Fibonacci Phyllotaxis: Models, Data and Alan Turing

jonathan@swintons.net

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Novel Fibonacci and non-Fibonacci structure in the sunflower: results of a citizen science experiment

Jonathan Swinton[1], Erinma Ochu[2] and The MSI Turing's Sunflower Consortium[3]

[1]Deodands Ltd

[2]University of Manchester Centre for the History of Science, Technology and Medicine

[3]Listed in the electronic supplementary material

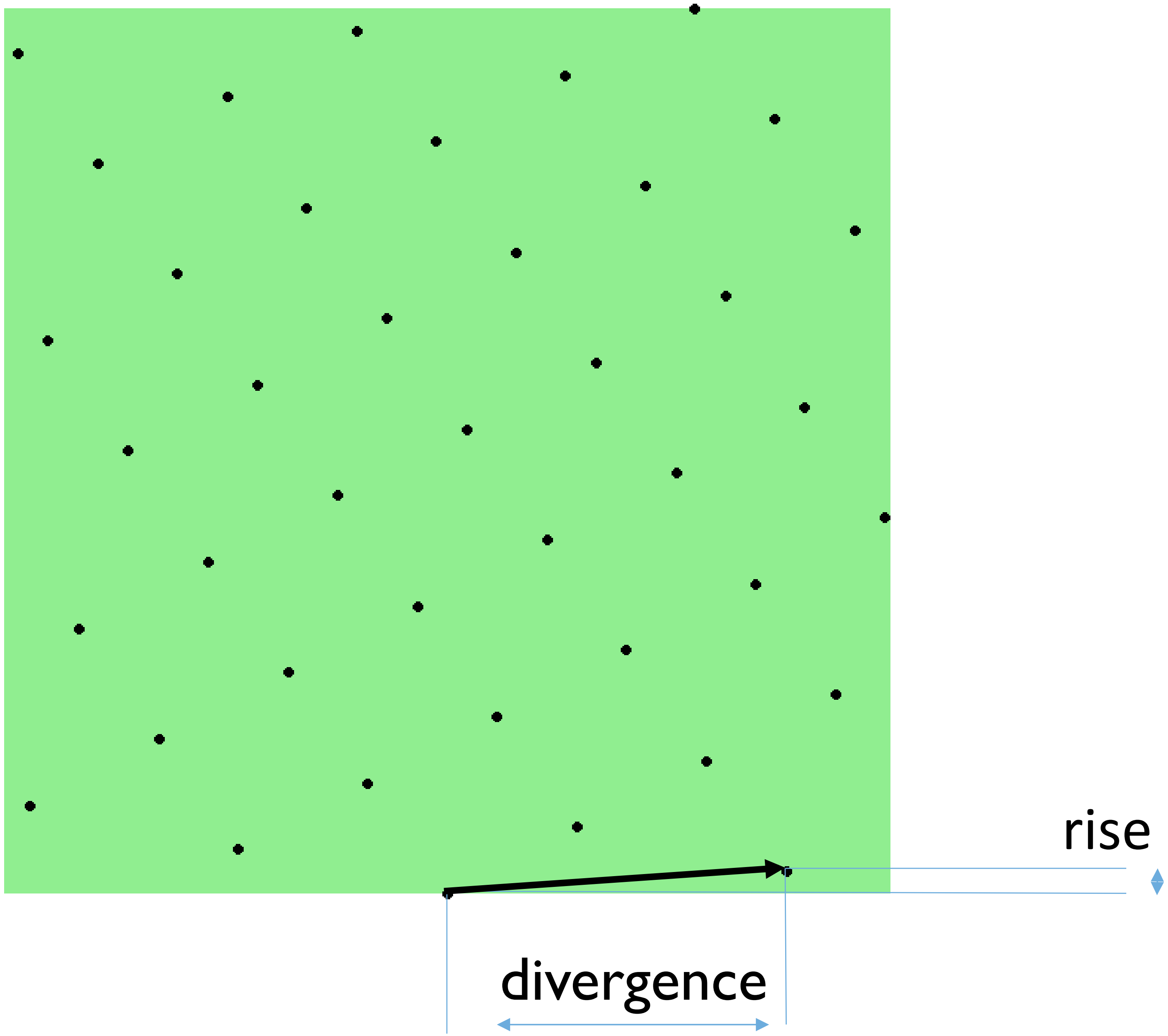
D'Arcy Thompson

The Fibonacci series…and the hypothesis of its introduction into plant-structure through natural selection, are all matters which deserve no place in the plain study of botanical phenomena…all such speculations as these hark back to a school of mystical idealism

