Calculus, Topology and 3D Printing

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Enhancing Mathematical Learning with 3D printing. SIAM Conference, September 30, 2016.

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Topology

Calculus/Geometry

Current Projects

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Introduction to Topology, Fall 2014



Current Projects

Models for Teaching, Summer 2015





3D printing in the classroom

- 1. Make manipulatives and visualizations.
 - Volumes by slices, washers or cylindrical shells.
 - Quadratic surfaces, other volumes (triple integrals).
 - Curves, knots, other geometric surfaces.
- 2. Students design and 3D print objects.
 - Use 3D printing in a class project.
 - Models should enhance mathematical understanding.
 - Projects include a writing component.
 - (Future) Build a course around 3D printing.



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Basics of 3D print design

- 1. Think about model you want to create.
- 2. Model it in *Mathematica* (maybe).
- 3. Import into, or design model entirely in 3D design software.
- 4. Export .stl file.
- 5. Open file in 3D printer's software, choose size, scale, supports, print quality, etc, producing a print file.
- 6. File is then printed by 3D printer (or not).

Software

- *Mathematica*, Maple (both \$).
- TinkerCAD, OpenSCAD (both free).
- Cinema4D (\$), Rhinoceros (\$), Blender (free).
- Meshlab, TopMod, Grasshopper, others.

Set up at W&L

IQ (Integrative and Quantitative) Center in Science Center.

- Classroom spaces:
 - Stereo 3D lab,
 - Computer Visualization Lab.
- 3D printing (Physical/Mechanical Lab)
 - FormLabs Form 1+ printer,
 - ProJect 260 3D System,
 - MakerBot Replicator 2X, uPrint SE, Series 1 Pro.
- Secret weapon: Dave Pfaff.





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Introduction to Topology, Fall 2014

Project - week 5

- Each person (group) designs a topological/mathematical object that can be 3D printed.
- Write a 1 page description of
 - the mathematics of the object,
 - how the object was designed in 3D, and how any design problems were solved.
- Can't download already existing objects.
- Grade is the same as a weekly homework assignment.

Notes

- 3 class periods spent on the project.
- Project came after material on quotient spaces.
- Project was an excuse to introduce some knot theory.

Topology

Calculus/Geometry

Current Projects

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Introduction to Topology, Fall 2014



What I learned

- This was the students favorite part of the course!
- Students need time to learn to use software.
 - Need explicit (project related) instructions on how to use Cinema 4D.
- Project was too open ended.
 - Give a list of suggestions of objects to design.
- Mathematical description of object was often lacking.
 - Adjust the writing requirement.
- Objects needed extra editing for clean 3D design.





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Summer 2015

W&L Summer Research Scholars: Emily Jaekle ('16) and Ryan McDonnell ('17).

Together we designed and wrote:

46 math models, 11 instruction sheets, 9 *Mathematica* worksheets, many blog entries.



Topology

Calculus/Geometry

Current Projects

Volumes by slices



Topology

Calculus/Geometry

Current Projects

Quadratic & Steinmetz surfaces







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Volumes with equations

Model design

- Design object (in Mathematica or in Cinema 4D).
- Add equations in Cinema 4D.
 - Find appropriate font, and thicken equations.
 - Use Bend tool and Boole tool to add equations.



Explaining Double Integrals



Geometry of curves and surfaces





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Calculus II Fall 2016

Project: Volumes by slices.

- Students work in groups (3 people).
- 3 class periods.
- 3D modeling.
 - Learn how to use Cinema 4D by building a volume of revolution (disk/washer method).
 - Build a more complex volume illustrating: cylindrical shell method and/or a volume by slices.
- Writing. For each volume:
 - Give details of mathematics of each slice.
 - Describe any 3D design problems and their solutions.
 - Compute integral to find the volume.
- We will 3D print a subset of the models.

Multivariable Calculus Spring 2017

- Existing course requirement for students to be familiar with a program like Maple or *Mathematica*.
- Currently have 6 *Mathematica* labs over the semester.
- Mathematica 11 is 3D printer friendly (?!)
- Existing labs can be adjusted for 3D printing.
- Try 2 smaller 3D printing projects
 - Curves (either level curves or spatial curves)
 - Volumes for integration.
- Watch this space!



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More information

My webpage: Math Visualization

All of these models are found on my Thingiverse page.

My blog: Visions in Math.



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Thank you

Dave Pfaff & the WLU IQ center,

and

Emily Jaekle ('15) & Ryan McDonnell ('16),

and

My Fall 2014 Math 341 Introduction to Topology class.

