# Simulation of an elastic sheet interaction with a non-Newtonian fluid by the LB-based immersed boundary method in 3D

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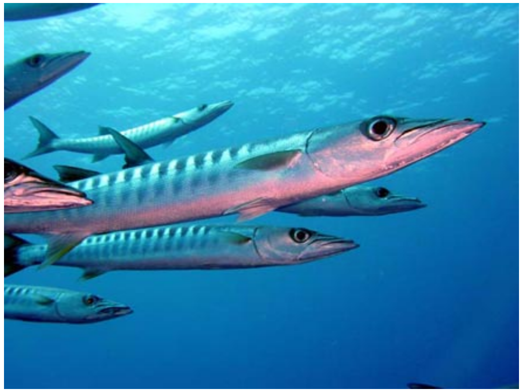
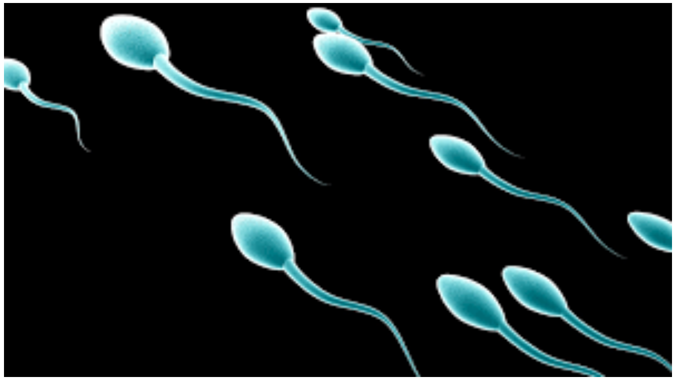
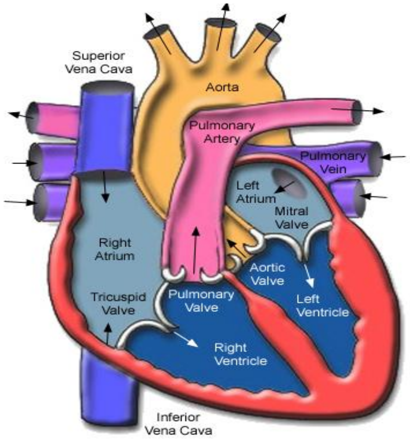
# Talk Schedule

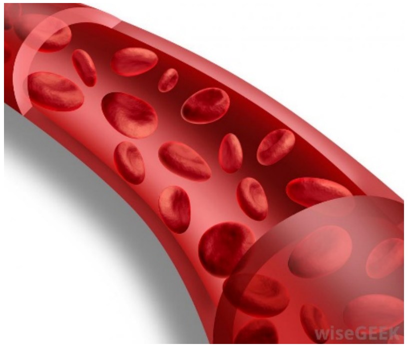
- Background
- Brief introduction of the LB Method
- Couple the LB method to the IB method for non-Newtonian fluid in 3D
- Some results of interaction of an elastic sheet with a non-Newtonian fluid 3D flow
- Summary and future work

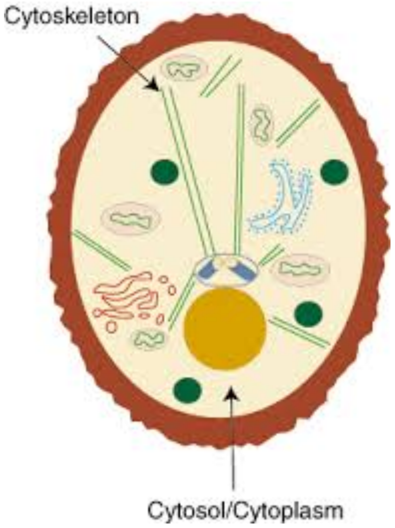
# Fluid-Flexible-Structure-Interaction







# Methods for Fluid-Structure-Interaction (incomplete list)

- ***Immersed Boundary Method***
- **C.S. Peskin, J. Comput. Phys. 25, pp.220 (1977)**
- **C.S. Peskin & D.M. McQueen, Contemp. Math. 141, pp.161 (1993).**
- **C.S. Peskin, Acta Numerica 11, 479 (2002).**
- **R. Mittal and G. Iaccarino, Annu. Rev. Fluid Mech. 37, pp. 239-261, (2005).**
- ***Immersed Interface Method***
- **R.J. LeVeque and Z.L. Li, SIAM J. Numer. Anal. 31, p.1019-1044 (1994).**
- **R.J. LeVeque and Z.L. Li, SIAM J. Sci. Comput. 18, p. 709-735 (1997).**
- **Z.L. Li and M.C. Lai, J. Comput. Phys. 171, p. 822-842 (2001).**
- **Z.L. Li, SIAM press, Philadelphia, (2006).**
- ***Immersed Finite Element***
- **L. Zhang, A. Gersternberger, X. Wang, and W.K. Liu, Comput. Methods Appl. Mech. Eng., 193 (2004).**
- **W.K. Liu, D.K. Kim, and S. Tang, Comput. Mech., DOI 10.1007/s00466-005-0018-5 (2005).**
- ***Material Point Method***
- **D.Sulsky, Z. Chen and H.L. Schreyer, Comput. Mech. Appl. Mech. Eng. 118, pp.179-197 (1994).**
- **D.Sulsky, S.J. Zhou and H.L. Schreyer, Comput. Phys. Commun. 87, pp.136-152 (1994).**
- ***Level Set Method***
- **T.Y. Hou, Z.L. Li, S. Osher, H.K. Zhao, J. Comp. Phys. 134, pp. 236-252 (1997).**
- **J. Xu, Z. Li, J. Lowengrub and H. Zhao, J. Comp. Phys. 212(2), pp. 590-616 (2006).**
- **G.H. Cottet and E. Maitre, C.R. Acad. Sci. Paris, Ser. I 338, pp.581-586 (2004).**
- **G.H. Cottet and E. Maitre, Math. Models & Methods in App. Sci. 16, pp. 415-438 (2006).**
- ***Fictitious Domain Method***
- **R. Glowinski, T. Pan, J. Periaux, Comp. Methods Appl. Mech. Eng. 111, (1994).**
- **R. Glowinski, T. Pan, J. Periaux, Comp. Methods Appl. Mech. Eng. 112 (1994).**
- **R. Glowinski, T. Pan, T. Hesla, D. Joseph, J. Periaux, J. Comput. Phys. 169, pp.363 (2001).**
- ***Arbitrary Lagrangian Eulerian Method***
- **T.J.R. Hughes, W. Liu, T.K. Zimmerman, Comput. Methods Appl. Mech. Eng. 29 (1981).**
- **J. Donea, S. Giuliani and J.P. Halleux, Comput. Methods Appl. Mech. Eng. 33, 689 (1982).**
- ***Method of Regularized Stokeslets*** **(R Cortez, SIAM J Sci Comput 2001)**

**Lattice Boltzmann method, Phase Filed Method, Front Tracking Method, ……**

**The immersed boundary method is originated by Charles Peskin (1992) and has become a popular practical and effective method for FSI problems**

- The boundary can be active (*beating heart, swimming sperm*) or passive (*flag-in-wind*)
- The boundary can be neutrally buoyant (*swimming fish*) or can have higher or lower density than surrounding fluids (*aggregated RBCs in flowing blood)*
- May be a body (eel, RBC) or a surface (flag, paper)
- May be open (flag) or closed (balloon)
- The boundary may be modeled by a collection of discrete elastic springs/fibers or by continuum/solid mechanics