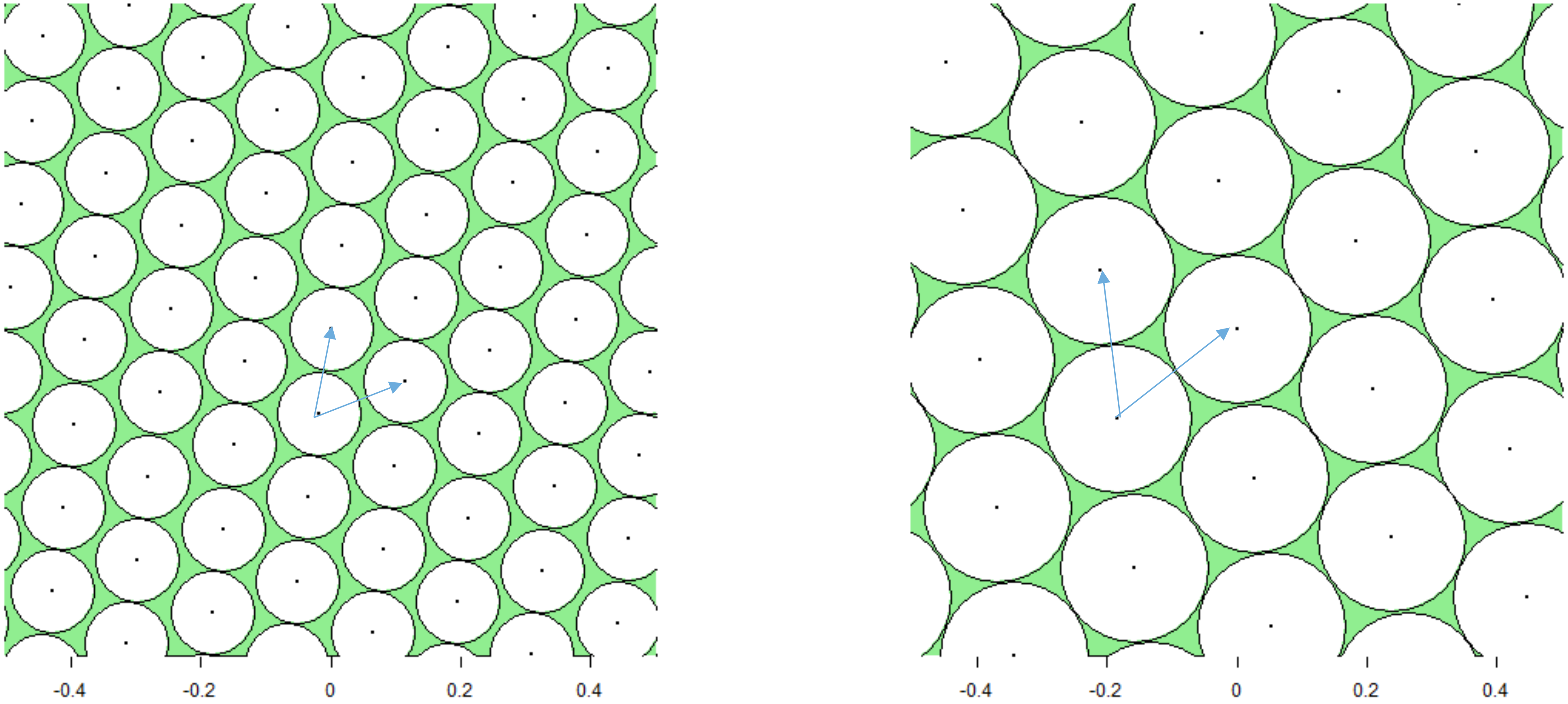
I’m going to talk about

- Mathematics of lattices
- Citizen science & Alan Turing: first computational biologist
- Opportunities for todays mathematical biologists

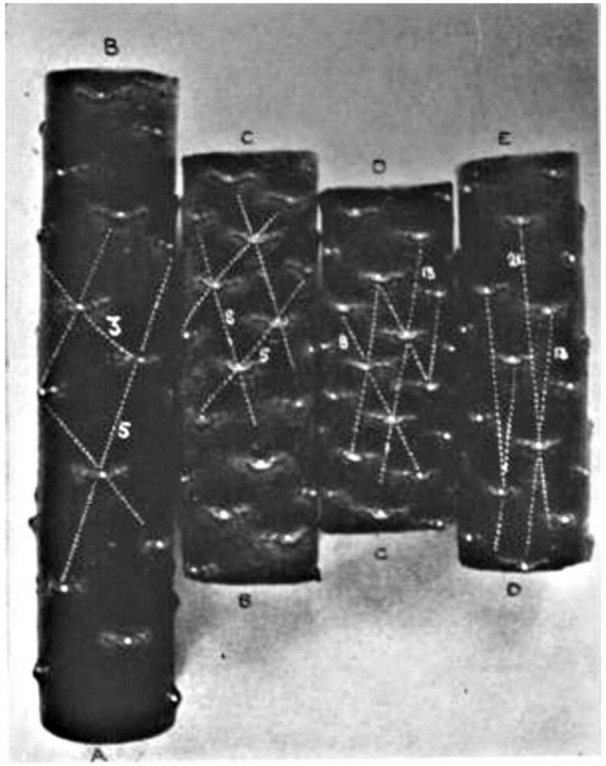
Lattice is on a cylinder has two parameters



Lattices on cylinders
- can define 'obvious' parastichies
- only some lattices are disk packings

