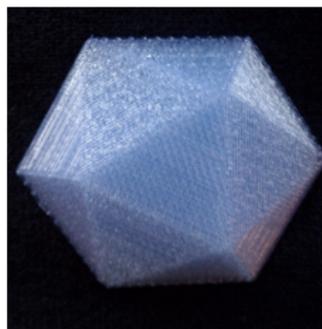


3D Printing Projects for Math Majors

Edward F. Aboufadel

Grand Valley State University



Outline of Talk: What Can a Math Major Do?

Mathematica

Outline of Talk: What Can a Math Major Do?

Mathematica

Project Ideas for Multivariable Calculus

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Mathematica

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Project Ideas for College Geometry

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Eugenia Cheng's Associahedron

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Other Project Ideas

Outline of Talk: What Can a Math Major Do?

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Project Ideas for Multivariable Calculus

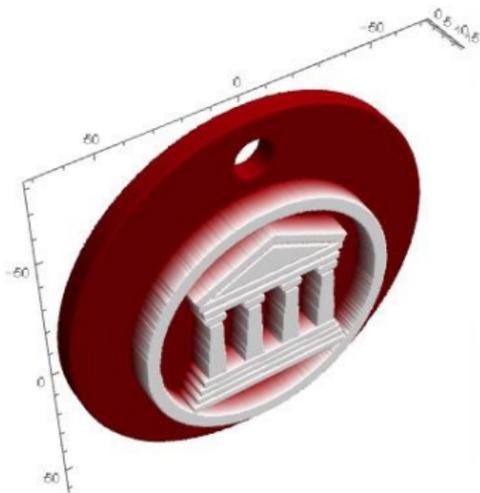
Project Ideas for College Geometry

Eugenia Cheng's Associahedron

Other Project Ideas

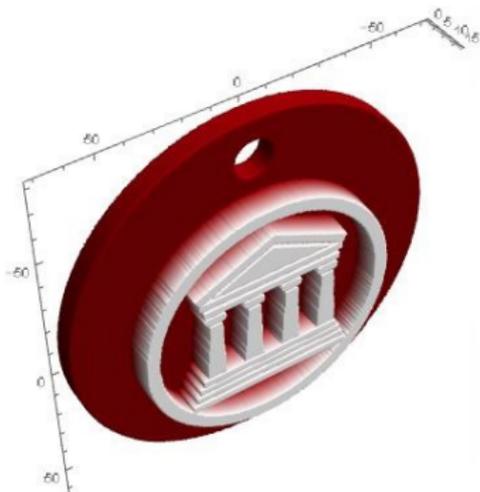
For Further Information

Mathematica



Generate 3D objects and export as STL

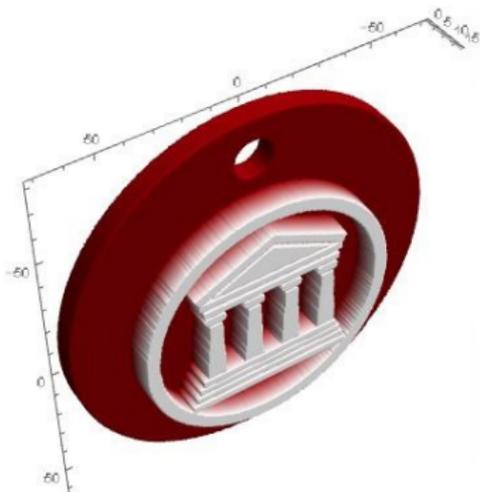
Mathematica



Generate 3D objects and export as STL

Define and render $z = f(x, y)$

Mathematica

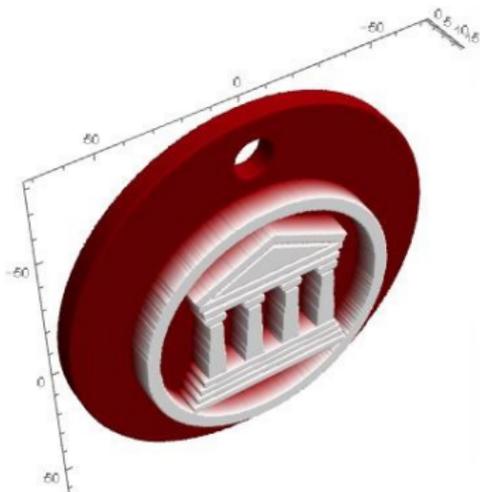


Generate 3D objects and export as STL

Define and render $z = f(x, y)$

Import and process images for use in designs

Mathematica



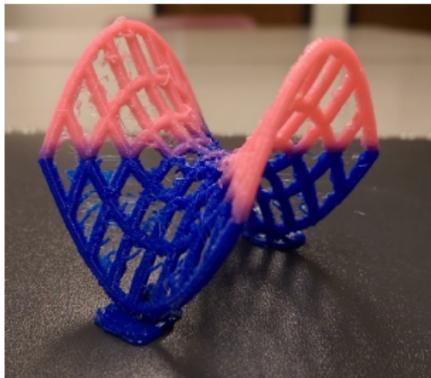
Generate 3D objects and export as STL

Define and render $z = f(x, y)$

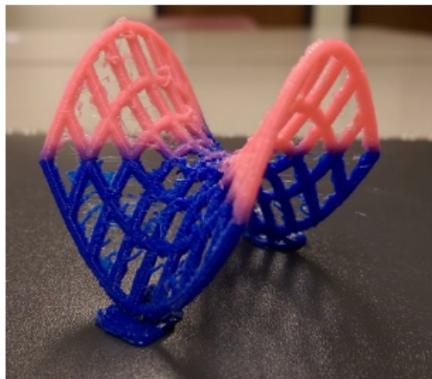
Import and process images for use in designs

GVSU has a site license

Multi. Calc: $z = f(x, y)$ surfaces

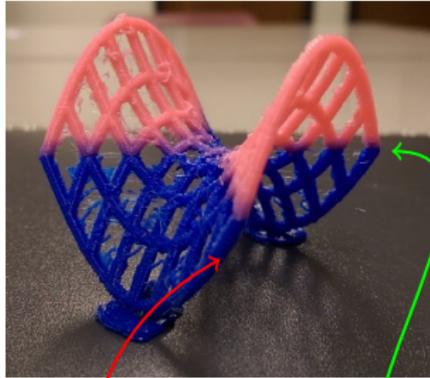


Multi. Calc: $z = f(x, y)$ surfaces



Inspired by David Bachman's designs.