# Different versions of the IB method (incomplete list)

**Original version** (Peskin 1972,1977,Peskin & McQueen 1993,1995,1996)

**Vortex-method version** (McCracken & Peskin 1980)

**Volume-conserved version** (Peskin & Printz 1993, Rosar & Peskin 2001)

**Adaptive mesh version** (Roma, Peskin & Berger 1999, Griffth & Peskin 2007)

**Second-order version** (Lai & Peskin 2000, Griffth & Peskin 2005)

**Multigrid version** (Fogelson & Zhu, Zhu & Peskin 2002)

**Penalty version** (Kim & Peskin 2006, Huang, Shin, Sung 2007, Huang Chang, Sung 2011)

**Stochastic version** (Atzberger, Kramer & Peskin 2006)

**Kirchhoff rod version** (Lim, Ferent, Wang & Peskin 2008)

**Viscoelastic fluid version** (Chrispell, Cortez, Khismatullin, Fauci 2011, Chrispell, Fauci, Shelley 2013)

**Implicit version** (Tu & Peskin 1992,Mayo & Peskin 1993, Fauci & Fogelson 1993,Taira & Colonius 2006,Mori & Peskin 2008, Hao & Zhu 2010,2011)

**Finite element version** (B Griffith & XY Luo 2014, Hua, Zhu & Lu 2015)

**Lattice Boltzmann version** (Zhu, He, et al. 2010)

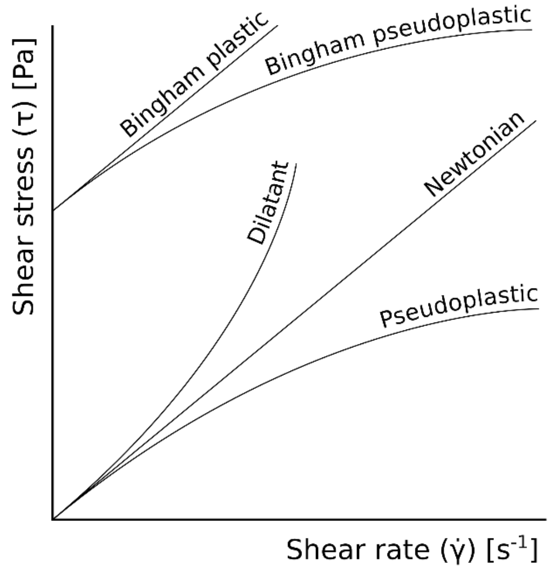
**.......**

# Non-Newtonian fluids are very common

- Natural substances: magma, lava, gums, extracts
- Slurries: cement slurry, paper pulp
- Human made: Soap solution, polymer solution, paint, cosmetics, toothpaste …
- Food: ketchup, jam, soup, yogurt
- Biological fluids: blood, cytoplasm, saliva, synovial fluid

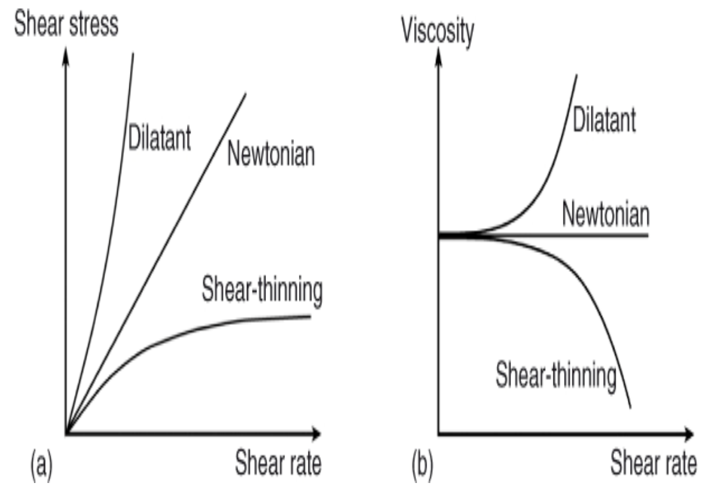
Newtonian: deviatoric stress $\boldsymbol{\tau}=2\mu\boldsymbol{D}$**,** the rate of strain $\mathbf{D}=1/2\,(\boldsymbol{u}_{i,j}+\boldsymbol{u}_{j,i})$

# Features of non-Newtonian fluids




- Shear-rate-dependent viscosity
- Normal stress differences
- History-dependent (memory effect)

# Power-law fluids: $\mu = m\,(\gamma)\ \uparrow n-1$




- n<1 Shear thinning: paint, ketchup, blood, cytoplasm
- n>1 Shear thickening: oobleck (cornstarch-water mixture)
- n=1 Newtonian

# N-S equations for non-Newtonian fluids by lattice Boltzmann method

**A) Why LB method for Navier-Stokes?**

1) Lattice Boltzmann method solves a series of scalar differential equations
2) Relationship between pressure and density
3) Clear physical interpretation of the scheme and easy handling of complex rigid boundary
4) Natural for parallelization
5)easier to model extra physics in a flow problem

**B) A brief introduction to the LB method**

**C) Lattice-Boltzmann based IB method**

# Some References for the LBM

**Books on the Lattice Boltzmann Method**:

D. A. Wolf-Gladrow,``Lattice-gas cellular automata and lattice Boltzmann Models -- an introduction'', Springer, Berlin, (2000).

S. Succi, ``The lattice Boltzmann equation'', Oxford Univ Press, Oxford (2001)

M.C. Sukop and D.T. Thorne, Jr., ``Lattice Boltzmann Modeling: an introduction for geoscientists and engineers'', Springer, Berlin, (2006).

Zhaoli Guo and Cuguang Zheng, ``Theory and applications of Lattice Boltzmann method'', Chinese Science Publisher, Beijing (2008).

Zhaoli Guo and Chang Shu, ``Lattice Boltzmann Method and its Applications in Engineering", World Scientific (2013).

Haibo Huang, Michael Sukop, and Xiyun Lu, ``Multiphase Lattice Boltzmann Methods: Theory and applications'', Wiley-Blackwell (2015)

**Widely cited paper on the LBM**:

L.-S. Luo. ``Unified Theory of the lattice Boltzmann models for nonideal gases'', Phys. Rev. Lett. 81: 1618 (1998).

S.Y. Chen, G.D. Doolen, ``Lattice Boltzmann Method for fluid flows'', *Annu Rev. Fluid Mech*., **30**, p329, (1998).