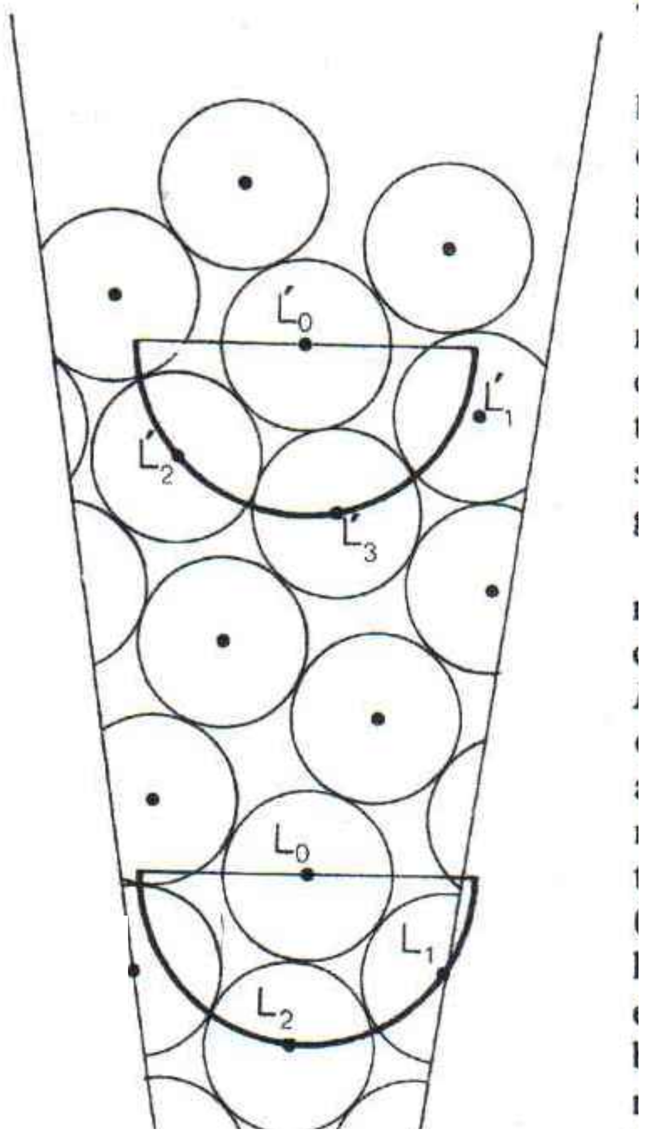


Model: lattice at each height on the stem is a quasi steady state solution of repeated node-placement decisions