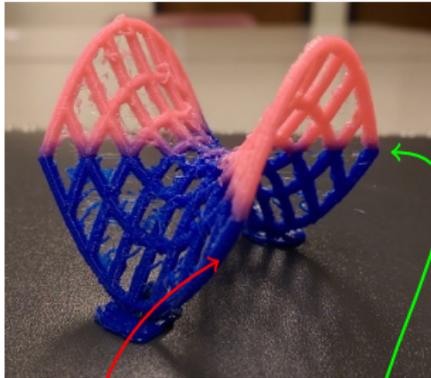
Multi. Calc: $z = f(x, y)$ surfaces



Inspired by David Bachman's designs.

Defined by a union of **tubes** and **small spheres** to generate a grid-like representation of the surface.

Multi. Calc: $z = f(x, y)$ surfaces



Inspired by David Bachman's designs.

Defined by a union of tubes and small spheres to generate a grid-like representation of the surface.

Implementation comes from Henry Segerman's "3D Printing for Mathematical Visualization". *Mathematica* code is available in Segerman's paper.

Multi. Calc: $z = f(x, y)$ surfaces



Created by Washington & Lee University student Emily Jaekle, working with Prof. Elizabeth Denne. Available on the Thingiverse.

Introduction

Project Ideas for Multivariable Calculus & Geometry

Eugenia Cheng's Associahedron

Other Project Ideas

Surfaces

The Star Project

Pendants & Pyramids

Mini Maker Faire

Multi. Calc: The Star Project – Joining Several Planes



Multi. Calc: The Star Project – Joining Several Planes

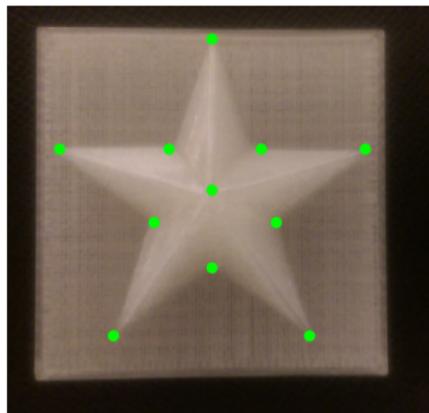
Thank you to Lindsay Czap
(GVSU student)



Multi. Calc: The Star Project – Joining Several Planes

Thank you to Lindsay Czap
(GVSU student)

Key coordinates

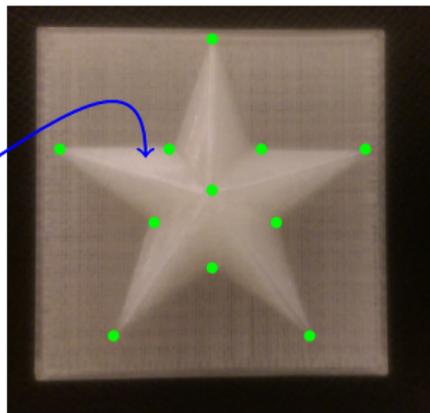


Multi. Calc: The Star Project – Joining Several Planes

Thank you to Lindsay Czap
(GVSU student)

Key coordinates

One of ten planes



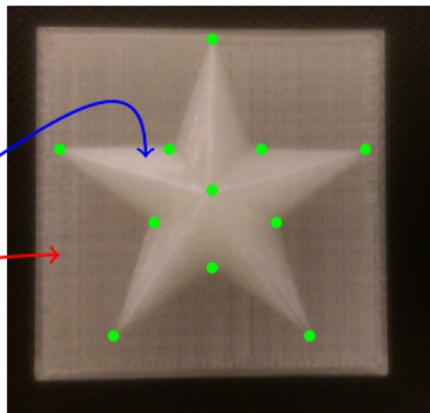
Multi. Calc: The Star Project – Joining Several Planes

Thank you to Lindsay Czap
(GVSU student)

Key coordinates

One of ten planes

Base



Introduction

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Multi. Calc: The Star Project – Joining Several Planes

Multi. Calc: The Star Project – Joining Several Planes

To find the equation of the plane through (x_0, y_0, z_0) , (x_1, y_1, z_1) , and (x_2, y_2, z_2) :

$$\det \begin{bmatrix} x & y & z & 1 \\ x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \end{bmatrix} = 0$$

and solve for z .

Multi. Calc: The Star Project – Joining Several Planes

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and solve for z .

Also: For each plane, need to restrict the domain to a triangle.

Can use `RegionFunction` option, or three tests with determinants and the sides of the triangles.

Multi. Calc: The Star Project – Joining Several Planes

To find the equation of the plane through (x_0, y_0, z_0) , (x_1, y_1, z_1) , and (x_2, y_2, z_2) :

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and solve for z .

