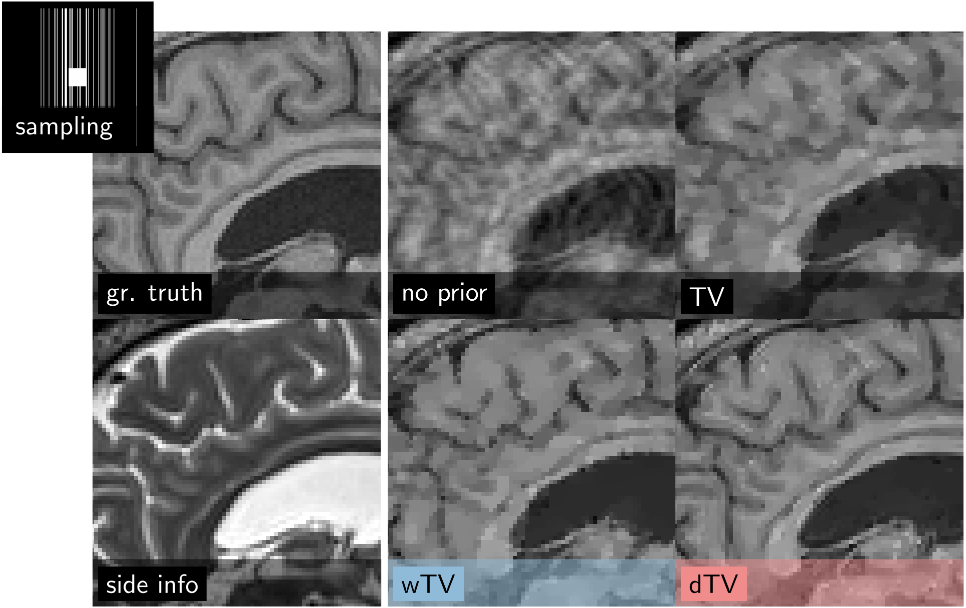

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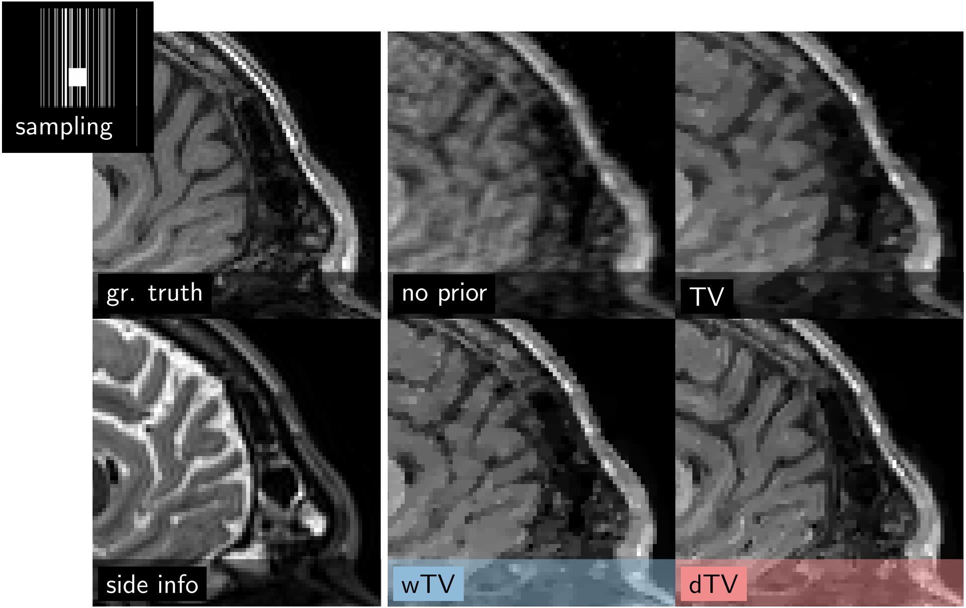