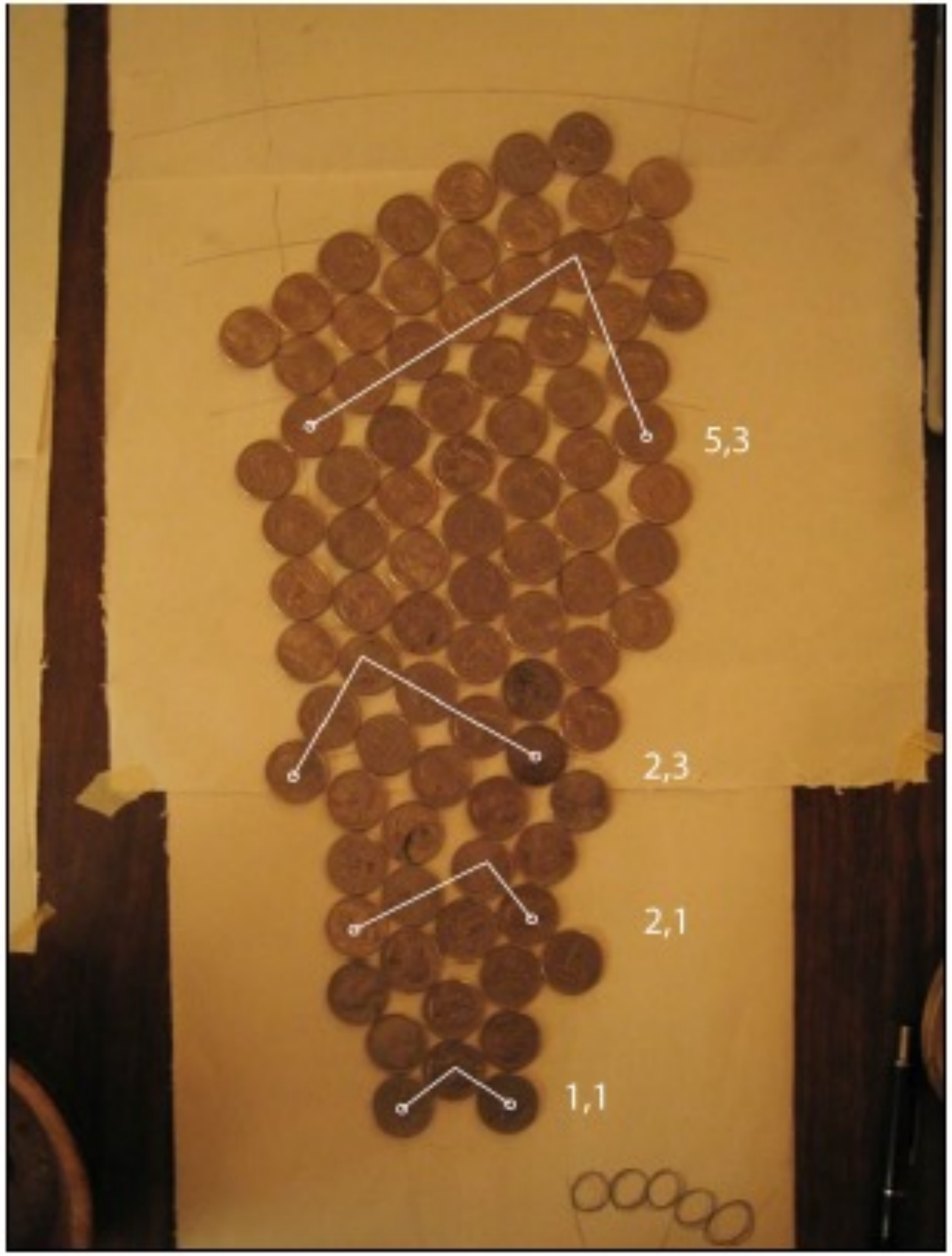


Church 1904

Node placement models

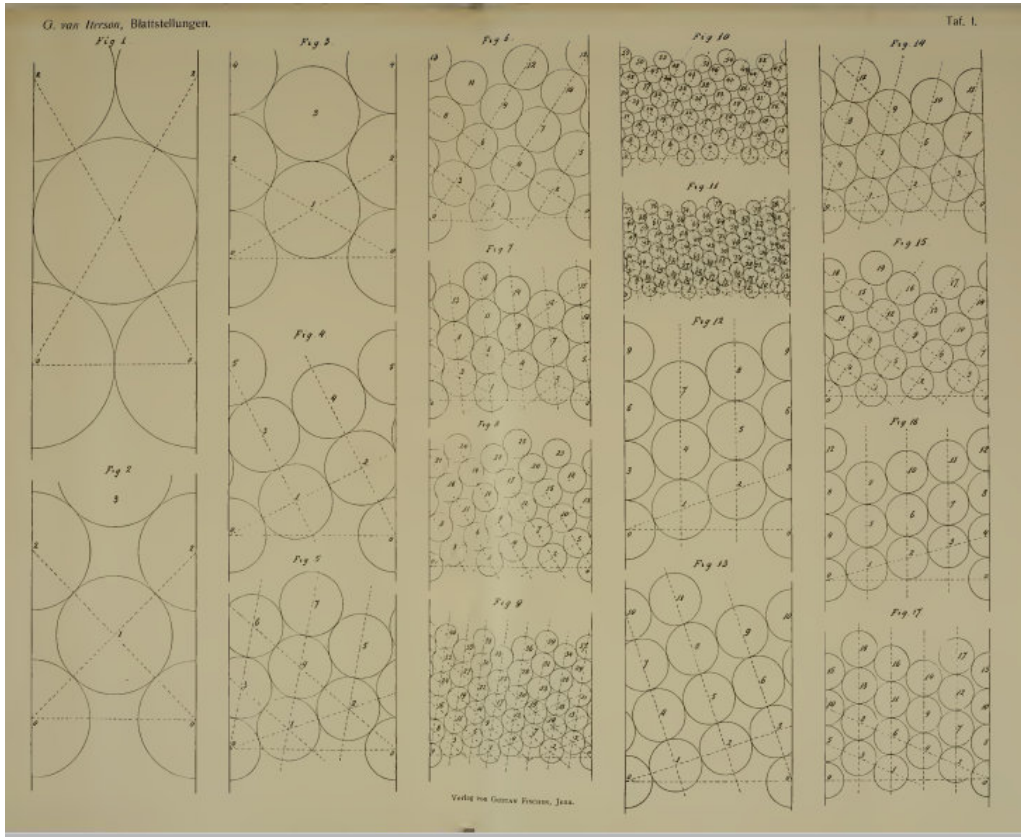


Mitchison 1977

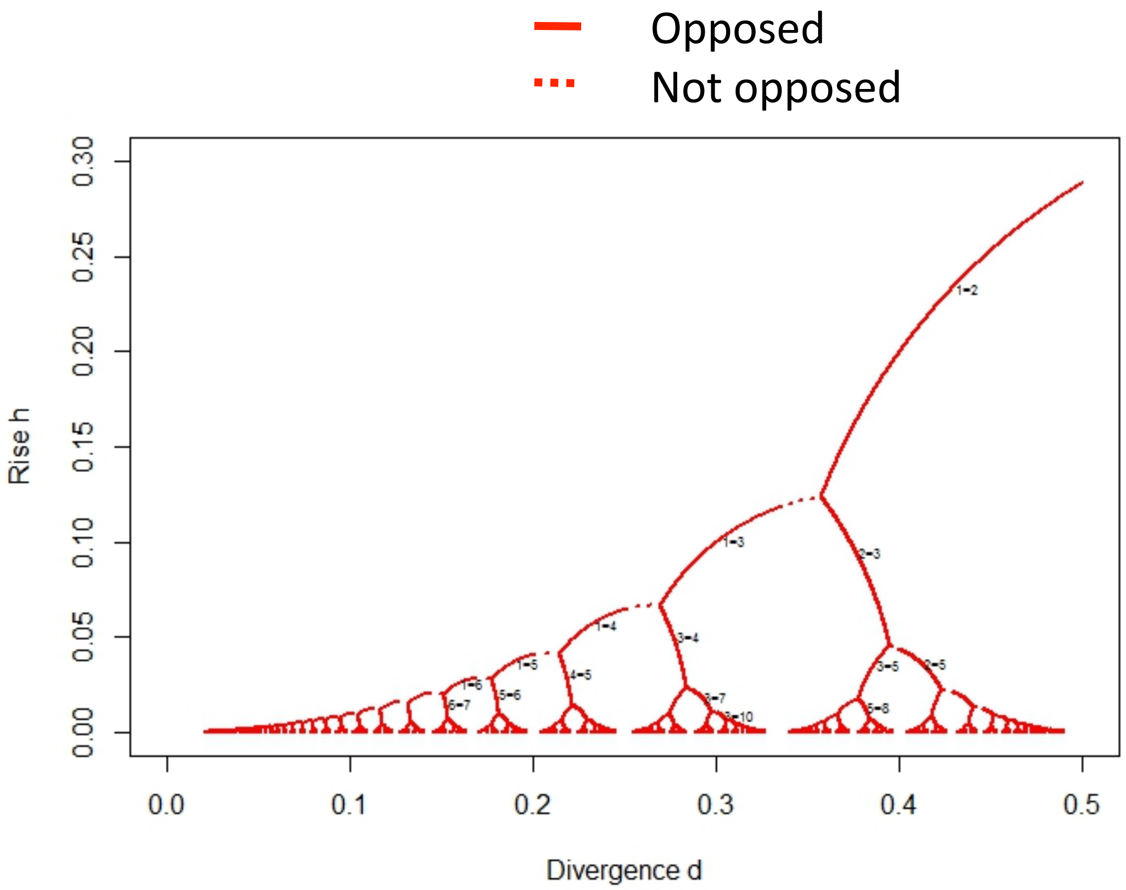


Atela 2011

The van Iterson diagram – packed disk lattices



van Iterson 1907