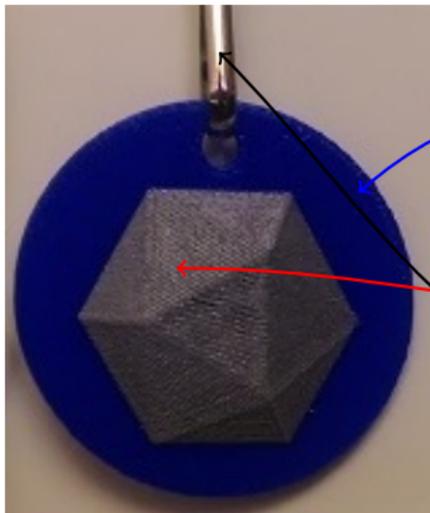
Also: For each plane, need to restrict the domain to a triangle. Can use `RegionFunction` option, or three tests with determinants and the sides of the triangles.

The base is generated separately, and all the pieces are joined together with `Show`.

Multi. Calc: MAA Pendant



Multi. Calc: MAA Pendant



Base: a cylinder, with a restricted domain to create the hole for the clip

One of ten planes

Lanyard ordered from Amazon

College Geometry: n -gonal Pyramids



College Geometry: n -gonal Pyramids

Base is a regular polygon



College Geometry: n -gonal Pyramids

Base is a regular polygon

Vertices are on the unit circle –
good trigonometry review

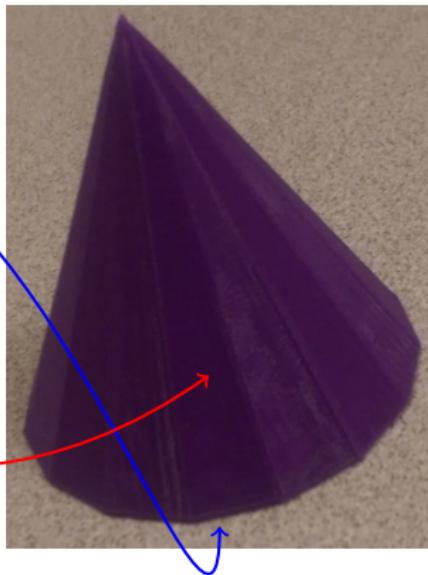


College Geometry: n -gonal Pyramids

Base is a regular polygon

Vertices are on the unit circle –
good trigonometry review

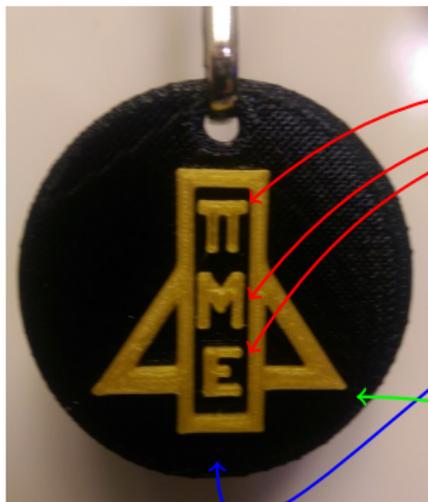
Several Polygon commands
used to create sides



College Geometry: Pi Mu Epsilon Pendant



College Geometry: Pi Mu Epsilon Pendant



Start with a cylinder for the base.

In 2D, each letter is a (irregular) dodecahedron.

Many polygons needed to define the sides perpendicular to the xy -plane

The angles in this isosceles triangle? Location of the third vertex?

College Geometry: Mini Maker Faire

A. Make a geometric object from a set of choices (including “Make A Counter Offer”).

College Geometry: Mini Maker Faire

- A. Make a geometric object from a set of choices (including “Make A Counter Offer”).
- B. Extend your work creatively to create another object.

College Geometry: Mini Maker Faire

A. Make a geometric object from a set of choices (including “Make A Counter Offer”).

B. Extend your work creatively to create another object.

C. Write a short paper (3-5 pages) describing your work, with mathematical details. I expect you'll have to do a bit of research on your topic to make your paper top-notch.

College Geometry: Mini Maker Faire

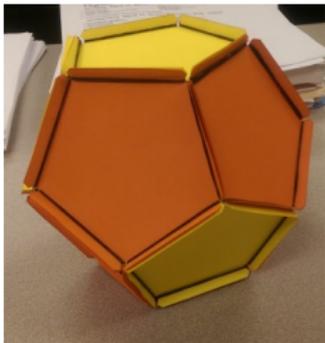
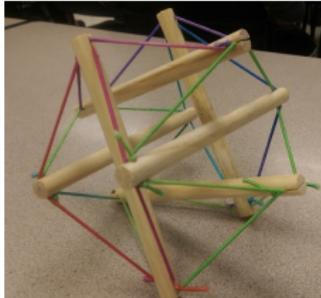
A. Make a geometric object from a set of choices (including “Make A Counter Offer”).

B. Extend your work creatively to create another object.

C. Write a short paper (3-5 pages) describing your work, with mathematical details. I expect you'll have to do a bit of research on your topic to make your paper top-notch.