**LBM is a fast growing area**

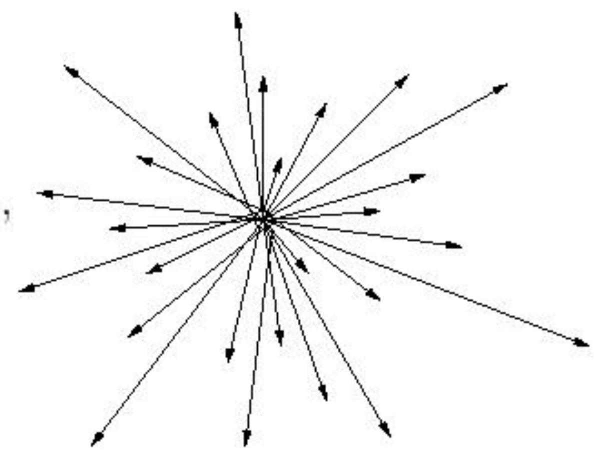
## I) Single Component LBM

**Boltzmann Equation (1872), PDE for velocity distribution f(x, $\boldsymbol{\xi}$, t)**

$$\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}+\frac{\mathbf{F}(\mathbf{x},t)}{m}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \boldsymbol{\xi}}=Q(f,f)$$

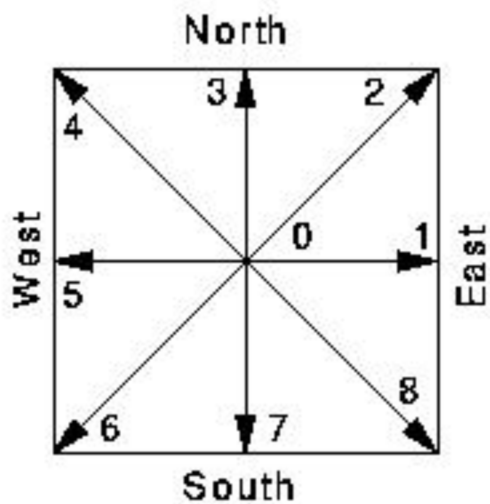
**BGK model, 1954**

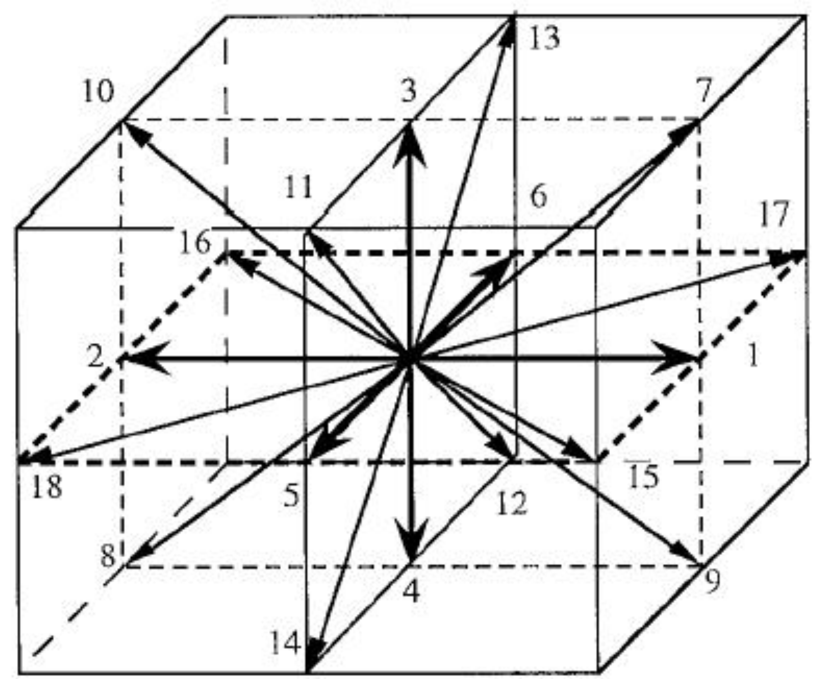
$$\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}=-\frac{1}{\tau}(f(\mathbf{x},\boldsymbol{\xi},t)-f^0(\mathbf{x},\boldsymbol{\xi},t)),$$



$\boldsymbol{\xi}$ **can be discretized by** $\{\boldsymbol{\xi}_j, j=0,1,2,...n.\}$

# Two widely used lattice Boltzmann models (left: D2Q9  right:D3Q19)



Discrete lattice BGK model

$$\frac{\partial f_j(\mathbf{x},t)}{\partial t} + \boldsymbol{\xi}_j \cdot \frac{\partial f_j(\mathbf{x},t)}{\partial \mathbf{x}} = -\frac{1}{\tau}(f_j(\mathbf{x},t) - f_j^0(\mathbf{x},t))$$

Discretization in time

$$\boldsymbol{\xi} = \frac{d\mathbf{x}}{dt}$$

$$\frac{df(\mathbf{x},t)}{dt} = -\frac{1}{\tau}(f(\mathbf{x},t) - f^0(\mathbf{x},t))$$

The lattice Boltzmann equation (LBE)

$$f_j(\mathbf{x} + \boldsymbol{\xi}_j, t+1) = f_j(\mathbf{x},t) - \frac{1}{\tau}(f_j(\mathbf{x},t) - f_j^0(\mathbf{x},t))$$

$$\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}+\frac{\mathbf{F}(\mathbf{x},t)}{m}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \boldsymbol{\xi}}=Q(f,f)$$

Simplified Boltzmann equation by BGK

$$\tfrac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\tfrac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}=-\tfrac{1}{\tau}(f(\mathbf{x},\boldsymbol{\xi},t)-f^0(\mathbf{x},\boldsymbol{\xi},t))$$

Discrete lattice BGK equation

$$\tfrac{\partial f_j(\mathbf{x},t)}{\partial t}+\boldsymbol{\xi}_j\cdot\tfrac{\partial f_j(\mathbf{x},t)}{\partial \mathbf{x}}=-\tfrac{1}{\tau}(f_j(\mathbf{x},t)-f_j^0(\mathbf{x},t))$$

The lattice Boltzmann equation (LBE)

$$f_j(\mathbf{x}+\boldsymbol{\xi}_j,t+1)=f_j(\mathbf{x},t)-\tfrac{1}{\tau}(f_j(\mathbf{x},t)-f_j^0(\mathbf{x},t))$$

# Physical interpretation of lattice Boltzmann method

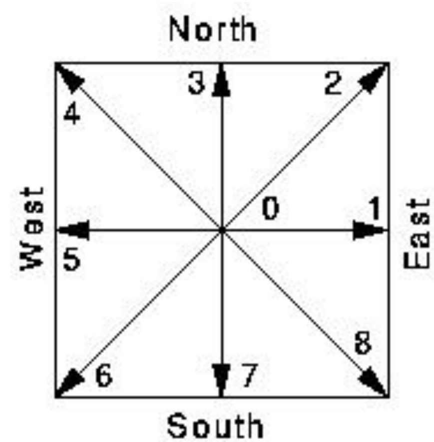
$$f_j(\mathbf{x} + \boldsymbol{\xi}_j, t+1) = f_j(\mathbf{x}, t) - \frac{1}{\tau}(f_j(\mathbf{x}, t) - f_j^0(\mathbf{x}, t))$$

1) Collision

$$f_j^*(\mathbf{x}, t) = f_j(\mathbf{x}, t) - \frac{1}{\tau}(f_j(\mathbf{x}, t) - f_j^0(\mathbf{x}, t))$$

2) Streaming

$$f_j(\mathbf{x} + \boldsymbol{\xi}_j, t+1) = f_j^*(\mathbf{x}, t)$$




3)no-slip BC by bounce-back

# Literature on using LB method in the IB method

Z.G. Feng and E.E. Michaelides, ``The immersed boundary-lattice Boltzmann method for solving fluid-particles interaction problems'', *J. Comput.Phys*. 195, 602-628 (2004).

Z.G. Feng and E.E. Michaelides, ``Proteus: a direct forcing method in the simulations of particulate flows'', *J. Comput. Phys*. 202, 20-51 (2005).

X.D. Niu, C. Shu, Y.T. Chew and Y. Peng,``A momentum exchange-based immersed boundary-lattice Boltzmann method for simulating incompressible viscous flows'', *Physics Letters A*, 354, p.173-182 (2006).