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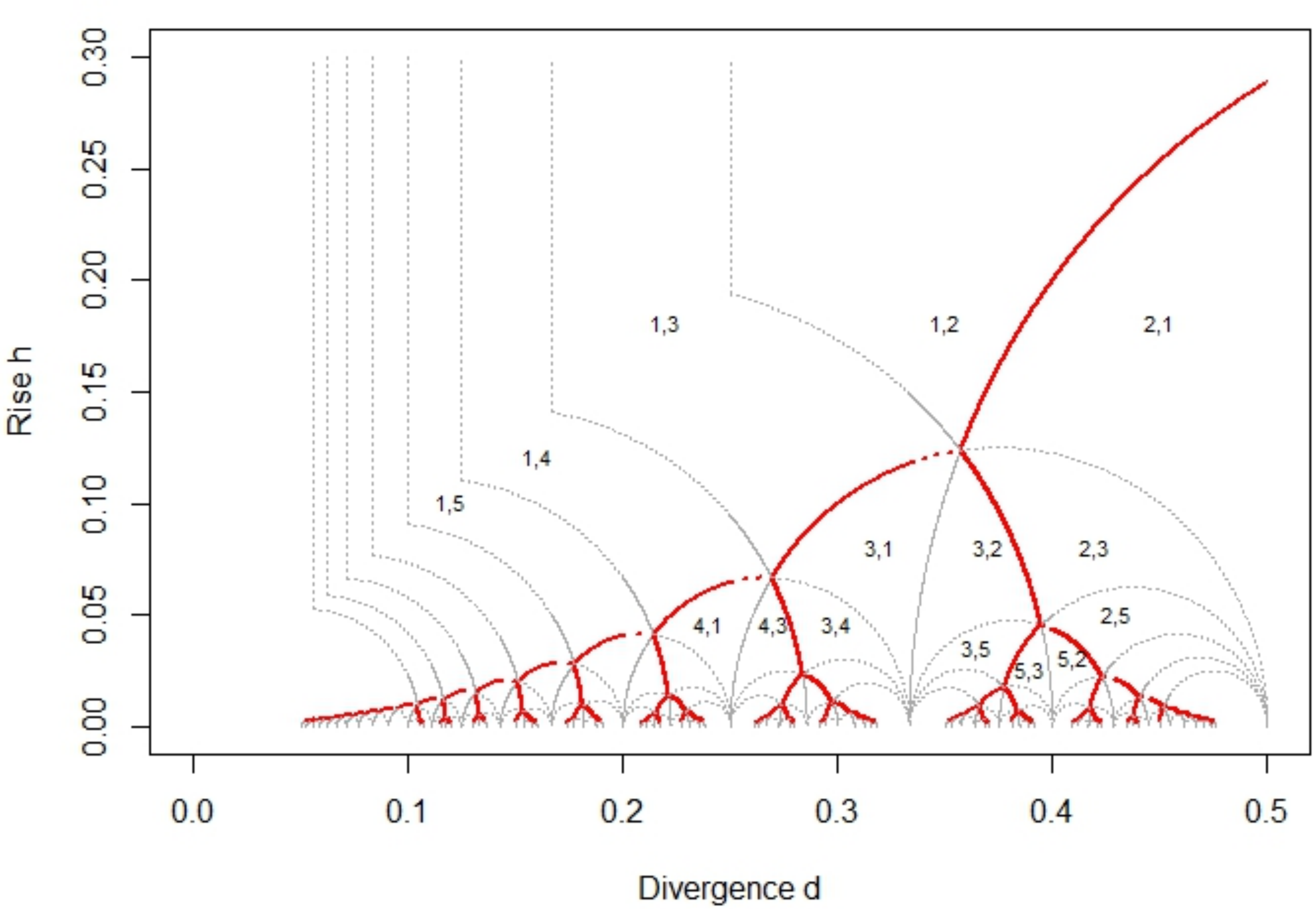
Lattice space categorised by primary and secondary parastichies

- Every relatively prime pair of integers appears once
- Observation of parastichy pairs constrains divergence

Primary and secondary parastichy vectors (m,n) equal in length (ie disk packing)