D. Give a five-minute presentation about your project to the class during the week of April 13.

College Geometry: Mini Maker Faire



Eugenia Cheng



Eugenia Cheng

Mathematician, pianist, minor
celebrity



Eugenia Cheng

Mathematician, pianist, minor
celebrity

University of Sheffield & the
School of Art Institute of
Chicago



Eugenia Cheng

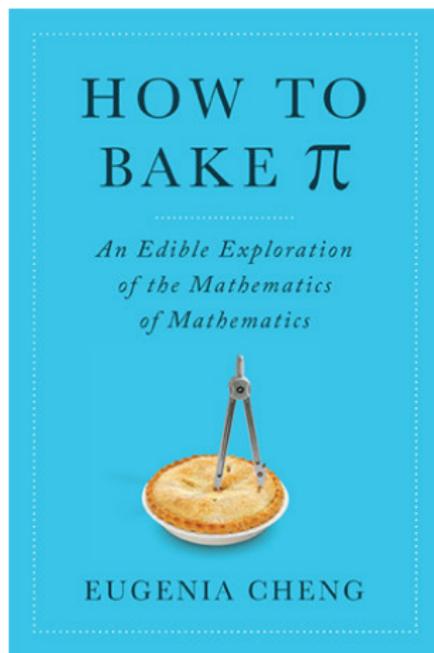
Mathematician, pianist, minor
celebrity

University of Sheffield & the
School of Art Institute of
Chicago

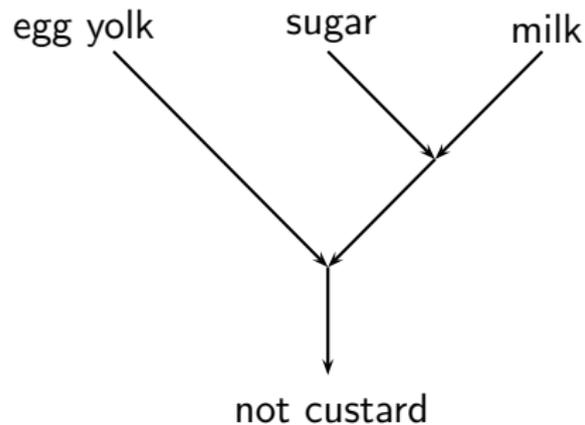
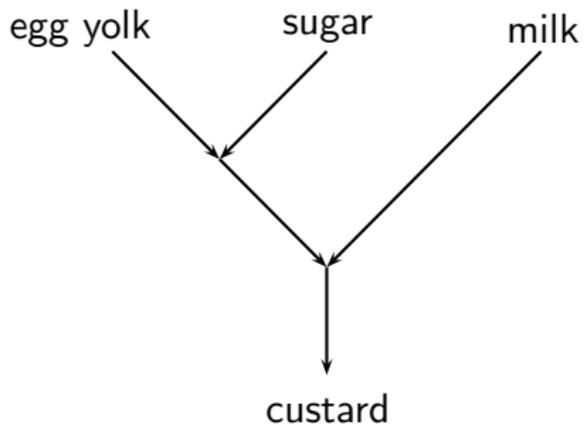
Specialist in category theory