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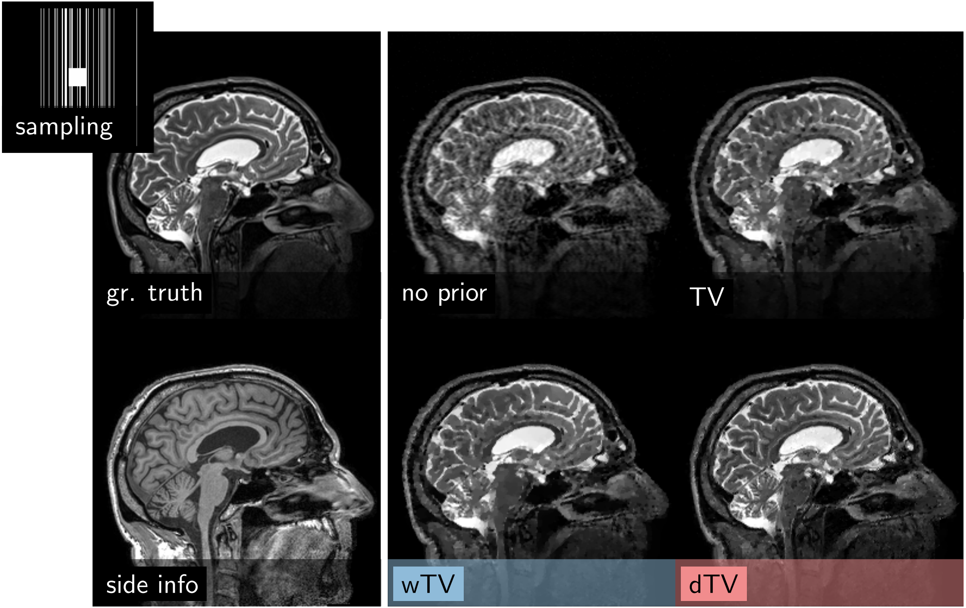

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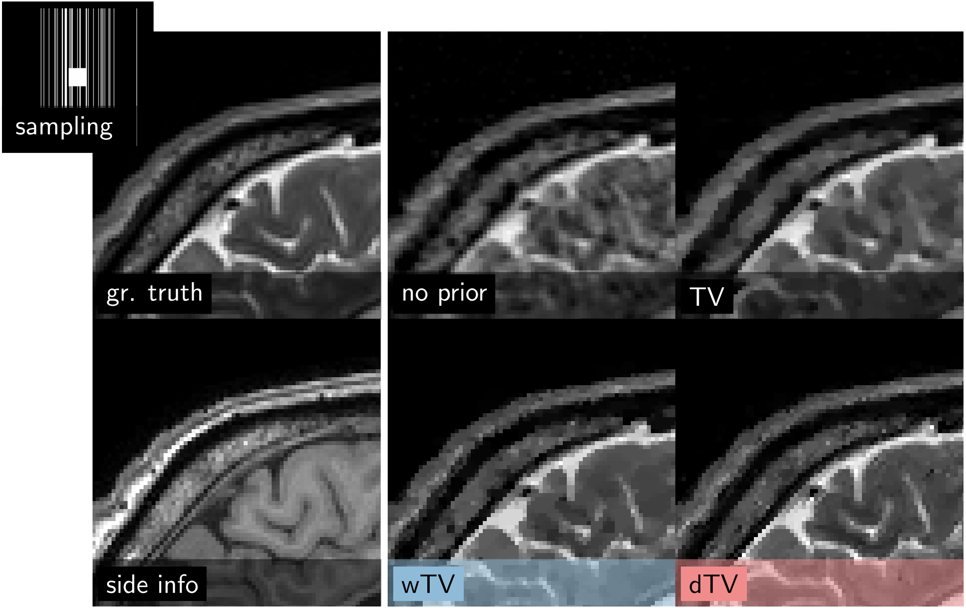

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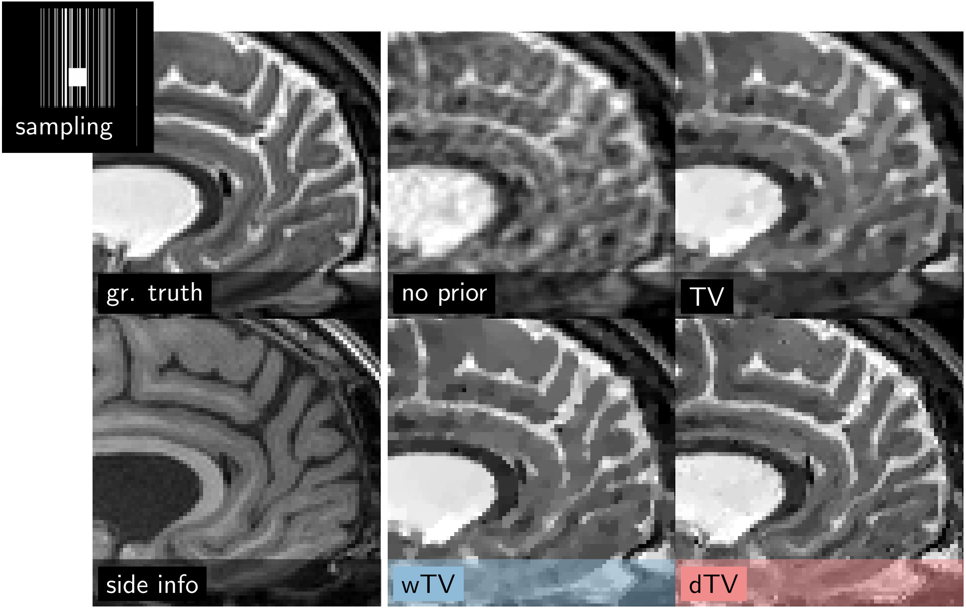