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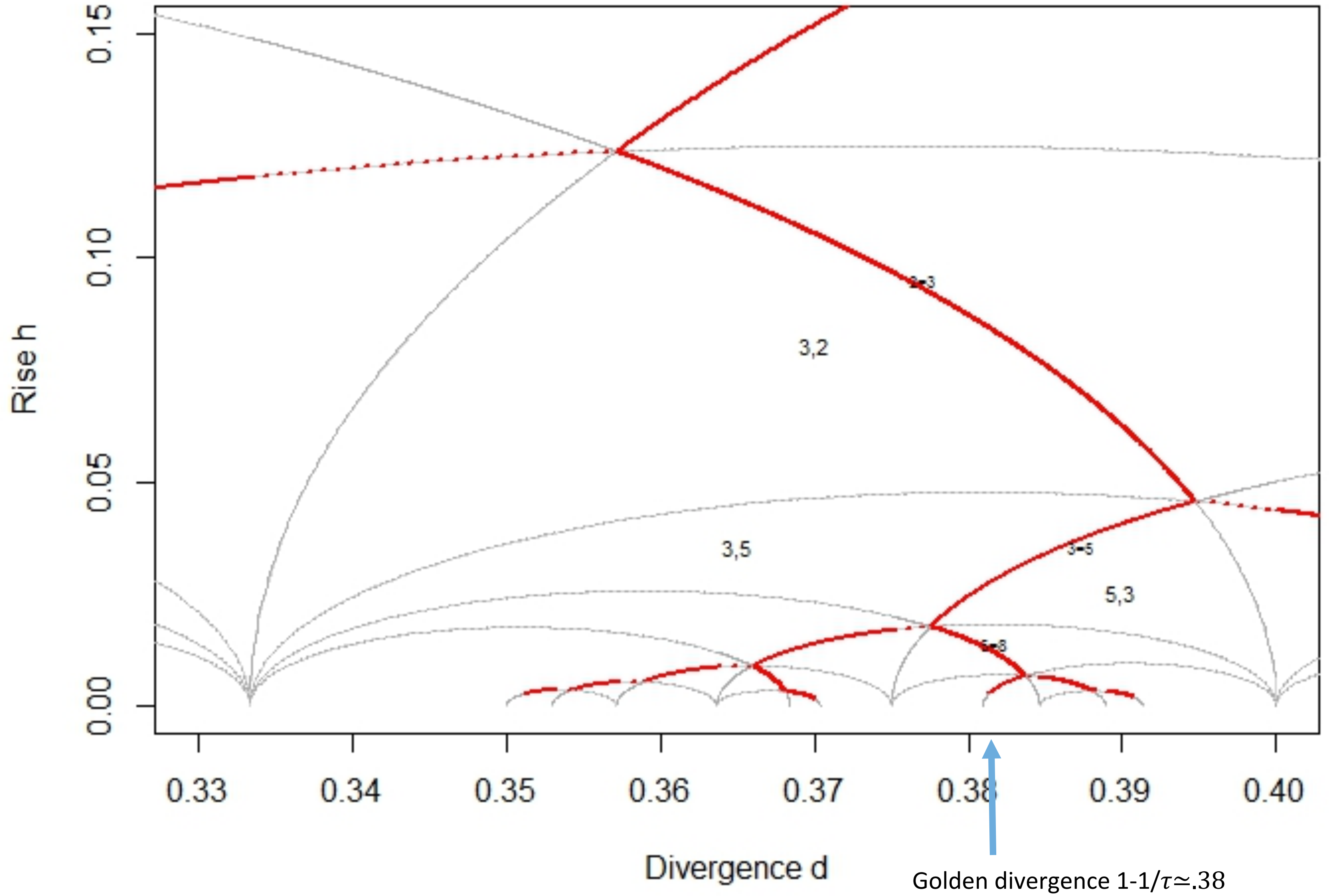
Branch points: (m,n) splits into (n,m+n), and (m, m+n)



A path to Fibonacci phyllotaxis

- A model for new node placement that locally produces regular lattices
- A constraint that causes these lattices to be disk-packing
  - Keeps us on the van Iterson tree
- A smoothly changing parameter that gradually increases the complexity of the lattice
- A constraint that causes the Fibonacci property to be preserved at each bifurcation
  - Turing: 'The hypothesis of geometrical phyllotaxis'

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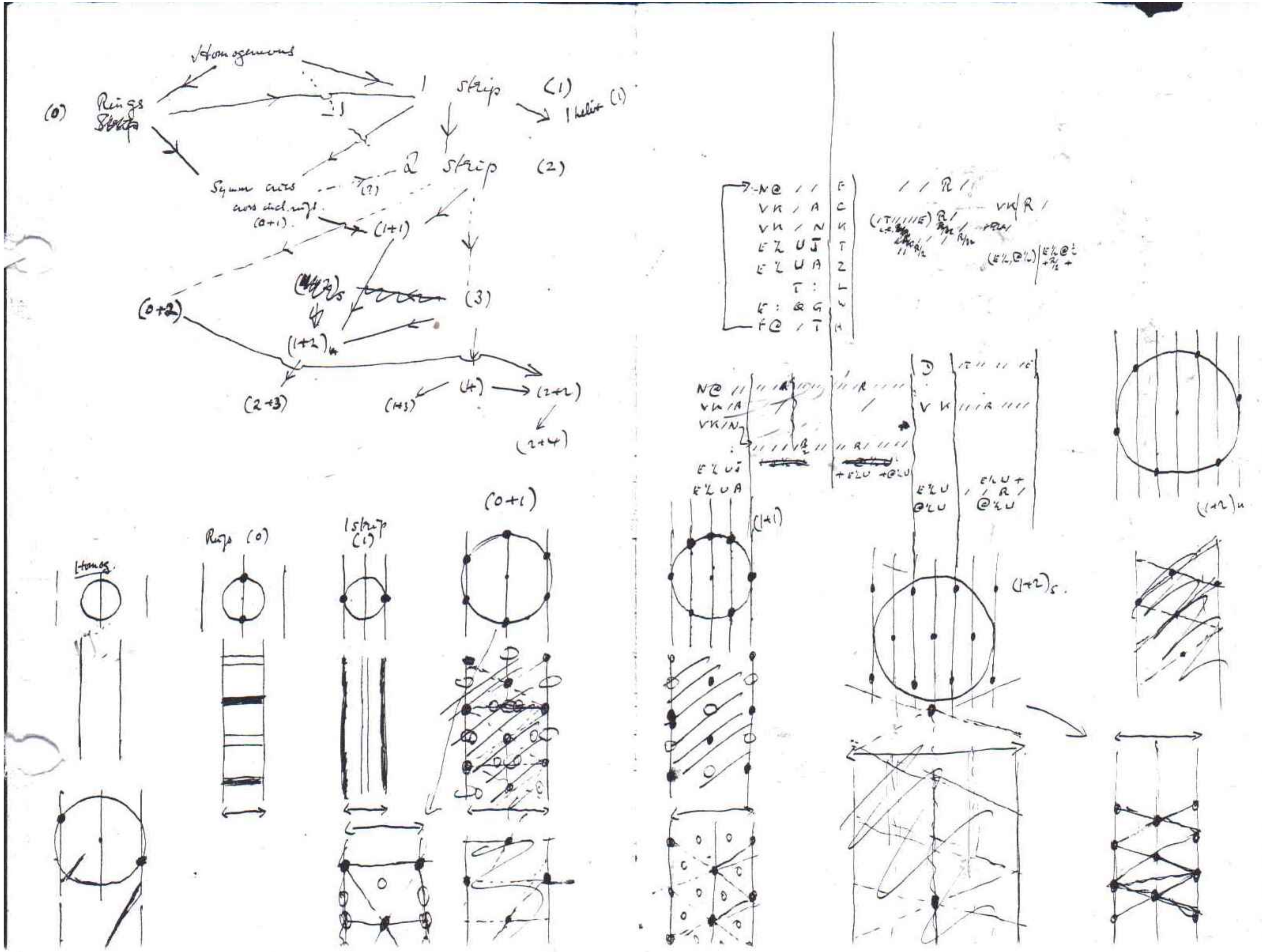