Y.Peng, C. Shu, Y.T. Chew, X.D. Niu, and X.Y. Lu,``Application of multi-block approach in the immersed boundary-lattice Boltzmann method for viscous fluid flows'', *J. Comp. Phys*., 218, p.460-478 (2006).

Y. Sui, Y.T. Chew, P. Roy and H.T. Low,``A hybrid immersed boundary and multi-block lattice Boltzmann method for simulating fluid and moving-boundaries interactions'', *Int. J. Numer. Meth. Fluids*, 53, p.1727-1754 (2007).

Y. Peng and L.-S. Luo, "A comparative study of immersed-boundary and interpolated bounce-back methods in LBE", Prog. in Comput. Fluid. Dyn. 8 (1-4), pp.156-167 (2008).

Zhu, He, et al. 2010, Hao & Zhu 2010,2011 (both explicit and implicit)

X Wang, C. Shu, J. Wu & LM Yang, Computer & Fluids, 100, pp. 165-175 (2014)

# Some features of our LB-IB method

1) Our math formulation is for any deformable immersed boundary whose motion is governed by LB equation; while in other existing works, the math formulation is for rigid particle/object whose motions are either prescribed or governed by the Newton's 2nd law.
2) Our formulation is for both Newtonian and non-Newtonian fluid-deformable structure interaction; existing works are for Newtonian flows.

# IB formulation by the LBM

$$\frac{\partial g(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial g(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}+\mathbf{f}(\mathbf{x},t)\cdot\frac{\partial g(\mathbf{x},\boldsymbol{\xi},t)}{\partial \boldsymbol{\xi}}=-\frac{1}{\tau}(g(\mathbf{x},\boldsymbol{\xi},t)-g^{(0)}(\mathbf{x},\boldsymbol{\xi},t)),$$

$$\rho(\mathbf{x},t)=\int g(\mathbf{x},\boldsymbol{\xi},t)d\boldsymbol{\xi},$$

$$(\rho\mathbf{u})(\mathbf{x},t)=\int g(\mathbf{x},\boldsymbol{\xi},t)\boldsymbol{\xi}d\boldsymbol{\xi}$$

$$\mathbf{F}(\boldsymbol{\alpha},t)=-\frac{\partial\mathcal{E}}{\partial\mathbf{X}}=-\frac{\partial(\mathcal{E}_s+\mathcal{E}_b)}{\partial\mathbf{X}}$$

$$\mathbf{f}_{ib}(\mathbf{x},t)=\int\mathbf{F}(\boldsymbol{\alpha},t)\delta(\mathbf{x}-\mathbf{X}(\boldsymbol{\alpha},t))d\boldsymbol{\alpha}$$

$$\mathbf{U}(\boldsymbol{\alpha},t)=\int\mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(\boldsymbol{\alpha},t))d\mathbf{x}$$

$$\frac{\partial\mathbf{X}}{\partial t}(\boldsymbol{\alpha},t)=\mathbf{U}(\boldsymbol{\alpha},t)$$

# Incorporate power-law in LB

- $\nu = 2\tau - 1/6$
- $\mu = m\,(\gamma)^{n-1}$
- $\gamma = \sqrt{2 S_{\alpha\beta} S_{\beta\alpha}}$
- $S_{\alpha\beta} = -3/2\tau \sum_{i=0}^{i=18} e_{i\alpha} e_{i\beta} f_i^{(1)}$
- $f_i^{(1)} = f_i - f_i^{(0)}$

# Algorithm of the 3D IB method by the LBM

$\mathbf{X}^n$ $\quad g_j(\mathbf{x}+\boldsymbol{\xi}_j, t+1) = g_j(\mathbf{x}, t) - \frac{1}{\tau}(g_j(\mathbf{x}, t) - g_j^0(\mathbf{x}, t))$

$\downarrow$

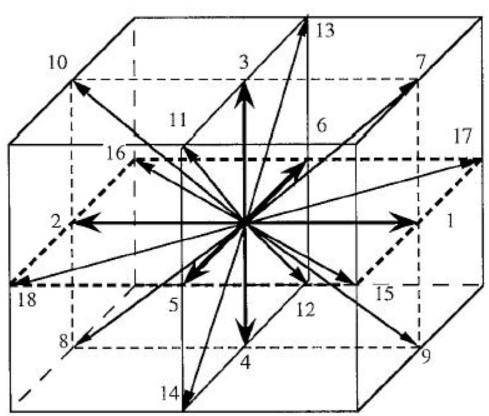
$\boldsymbol{\varepsilon}^n$ $\quad + (1 - \frac{1}{2\tau}) w_j (\frac{\boldsymbol{\xi}_j - \boldsymbol{u}^n}{c_s^2} + \frac{\boldsymbol{\xi}_j \cdot \boldsymbol{u}^n}{c_s^4}\, \boldsymbol{\xi}_j) \cdot \boldsymbol{f}^n$

$\downarrow$

$\mathbf{F}^n$

$\downarrow$

$$\mathbf{f}^n = \int \mathbf{F}^n\, \delta(\mathbf{x} - \mathbf{X}^n)\, d\alpha$$




$\mathrm{u}^{n+1}, \mathrm{p}^{n+1}$

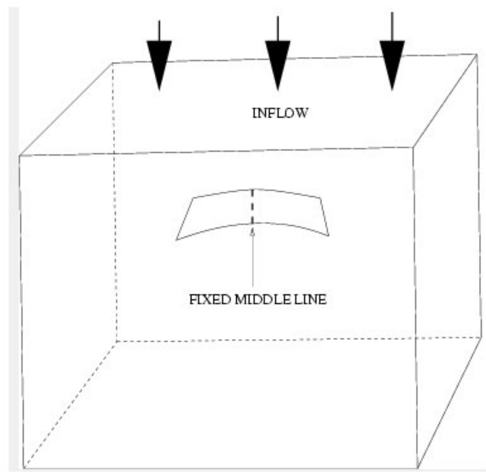
$\downarrow$

$$\mathbf{U}^{n+1} = \int \mathrm{u}^{n+1}\, \delta(\mathbf{x} - \mathbf{X}^n)\, d\mathbf{x}$$

$\downarrow$

$\mathbf{X}^{n+1}$

# A non-Newtonian viscous flow past an elastic sheet fixed at the midline



# Some Computational Results

- 1)Cd versus n: μ = m $(\gamma)\uparrow n-1$

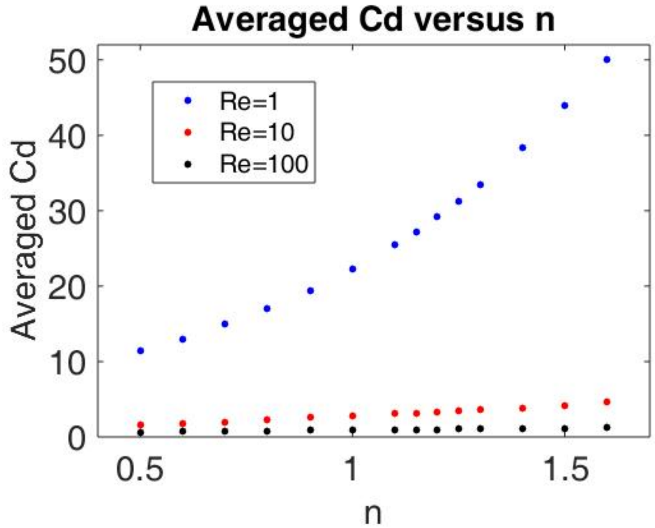
- 2)Cd versus Re = $\rho U\uparrow 2-n\, L\uparrow n\,/m$