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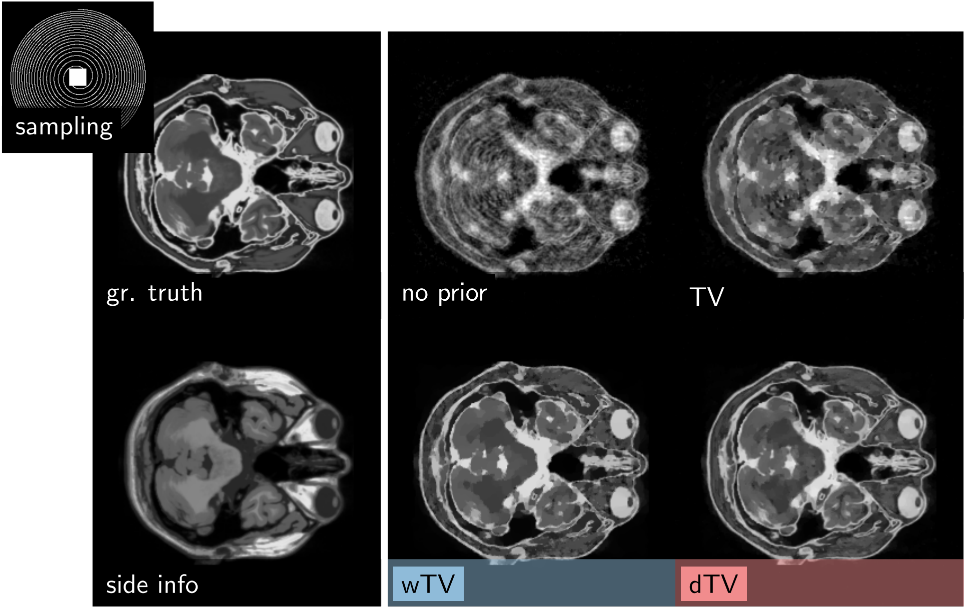


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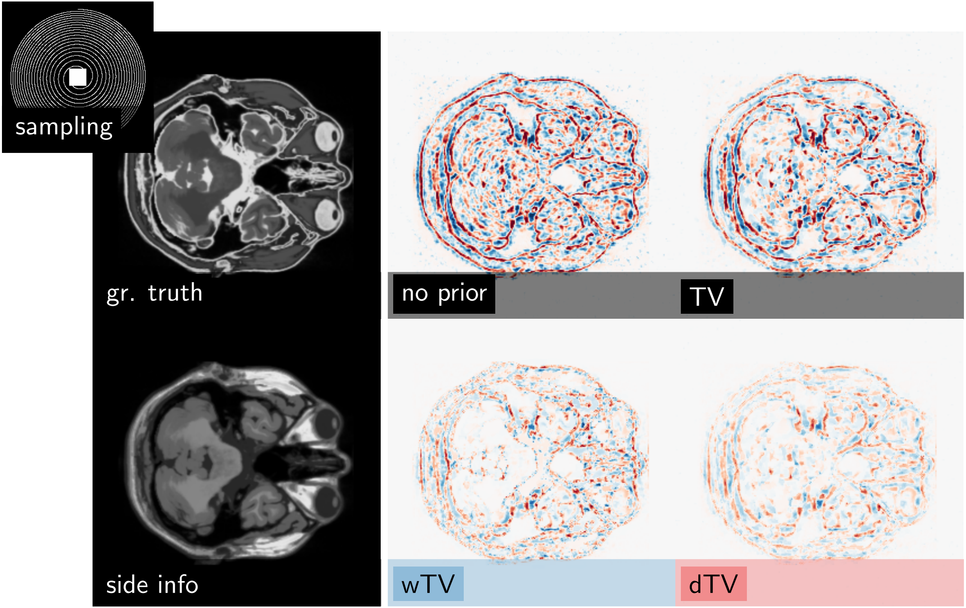
# Results