An Edible Exploration of the Mathematics of Mathematics

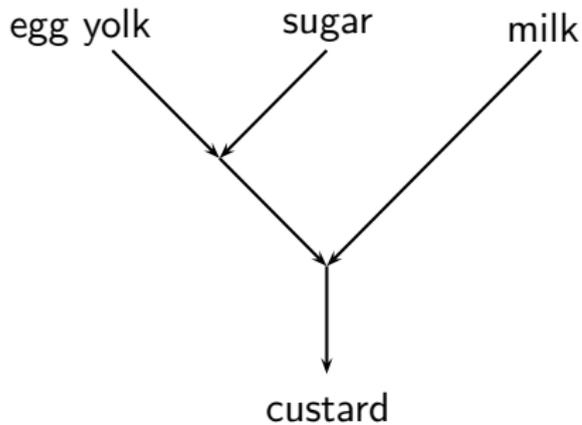


Mixing Ingredients to Make Custard

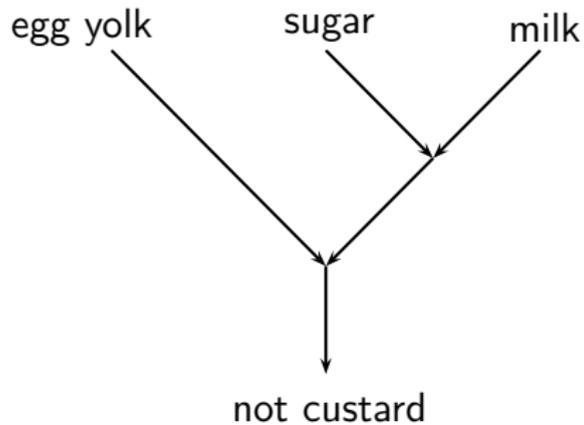


Mixing Ingredients to Make Custard

$(es)m$



$e(sm)$



Mixing Five Ingredients

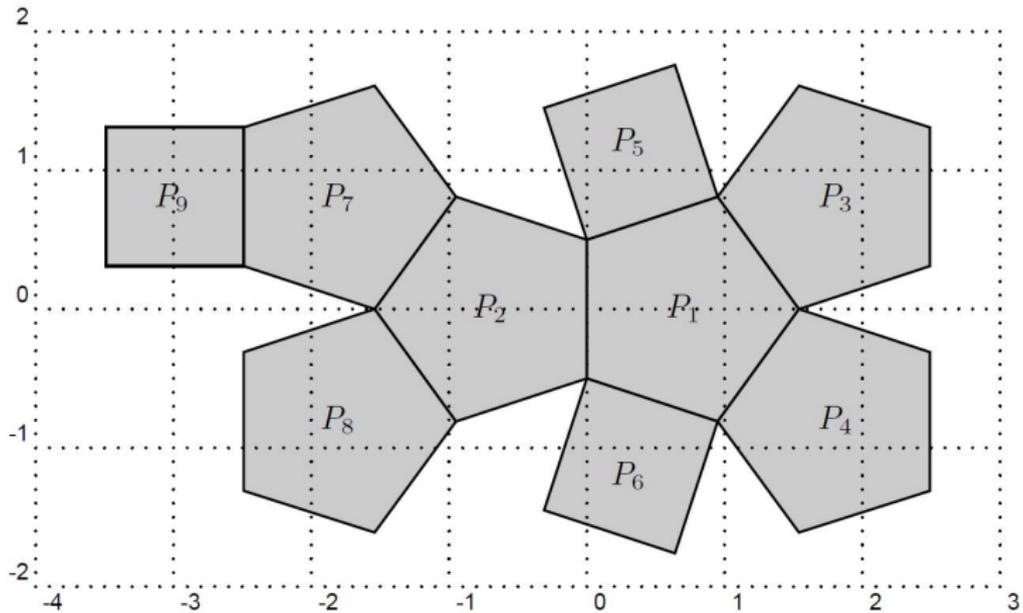
<i>Label</i>	<i>Coordinates</i>	<i>Tree</i>	<i>Label</i>	<i>Coordinates</i>	<i>Tree</i>
<i>A</i>	$(0, \frac{1}{2})$	(12)(3(45))	<i>I</i>	$(\alpha + \gamma, \beta)$	(1(2(34)))5
<i>B</i>	$(\alpha, \beta + \frac{1}{2})$	(12)((34)5)	<i>I'</i>	$(\alpha + \gamma, -\beta)$...
<i>C</i>	$(\gamma, 0)$	((12)(34))5	<i>I''</i>	$(-\alpha - \gamma - 1, \beta + 1)$...
<i>D</i>	$(\alpha, -\beta - \frac{1}{2})$	((((12)3)4)5)	<i>J</i>	$(\alpha + \gamma, \beta + 1)$	1((2(34))5)
<i>E</i>	$(0, -\frac{1}{2})$	((12)3)(45)	<i>J'</i>	$(-\alpha - \gamma, \beta + 1)$...
<i>F</i>	$(-\beta, -\alpha - \frac{1}{2})$	(1(23))(45)	<i>K</i>	$(\gamma, 2\beta + 1)$	1(2((34)5))
<i>F'</i>	$(-\alpha, -\beta - \frac{1}{2})$...	<i>K'</i>	$(\alpha - \beta, \alpha + \beta + \frac{1}{2})$...
<i>G</i>	$(\alpha - \beta, -\alpha - \beta - \frac{1}{2})$	((1(23))4)5	<i>K''</i>	$(-\gamma, 2\beta + 1)$...
<i>G'</i>	$(\gamma, -2\beta - 1)$...	<i>L</i>	$(-\beta, \alpha + \frac{1}{2})$	1(2(3(45)))
<i>G''</i>	$(-\gamma, -2\beta - 1)$...	<i>L'</i>	$(-\alpha, \beta + \frac{1}{2})$...
<i>H</i>	$(\alpha + \gamma, -\beta - 1)$	(1((23)4))5	<i>M</i>	$(-\gamma, 0)$	1((23)(45))
<i>H'</i>	$(-\alpha - \gamma - 1, \beta)$...	<i>N</i>	$(-\alpha - \gamma, \beta)$	1(((23)4)5)
<i>H''</i>	$(-\alpha - \gamma, -\beta - 1)$...	<i>N'</i>	$(-\alpha - \gamma, -\beta)$...

Defining Polyhedra

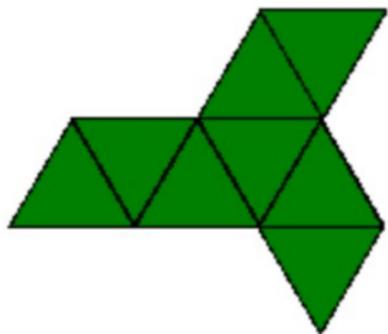
<i>Label</i>	<i>Coordinates</i>	<i>Tree</i>	<i>Label</i>	<i>Coordinates</i>	<i>Tree</i>
<i>A</i>	$(0, \frac{1}{2})$	(12)(3(45))	<i>I</i>	$(\alpha + \gamma, \beta)$	(1(2(34)))5
<i>B</i>	$(\alpha, \beta + \frac{1}{2})$	(12)((34)5)	<i>I'</i>	$(\alpha + \gamma, -\beta)$...
<i>C</i>	$(\gamma, 0)$	((12)(34))5	<i>I''</i>	$(-\alpha - \gamma - 1, \beta + 1)$...
<i>D</i>	$(\alpha, -\beta - \frac{1}{2})$	((12)3)4)5	<i>J</i>	$(\alpha + \gamma, \beta + 1)$	1((2(34))5)
<i>E</i>	$(0, -\frac{1}{2})$	((12)3)(45)	<i>J'</i>	$(-\alpha - \gamma, \beta + 1)$...
<i>F</i>	$(-\beta, -\alpha - \frac{1}{2})$	(1(23))(45)	<i>K</i>	$(\gamma, 2\beta + 1)$	1(2((34)5))
<i>F'</i>	$(-\alpha, -\beta - \frac{1}{2})$...	<i>K'</i>	$(\alpha - \beta, \alpha + \beta + \frac{1}{2})$...
<i>G</i>	$(\alpha - \beta, -\alpha - \beta - \frac{1}{2})$	((1(23))4)5	<i>K''</i>	$(-\gamma, 2\beta + 1)$...
<i>G'</i>	$(\gamma, -2\beta - 1)$...	<i>L</i>	$(-\beta, \alpha + \frac{1}{2})$	1(2(3(45)))
<i>G''</i>	$(-\gamma, -2\beta - 1)$...	<i>L'</i>	$(-\alpha, \beta + \frac{1}{2})$...
<i>H</i>	$(\alpha + \gamma, -\beta - 1)$	(1((23)4))5	<i>M</i>	$(-\gamma, 0)$	1((23)(45))
<i>H'</i>	$(-\alpha - \gamma - 1, \beta)$...	<i>N</i>	$(-\alpha - \gamma, \beta)$	1(((23)4)5)
<i>H''</i>	$(-\alpha - \gamma, -\beta - 1)$...	<i>N'</i>	$(-\alpha - \gamma, -\beta)$...