- 3)Drag Scaling (D versus $\eta$)

  D=Cd $\eta\uparrow 2$ $\qquad\qquad$ $\eta=1/\sqrt{K\downarrow b}$

- 4)Short movies

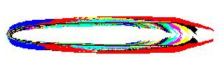
# Cd versus n (top) and Shape versus n (bottom) for Re = 1, 10, 100




Re =1

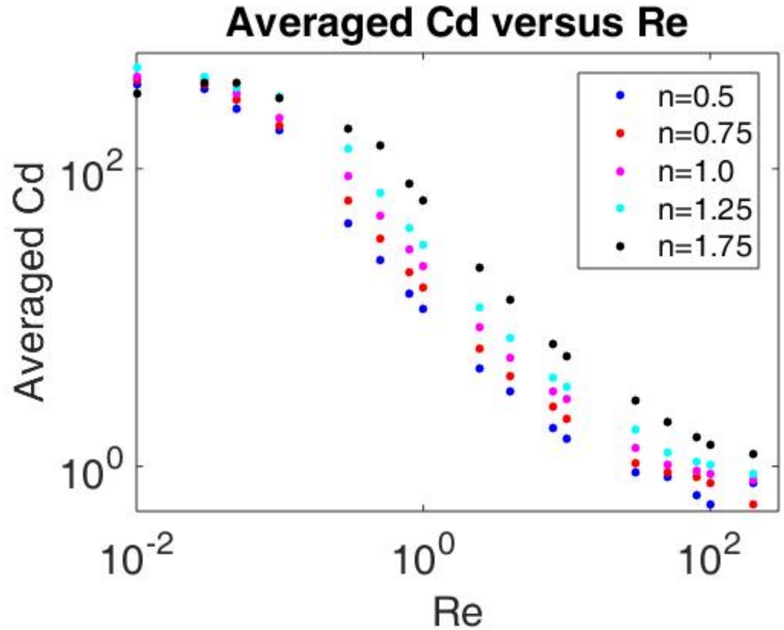
Re = 10

Re = 100



Cd increases with n; viscous force dominates drag.

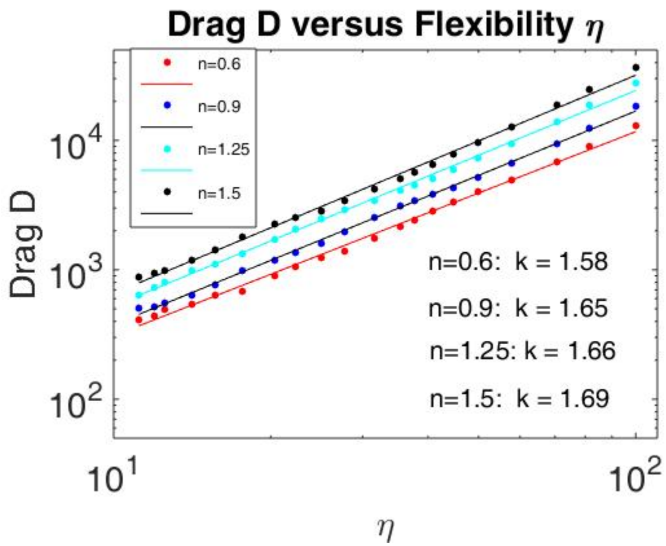
# Cd versus Re (top) and Shape versus Re (bottom) for five values of n




n=0.5 n=0.75 n=1.0 n=1.25 n=1.75

Cd decreases with Re; viscous force dominates drag

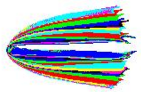
# Drag Scaling (top) and sheet Shape (bottom) for Re = 10