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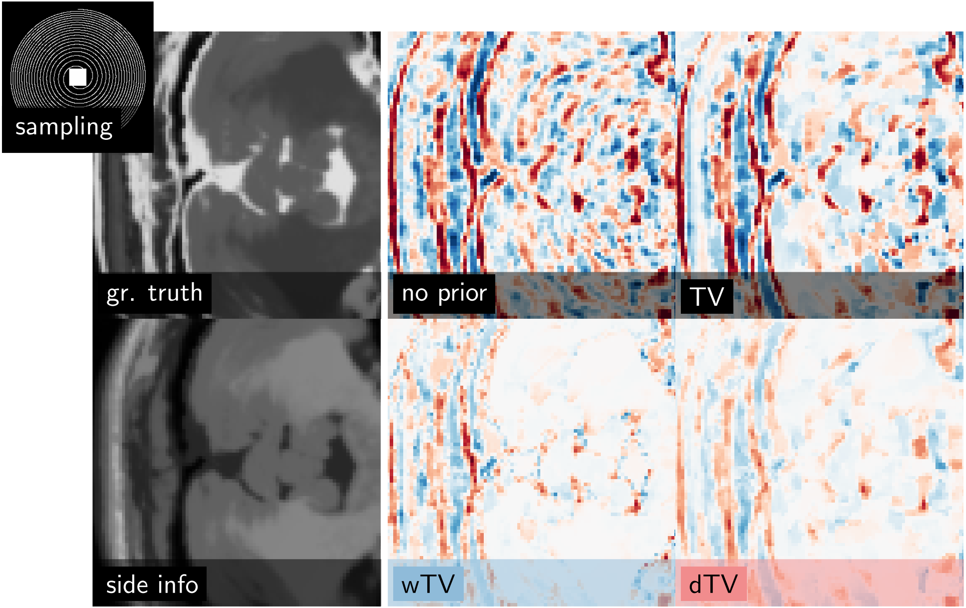

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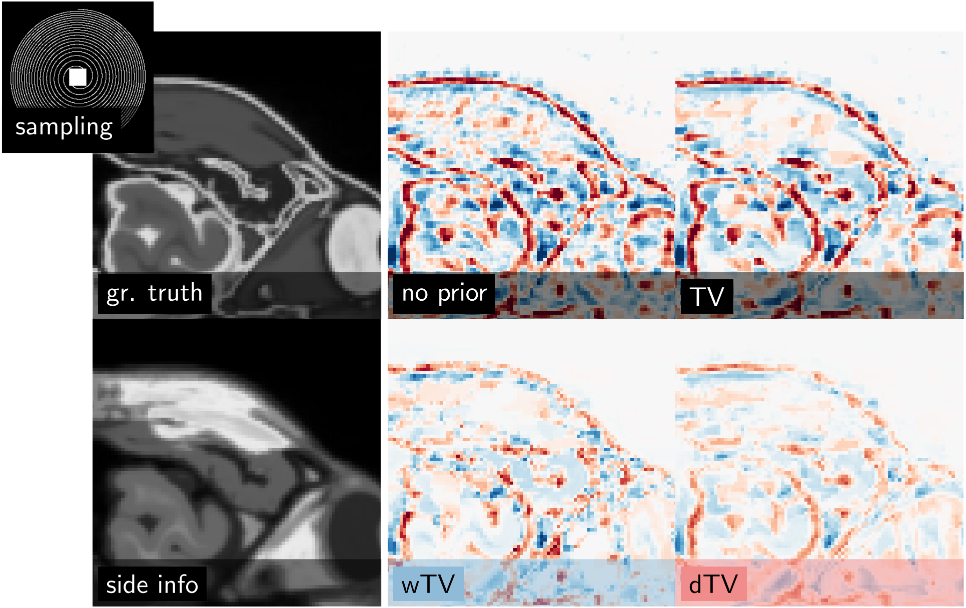