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At present I am not working on the problem at all, but on my mathematical theory of embryology, which I think I described to you at one time. This is yielding to treatment, and it will so far as I can see, give satisfactory explanations of -

i) Gastrulation.

ii) Polyogonally symmetrical structures, e.g., starfish, flowers.

iii) Leaf arrangement, in particular the way the Fibonacci series (0, 1, 1 = 2, 3, 5, 8, 13,.......) comes to be involved.

iv) Colour patterns on animals, e.g., stripes, spots and dappling.

v) Patterns on nearly spherical structures such as some Radiolaria, but this is more difficult and doubtful.

I am really doing this now because it is yielding more easily to treatment. I think it is not altogether unconnected with the other problem. The brain structure has to be one which can be

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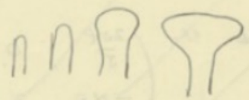
$\frac{\partial U}{\partial t} = \phi(\nabla^2)U + GU^2 - HU(\psi(\nabla^2)U^2) + I(r,\eta)U$

$I(r,\eta) = I_0 - \frac{k_0^2\eta^2}{\sigma^2}$

$\phi(\nabla^2) = I_2\left(1+\frac{\nabla^2}{k_0^2}\right)^2$

$\psi(\nabla^2) = \frac{1}{1-\frac{\nabla^2}{R^2k_0^2}}$

$(I_2)\ (k_0)\ (G)\ I_0\ \sigma\ H\ R\ e$

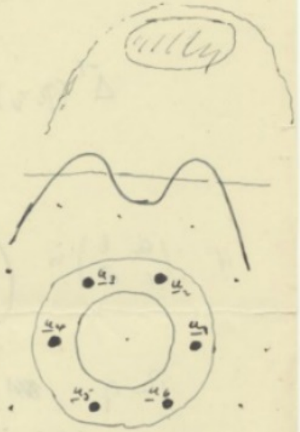


$$\frac{\partial U}{\partial t} = (1+\nabla^2)^2 U + U^2 - HU\frac{U^2}{1-\nabla^2/R^2} + \left(I - \frac{\eta^2}{\sigma^2}\right)U$$

$$U = \sum_{\underline{v}\in L} \eta_{\underline{v}}\, e^{i(\underline{v},\underline{x})} \qquad U^2 = \sum\left(\sum_{\underline{u}+\underline{v}=\underline{w}} \eta_{\underline{u}}\eta_{\underline{v}}\right) e^{i(\underline{w},\underline{x})}$$

As an approx. $\frac{U^2}{1-\nabla^2/R^2} = \sum|\eta_v|^2 = V$ (R small) and $\sigma = \infty$

$$\frac{d\eta_{\underline{v}}}{dt} = \left((1-v^2)^2 + I_0 - HV\right)\eta_{\underline{v}} + \sum_{\underline{u}+\underline{w}=\underline{v}} \eta_{\underline{u}}\eta_{\underline{w}}$$



A lattice pattern remains a lattice.

Suppose only $\eta = \eta_{\underline{u}_1} = \dots = \eta_{\underline{u}_6}$ different from zero

P.T.O.

$\frac{d\eta}{dt} = (I_0 - HV)\eta + 2\eta^2 \qquad V = 6\eta^2$

$= \eta(I_0 + 2\eta - 6H\eta^2) = F(\eta)$ say.

Solution $\eta^{(0)}$, $F(\eta^{(0)}) = 0$

Stability if $F'(\eta^{(0)}) < 0$. Roots alternately stable + unstable.

One root at $\eta = 0$, also at $\eta = \frac{1}{12H}\left(\sqrt{1+6HI_0}+1\right)$

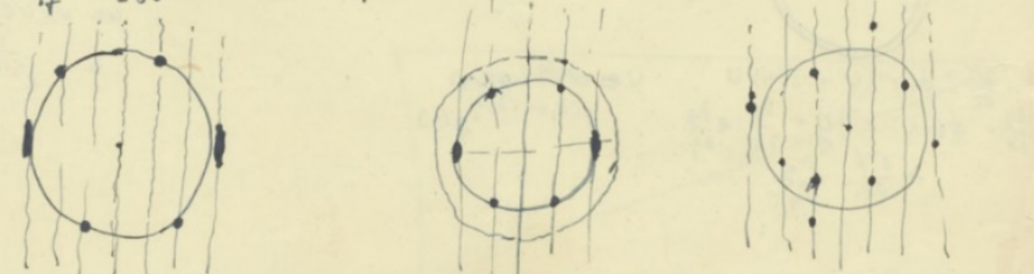
$F'(\eta_0) \to -\infty$ at $\eta \to \pm\infty$.