n=0.6

n=0.9

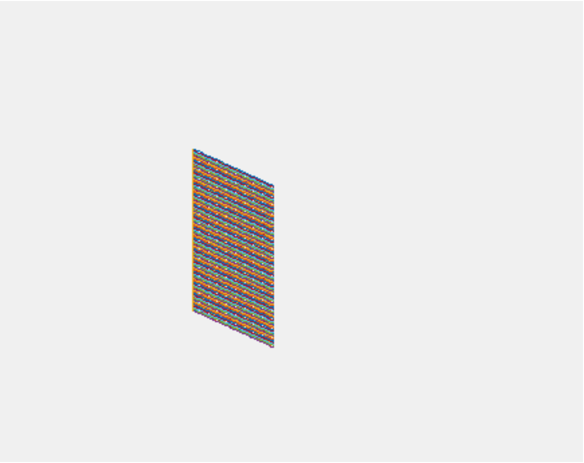
n=1.25



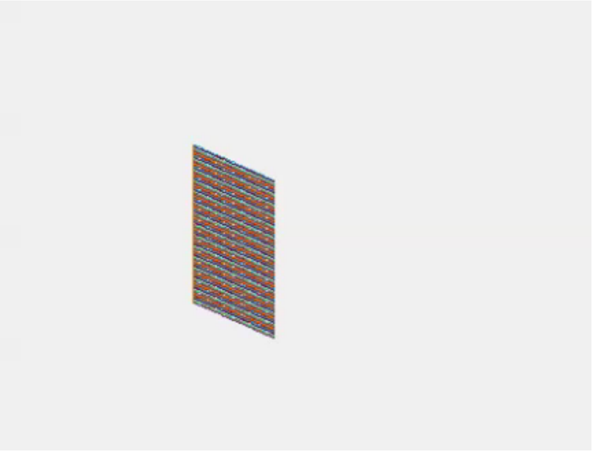
n=1.5

Total drag scales approximately as 1.6 power of inflow speed

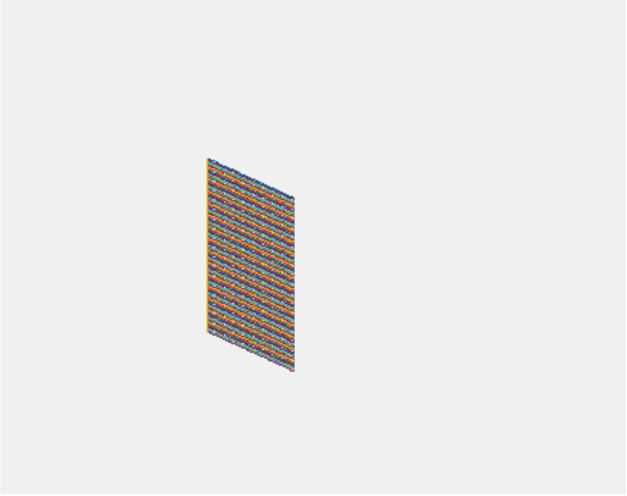
# Movies showing sheet deformation



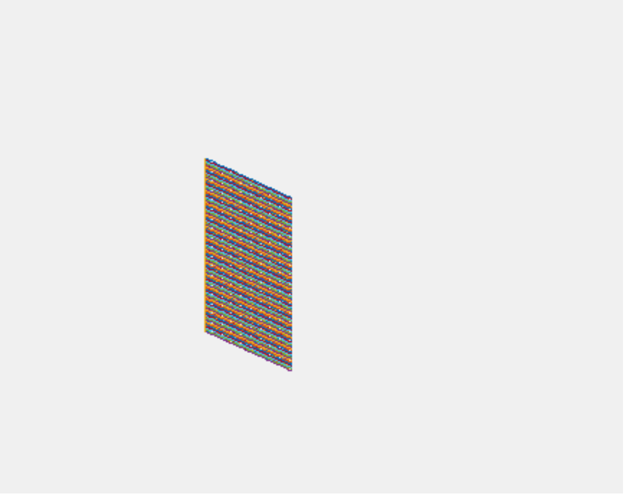
n=0.5



n=0.8



n=1.25



n=1.75

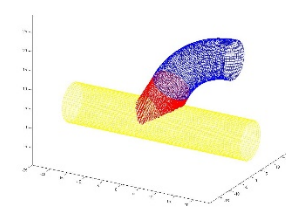
# Summary

1) We have successfully coupled the lattice Boltzmann D3Q19 model with power-law non-Newtonian fluids to the IB method in three dimensions.
2) Drag coefficient Cd increases with the exponent n and decreases with Re.
3) Viscous force dominates the drag of the sheet.
4) Total drag of the sheet scales approximately as 1.6 power of the inflow speed for Re=10, in contrast with the approximate 1 for a Newtonian fluid, and 2 for a rigid body in a Newtonian fluid.

# Ongoing and Future Work

1) Extension to non-Newtonian fluids described by the Oldroyd-B model and more generally the FENE-P model.
2) Spectral/hp elements to model the thin-walled structures with large deformation, large displacement, and large rotation, i.e. consider both material and geometry nonlinearities. (with S Dong and F Song)

3) Applications to blood flows during hemodialysis



# Acknowledgement

**Thanks to support from NSF (DMS-1522554).**

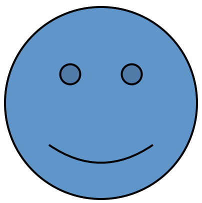
**Thanks to the organizers!**

**Thank you for your attentions!**

Happy Birthday, Charlie!

# THE END

## THANK YOU for your attention !



## Linearization of operators $F, S, U, I$

$\tilde{F}: \boldsymbol{X}^{n+1} \rightarrow \boldsymbol{F}^{n+1}$

$$(\mathbf{F}_s)_l = \frac{K_s}{\Delta\alpha_1} \sum_{m=1}^{n_f-1} (|\mathrm{X}_{m+1}^n - \mathrm{X}_m^n| - \Delta\alpha_1) \frac{\mathrm{X}_{m+1}^{n+1} - \mathrm{X}_m^{n+1}}{|\mathrm{X}_{m+1}^n - \mathrm{X}_m^n|} (\delta_{ml} - \delta_{m+1,l})$$

$\tilde{S}$: $\boldsymbol{f}^{n+1} = \int \boldsymbol{F}^{n+1} \delta(\mathrm{x} - \mathbf{X}^{\mathrm{n}})\, \mathrm{d}\boldsymbol{\alpha}$

$L$:

$$g_j(\mathbf{x}+\boldsymbol{\xi}_j, t+1) = g_j(\mathbf{x}, t) - \frac{1}{\tau}(g_j(\mathbf{x}, t) - g_j^0(\mathbf{x}, t)) + (1 - \frac{1}{2\tau}) w_j (\frac{\boldsymbol{\xi}_j - \boldsymbol{u}^n}{c_s^2} + \frac{\boldsymbol{\xi}_j \cdot \boldsymbol{u}^n}{c_s^4} \boldsymbol{\xi}_j) \cdot \boldsymbol{f}^{n+1}$$

$\tilde{U}$: $g_j^{n+1} \rightarrow \boldsymbol{u}^{n+1}$

$$\rho(\mathbf{x}, t+1) = \sum_j g_j^n(\mathbf{x}),$$

$$(\rho\mathbf{u})(\mathbf{x}, t+1) = \sum_j \boldsymbol{\xi}_j g_j^{n+1}(\mathbf{x}) + \frac{\mathbf{f}^{n+1}(\mathbf{x})}{2}.$$

$\tilde{I}$: $\boldsymbol{U}^{n+1} = \int \boldsymbol{u}^{n+1}\, \delta(\mathrm{x} - \boldsymbol{X}^n) \mathrm{dx}$

## A linear system of algebraic equations
($\Delta$t = 1 in the LBM)

$$\tilde{I}\tilde{U}L\tilde{S}\tilde{F}\boldsymbol{X}^{n+1} = \boldsymbol{X}^{n+1} - \boldsymbol{X}^n$$

# Summary of the implicit algorithm

$$IULSF\mathbf{X}^{n+1} = \mathbf{X}^{n+1} - \mathbf{X}^{n}$$

$$\tilde{I}\tilde{U}L\tilde{S}\tilde{F}\mathbf{X}^{n+1} = \mathbf{X}^{n+1} - \mathbf{X}^{n}$$

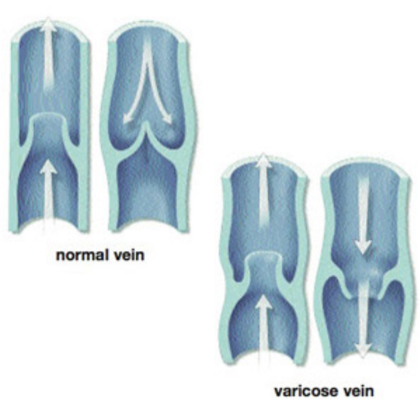
$$\mathbf{u}^{n+1}(\mathbf{x}, t) = ULSF\mathbf{X}^{n+1}$$

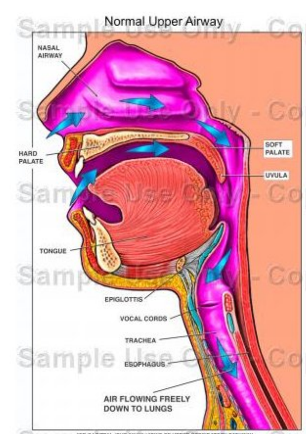
# Summary and future work

1) We have successfully coupled the lattice Boltzmann D3Q19 model to the IB method in three dimensions both explicitly and implicitly.

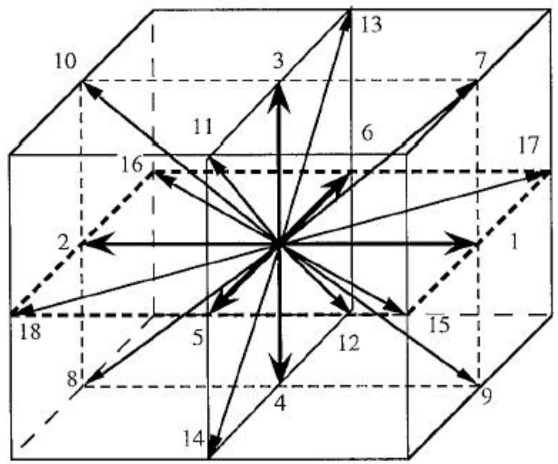
2) As an application of the hybrid method, we have found that the drag of a flexible sheet is approximately proportional to the inflow speed which is in contrast with the square law for a rigid body in a viscous flow.

3) Application of the hybrid method in biological flows such as flows past vein valves and soft plate in human airway.