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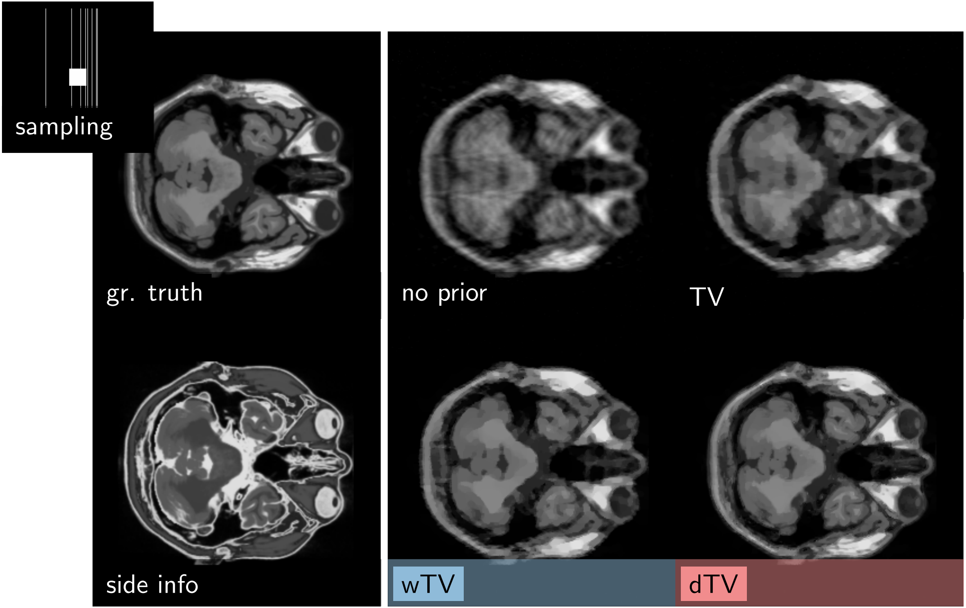


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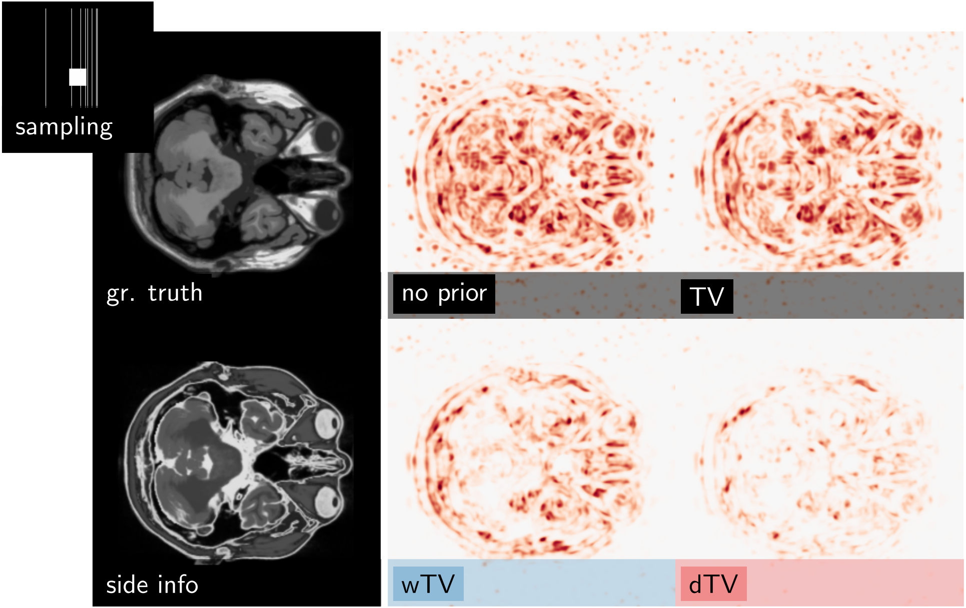


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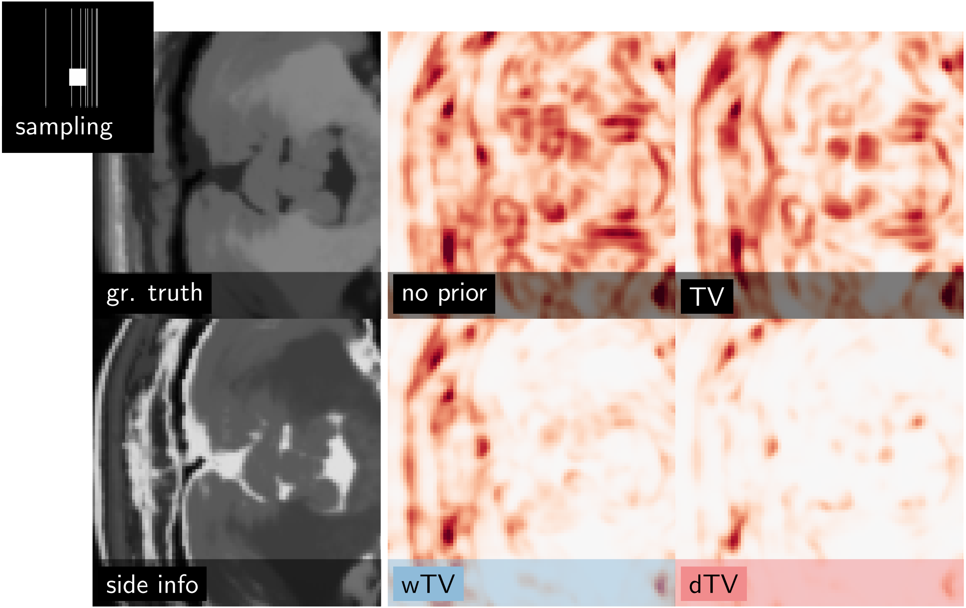
# Results