Note: $\alpha = \cos(18^\circ)$, $\beta = \sin(18^\circ)$, and $\gamma = \frac{\cot(36^\circ)}{2} + \frac{\csc(36^\circ)}{2} = \frac{1 + \cos(36^\circ)}{2 \sin(36^\circ)}$. Then, the polygons are: $P_1 = \{A, B, C, D, E\}$

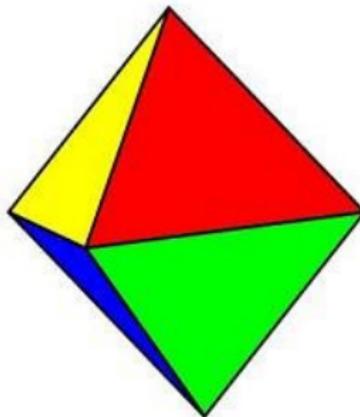
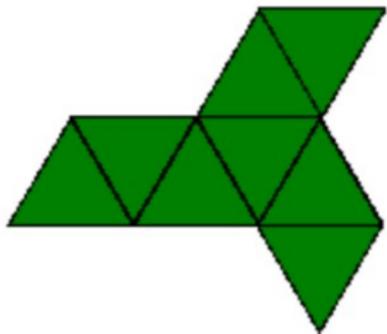
Cheng's Net