# D3Q19 LB Model




$$\boldsymbol{\xi}_j = \begin{cases} (0,0,0), & j=0 \\ (\pm 1,0,0),(0,\pm 1,0),(0,0,\pm 1), & j=1,2,...,6 \\ (\pm 1,\pm 1,0),(\pm 1,0,\pm 1),(0,\pm 1,\pm 1), & j=7,8,...,18. \end{cases}$$

$$g_j(\mathbf{x}+\boldsymbol{\xi}_j, t+1) = g_j(\mathbf{x},t) - \frac{1}{\tau}(g_j(\mathbf{x},t) - g_j^0(\mathbf{x},t)) + (1-\frac{1}{2\tau})w_j(\frac{\boldsymbol{\xi}_j - \mathbf{u}}{{c_s}^2} + \frac{\boldsymbol{\xi}_j \cdot \mathbf{u}}{{c_s}^4}\boldsymbol{\xi}_j) \cdot \mathbf{f}$$

$$\mathbf{f}_{ib}^n(\mathbf{x}) = \sum_{\boldsymbol{\alpha}} \mathbf{F}^n(\boldsymbol{\alpha})\delta_h(\mathbf{x} - \mathbf{X}^n(\boldsymbol{\alpha}))\Delta\boldsymbol{\alpha}$$

$$g_j^0(\mathbf{x},t) = \rho(\mathbf{x},t)w_j(1 + 3\boldsymbol{\xi}_j \cdot \mathbf{u}(\mathbf{x},t) + \frac{9}{2}(\boldsymbol{\xi}_j \cdot \mathbf{u}(\mathbf{x},t))^2 - \frac{3}{2}\mathbf{u}(\mathbf{x},t) \cdot \mathbf{u}(\mathbf{x},t))$$

# Compute Lagrange Force

$$\mathcal{E}_s = \frac{1}{2}K_s \sum_m (|D_{\alpha_1}\mathrm{x}| - 1)^2 \Delta\alpha_1 = \frac{1}{2}K_s \sum_{m=1}^{n_f-1} (\frac{|\mathbf{X}_{m+1} - \mathbf{X}_m|}{\Delta\alpha_1} - 1)^2 \Delta\alpha_1$$

$$(\mathbf{F}_s)_l = \frac{K_s}{\Delta\alpha_1} \sum_{m=1}^{n_f-1} (|\mathbf{X}_{m+1} - \mathbf{X}_m| - \Delta\alpha_1) \frac{\mathbf{X}_{m+1} - \mathbf{X}_m}{|\mathbf{X}_{m+1} - \mathbf{X}_m|} (\delta_{ml} - \delta_{m+1,l})$$

$$\mathcal{E}_b = \frac{1}{2}K_b \sum_m |D_{\alpha_1} D_{\alpha_1}\mathrm{X}|^2 \Delta\alpha_1 = \frac{1}{2}K_b \sum_{m=2}^{n_f-1} [\frac{|\mathbf{X}_{m+1} + \mathbf{X}_{m-1} - 2\mathbf{X}_m|^2}{(\Delta\alpha_1)^4}] \Delta\alpha_1$$

$$(\mathbf{F}_b)_l = \frac{K_b}{(\Delta\alpha_1)^4} \sum_{m=2}^{n_f-1} (\mathbf{X}_{m+1} + \mathbf{X}_{m-1} - 2\mathbf{X}_m)(2\delta_{ml} - \delta_{m+1,l} - \delta_{m-1,l})$$

$$\delta_{ml} = \begin{cases} 1, & \text{if} \quad m = l, \\ 0, & \text{if} \quad m \neq l. \end{cases}$$

$$\rho(\mathbf{x},t) = \sum_j g_j(\mathbf{x},t),$$

$$(\rho\mathbf{u})(\mathbf{x},t) = \sum_j \boldsymbol{\xi}_j g_j(\mathbf{x},t) + \frac{\mathbf{f}(\mathbf{x},t)}{2}.$$

$$\mathbf{U}^{n+1}(\boldsymbol{\alpha}) = \sum_{\mathbf{x}} \mathbf{u}^{n+1}(\mathbf{x})\delta_h(\mathbf{x} - \mathbf{X}^n(\boldsymbol{\alpha}))h^3$$

$$\frac{\mathbf{X}^{n+1}(\boldsymbol{\alpha}) - \mathbf{X}^n(\boldsymbol{\alpha})}{\Delta t} = \mathbf{U}^{n+1}(\boldsymbol{\alpha})$$

# Physical interpretation of lattice Boltzmann method

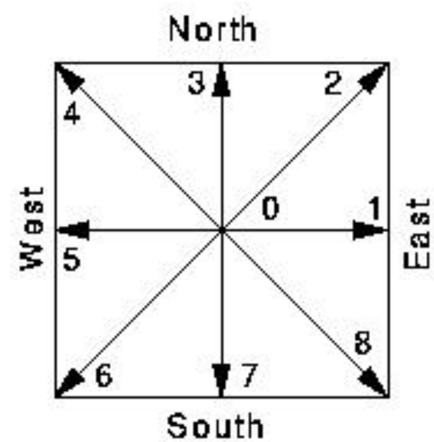
$$f_j(\mathbf{x}+\boldsymbol{\xi}_j, t+1) = f_j(\mathbf{x},t) - \frac{1}{\tau}(f_j(\mathbf{x},t) - f_j^0(\mathbf{x},t))$$

1) Collision

$$f_j^*(\mathbf{x},t) = f_j(\mathbf{x},t) - \frac{1}{\tau}(f_j(\mathbf{x},t) - f_j^0(\mathbf{x},t))$$

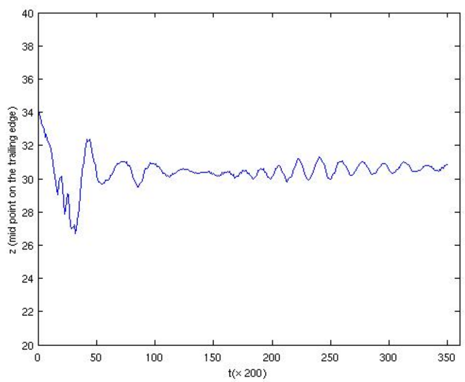
2) Streaming

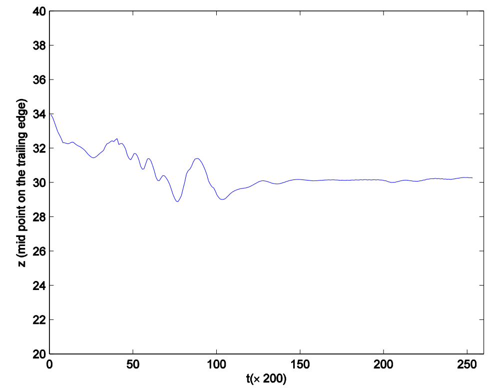
$$f_j(\mathbf{x}+\boldsymbol{\xi}_j, t+1) = f_j^*(\mathbf{x},t)$$