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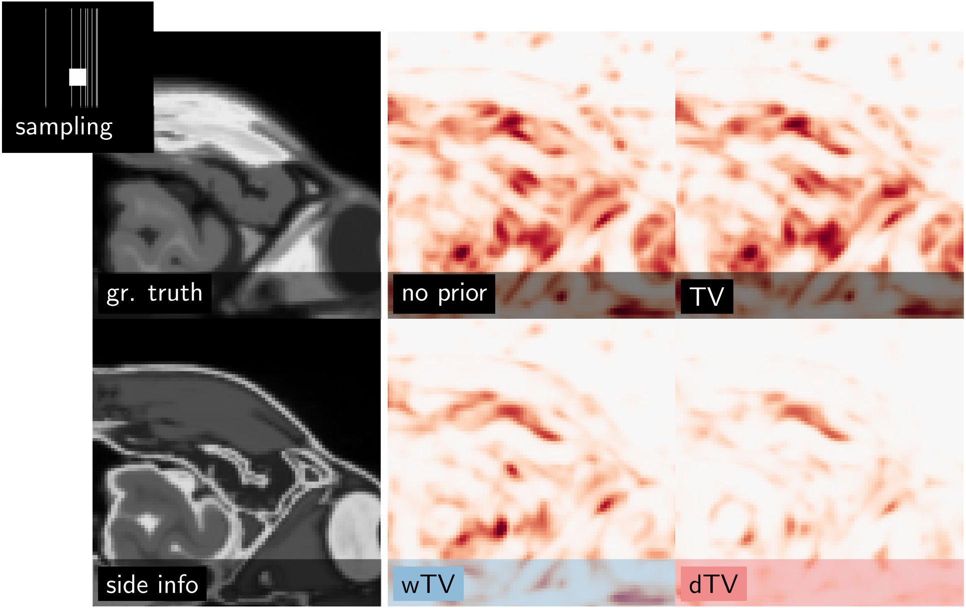
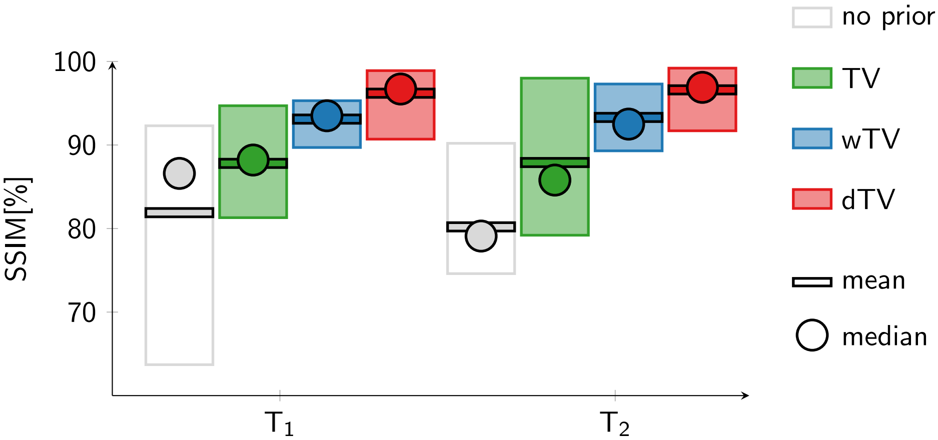


# Quantitative Results




- Range (min, max), mean and median over 12 data sets

# Conclusions

**Summary**

- extend total variation to include structural side information
- structure based on location and direction
- efficient convex optimization
- Results show it is beneficial to use side information; direction better than location

**Future**

- extend to joint reconstruction
- extend to more than two contrasts