A Simpler Net



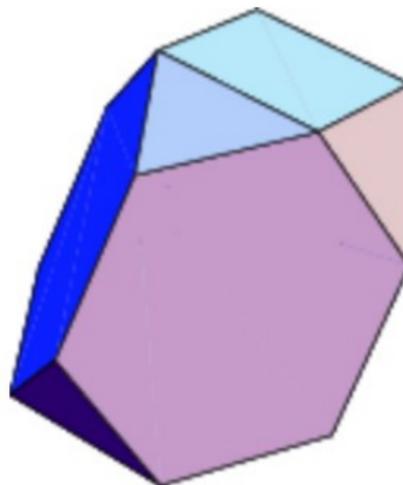
A Simpler Net



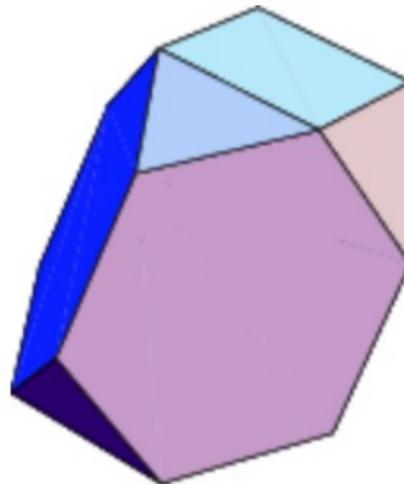
A More Complicated Net



A More Complicated Net

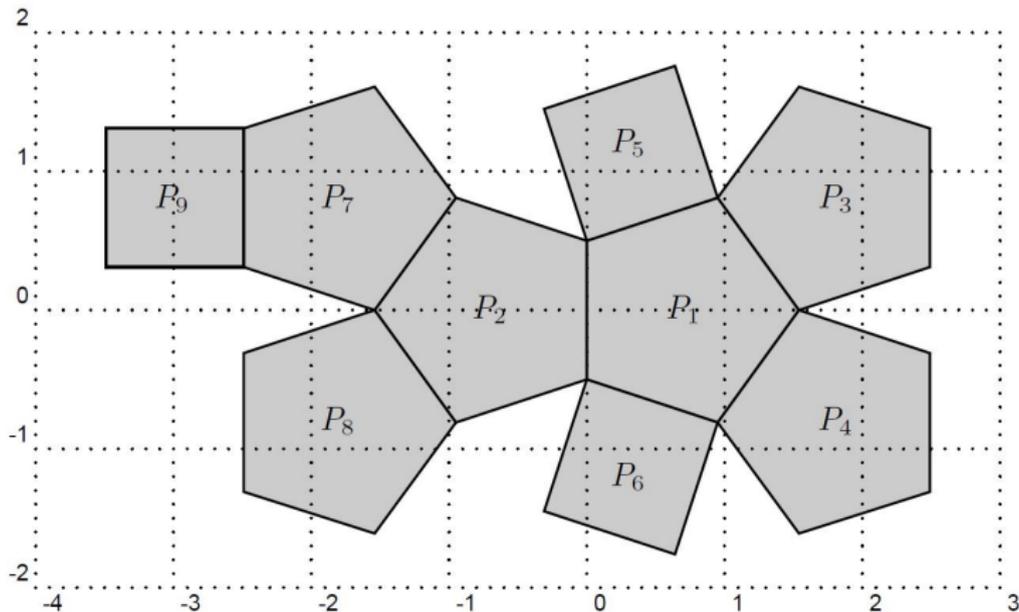


A More Complicated Net



Augmented Truncated
Tetrahedron

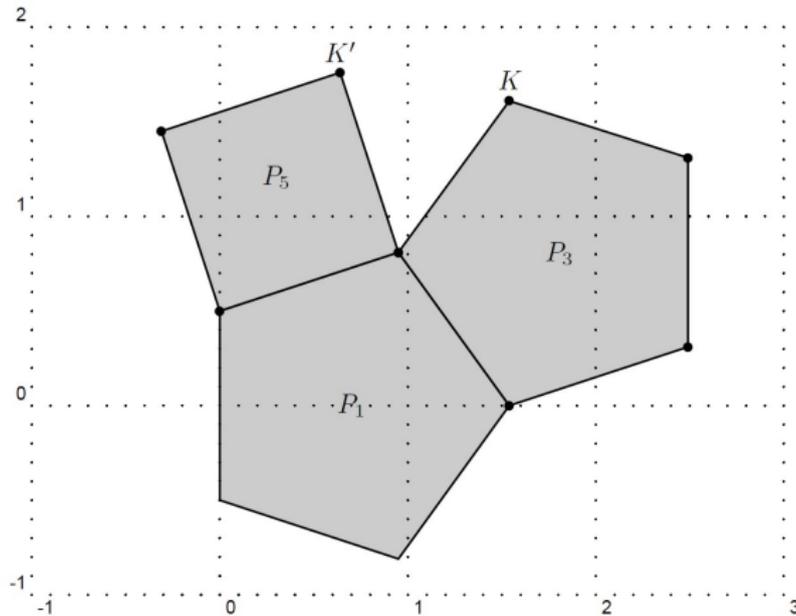
Cheng's Net



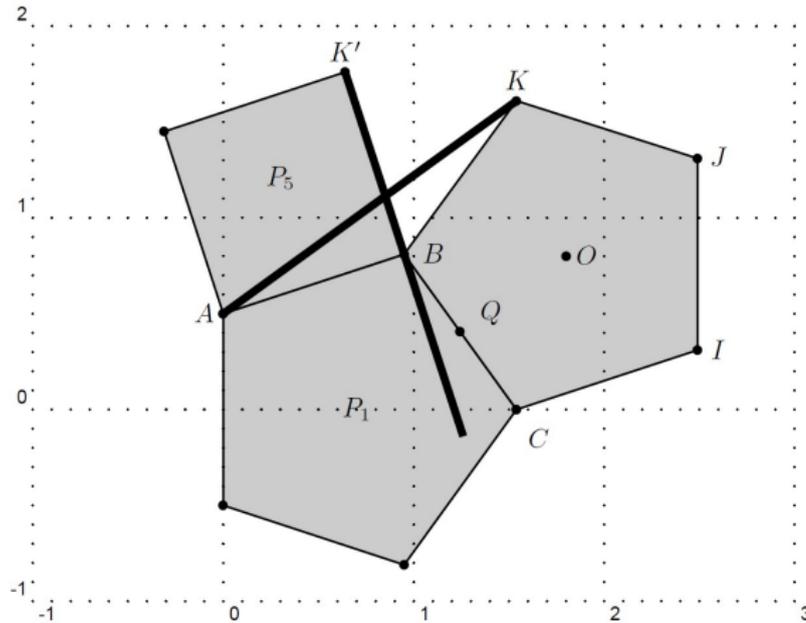
Wonky?

Just don't try to make it out of thick card stock, because it won't quite fit together – it needs to be made from paper that's a bit bendy, otherwise the pentagon and squares would have to be a bit wonky for them all to fit together. – E. Cheng