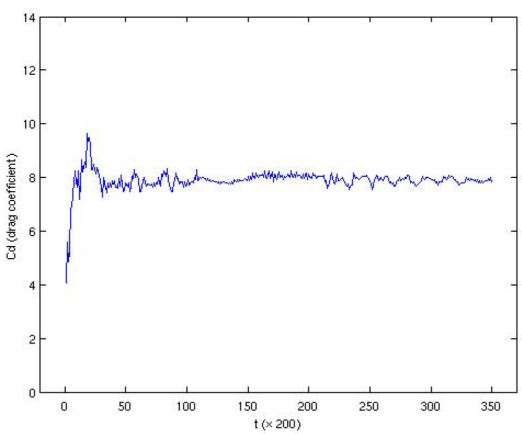


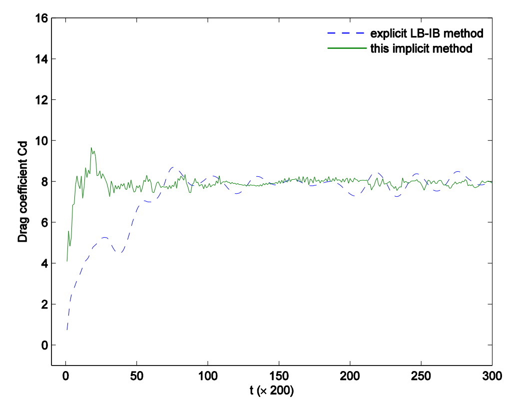
# 2) A flag flapping in wind -- simulated by the 3D implicit lattice Boltzmann based immersed boundary method

Position of trailing edge versus time (left &middle) and drag coefficient vs time (right)



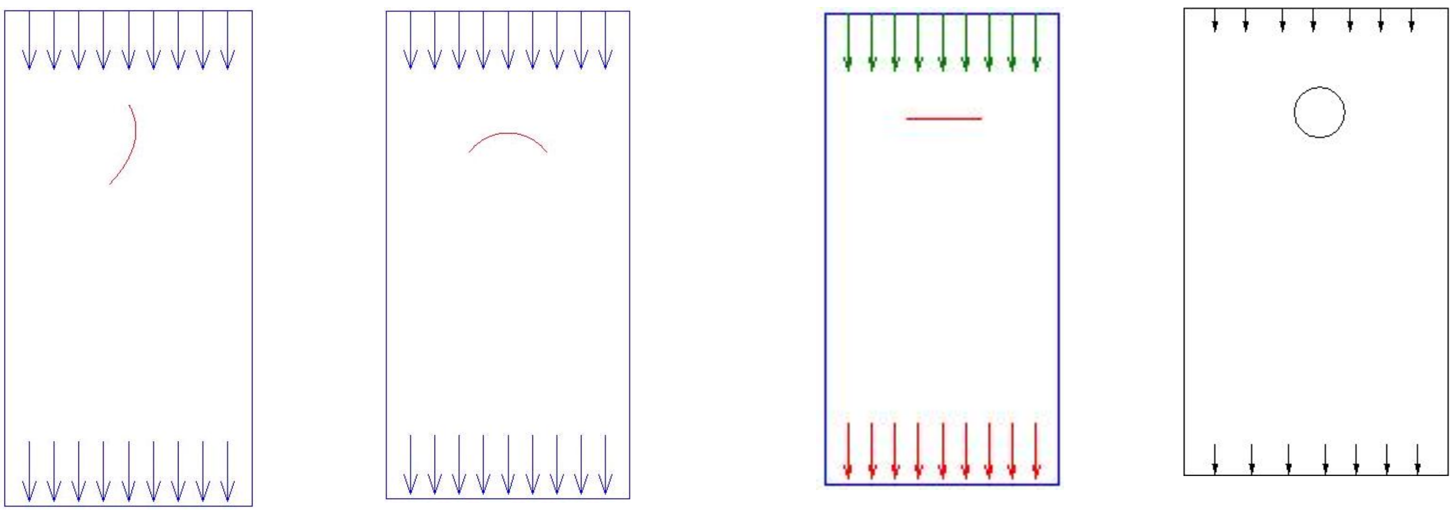



# An incomplete list of the application of the immersed boundary method (neutrally buoyant case)

- blood flow in the human heart (Peskin, McQueen)
- design of prosthetic cardiac valves (McQueen, Peskin, Yellin)
- platelet aggregation during blood clotting (Fogelson)
- cell and tissue deformation under shear flow (Bottino,Eggleton)
- wave propagation in the cochlea (Beyer)
- flow and transport in a renal arteriole (Arthurs et al.)
- aquatic animal locomotion (Fauci and Peskin)
- flow of suspensions (Fogelson,Peskin,Sulsky,Brackbill)
- valveless pumping (Jung and Peskin)
- flow in a collapsible tube (Rosar)
- ……

# Why Implicit IBM?



- E. Givelberg, Modeling elastic shells immersed in fluid, Comm. Pure Appl. Math. 57 (2004) 283309.

$$\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}+\frac{\mathbf{F}(\mathbf{x},t)}{m}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \boldsymbol{\xi}}=Q(f,f)$$

Simplified Boltzmann equation by BGK

$$\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial f(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}=-\frac{1}{\tau}(f(\mathbf{x},\boldsymbol{\xi},t)-f^{0}(\mathbf{x},\boldsymbol{\xi},t))$$

Discrete lattice BGK equation

$$\frac{\partial f_j(\mathbf{x},t)}{\partial t}+\boldsymbol{\xi}_j\cdot\frac{\partial f_j(\mathbf{x},t)}{\partial \mathbf{x}}=-\frac{1}{\tau}(f_j(\mathbf{x},t)-f_j^{0}(\mathbf{x},t))$$

The lattice Boltzmann equation (LBE)

$$f_j(\mathbf{x}+\boldsymbol{\xi}_j,t+1)=f_j(\mathbf{x},t)-\frac{1}{\tau}(f_j(\mathbf{x},t)-f_j^{0}(\mathbf{x},t))$$

# IB formulation by the LBM

$$\frac{\partial g(\mathbf{x},\boldsymbol{\xi},t)}{\partial t}+\boldsymbol{\xi}\cdot\frac{\partial g(\mathbf{x},\boldsymbol{\xi},t)}{\partial \mathbf{x}}+\mathbf{f}(\mathbf{x},t)\cdot\frac{\partial g(\mathbf{x},\boldsymbol{\xi},t)}{\partial \boldsymbol{\xi}}=-\frac{1}{\tau}(g(\mathbf{x},\boldsymbol{\xi},t)-g^{(0)}(\mathbf{x},\boldsymbol{\xi},t)),$$

$$\rho(\mathbf{x},t)=\int g(\mathbf{x},\boldsymbol{\xi},t)d\boldsymbol{\xi},$$

$$(\rho\mathbf{u})(\mathbf{x},t)=\int g(\mathbf{x},\boldsymbol{\xi},t)\boldsymbol{\xi}d\boldsymbol{\xi}$$

$$\mathbf{F}_d(\boldsymbol{\alpha},t)=-M(\boldsymbol{\alpha},t)\frac{\partial^2\mathbf{X}(\boldsymbol{\alpha},t)}{\partial t^2}$$

$$\mathbf{F}(\boldsymbol{\alpha},t)=-\frac{\partial\mathcal{E}}{\partial\mathbf{X}}=-\frac{\partial(\mathcal{E}_s+\mathcal{E}_b)}{\partial\mathbf{X}}$$

$$\mathbf{f}_{ib}(\mathbf{x},t)=\int\mathbf{F}(\boldsymbol{\alpha},t)\delta(\mathbf{x}-\mathbf{X}(\boldsymbol{\alpha},t))d\boldsymbol{\alpha}$$

$$\mathbf{U}(\boldsymbol{\alpha},t)=\int\mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(\boldsymbol{\alpha},t))d\mathbf{x}$$

$$\frac{\partial\mathbf{X}}{\partial t}(\boldsymbol{\alpha},t)=\mathbf{U}(\boldsymbol{\alpha},t)$$