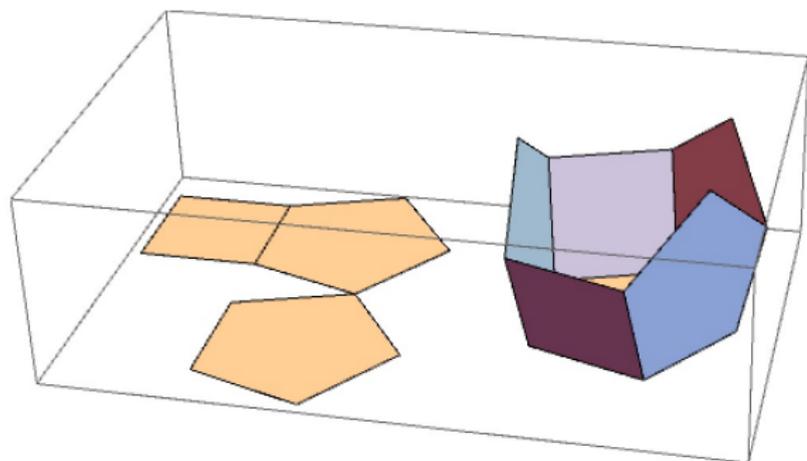
Folding Cheng's Net



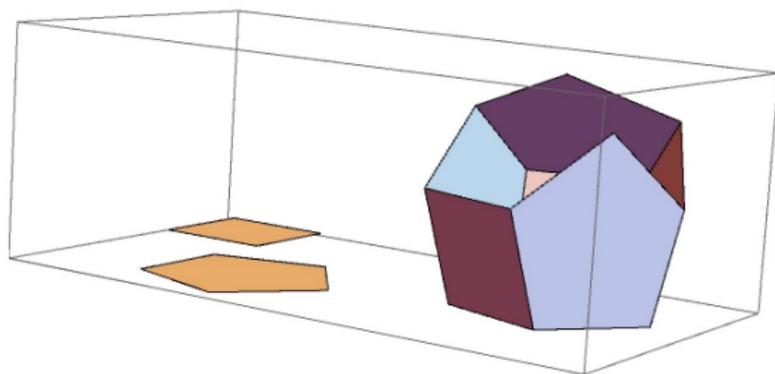
Folding Cheng's Net



Folding Cheng's Net



Folding Cheng's Net

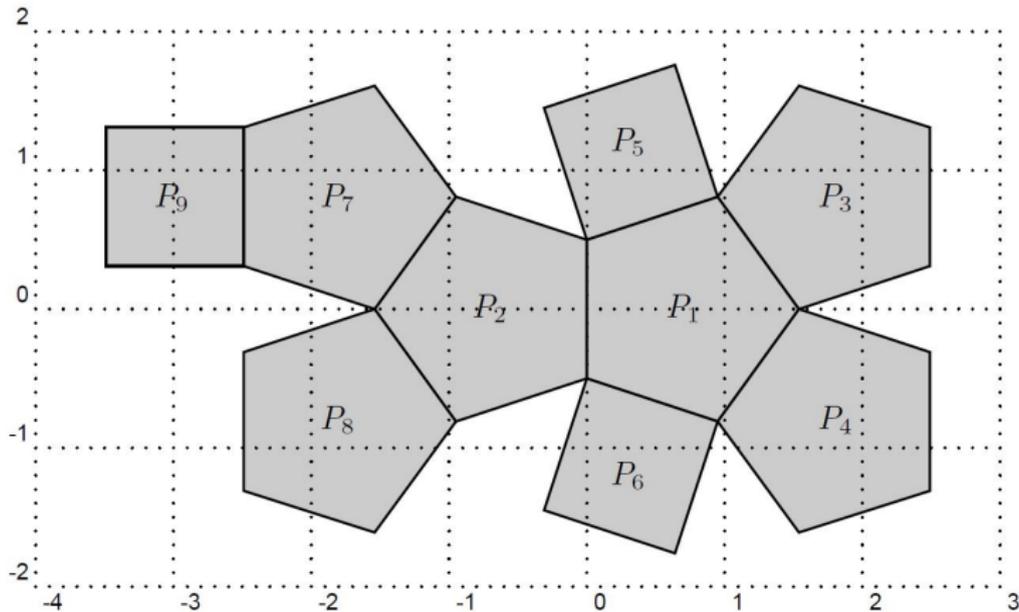


Cheng's Associahedron

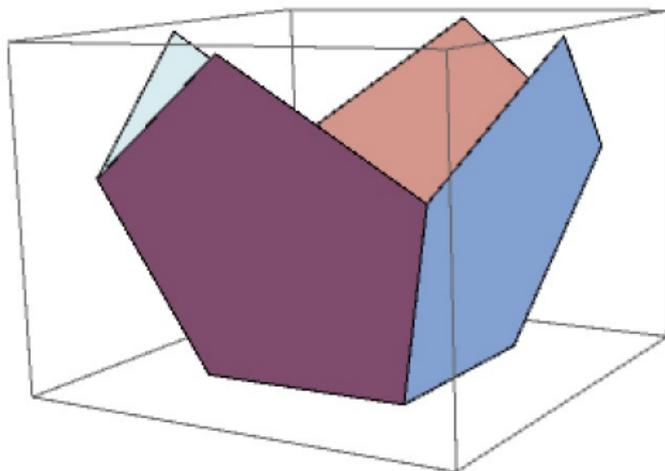


Thank you to Samantha Law
(GVSU student)

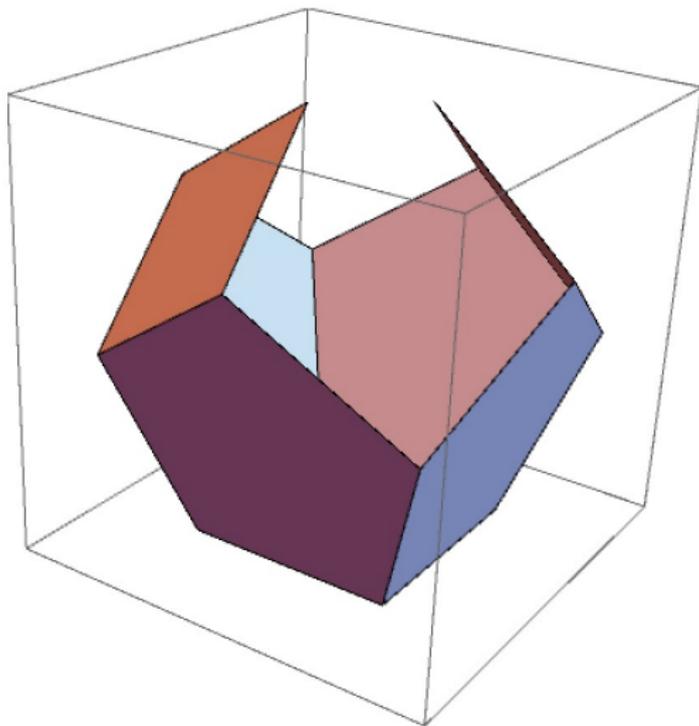
Cheng's Net



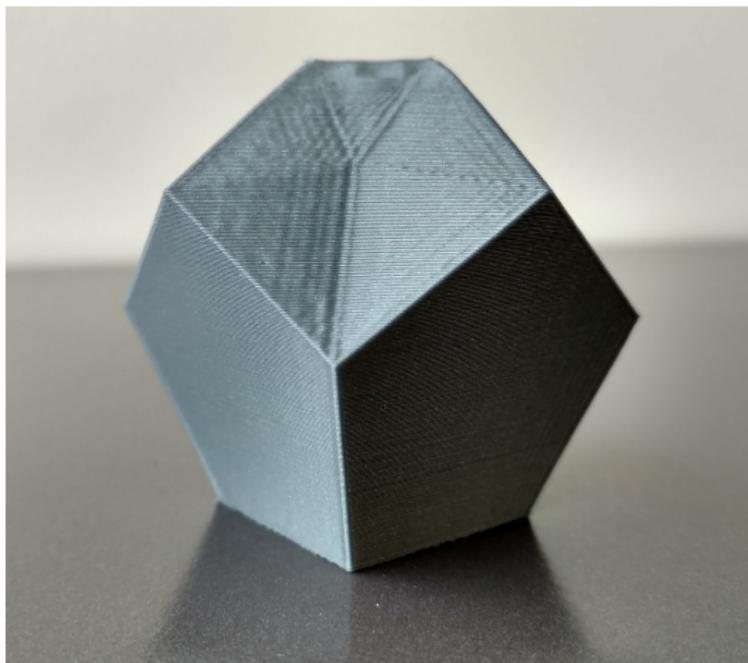
Alternate Build



Alternate Build



Alternate Build



3D Bliss



Vote for Number 6



Vote for Number 6

Load images into Mathematica



Vote for Number 6

Load images into Mathematica

Significant pre-processing (and Mathematica processing) of image is needed.



Vote for Number 6

Load images into Mathematica

Significant pre-processing (and Mathematica processing) of image is needed.

This hook is the difference of two cylinders



Other ideas for courses, independent studies

Linear transformations of 3D objects – effect on printability, printing speeds, volume used, etc.

Re-create work of Hart, Segerman, Knill, Taalman, etc.



For More Information



- Short “how-to” papers, STL files, some Mathematica code, and links to other sites (including my Thingiverse page): sites.google.com/site/aboufadelreu.