Therefore $\eta=0$ is unstable if $I_0 > 0$

If $1+6HI_0 < 0$ the only root is $\eta = 0$ and stable

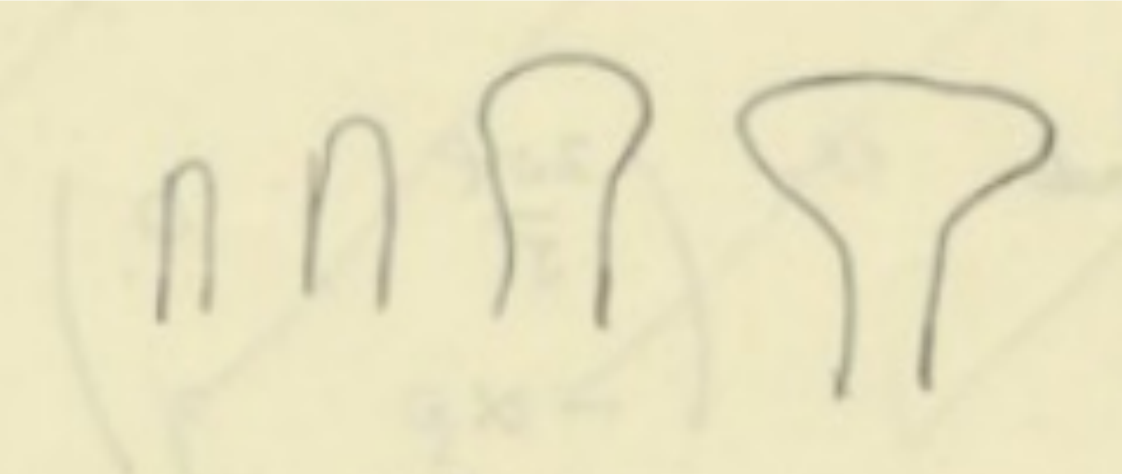
If $-\frac{1}{6H} < I_0 < 0$ then $\eta = 0$ is a stable root and also $\frac{1}{12H}\left(1+\sqrt{1+6HI_0}\right)$

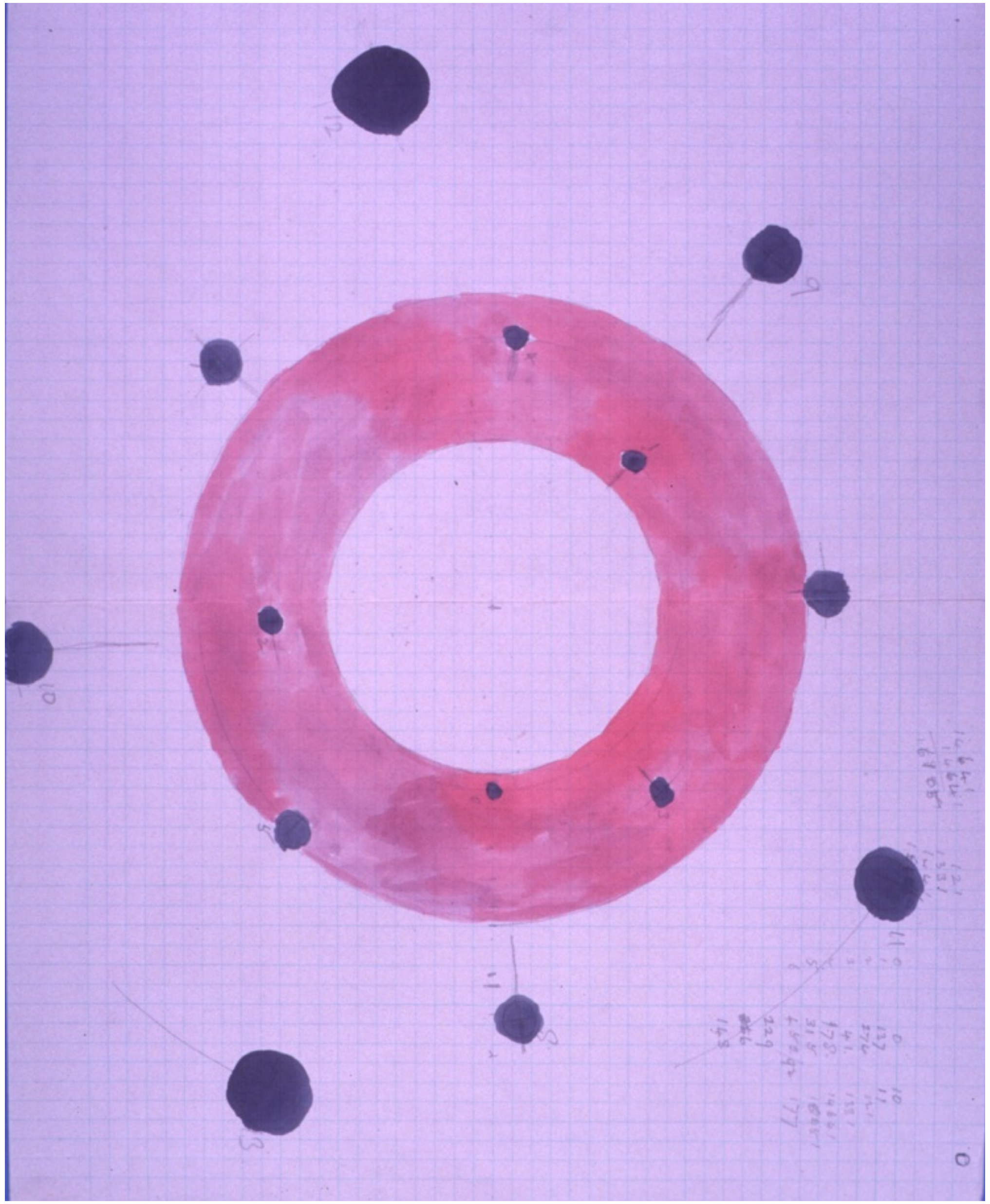
If $I_0 > 0$ then $\eta = 0$ is unstable and $\frac{1}{12H}\left(1\pm\sqrt{1+6HI_0}\right)$ are stable.

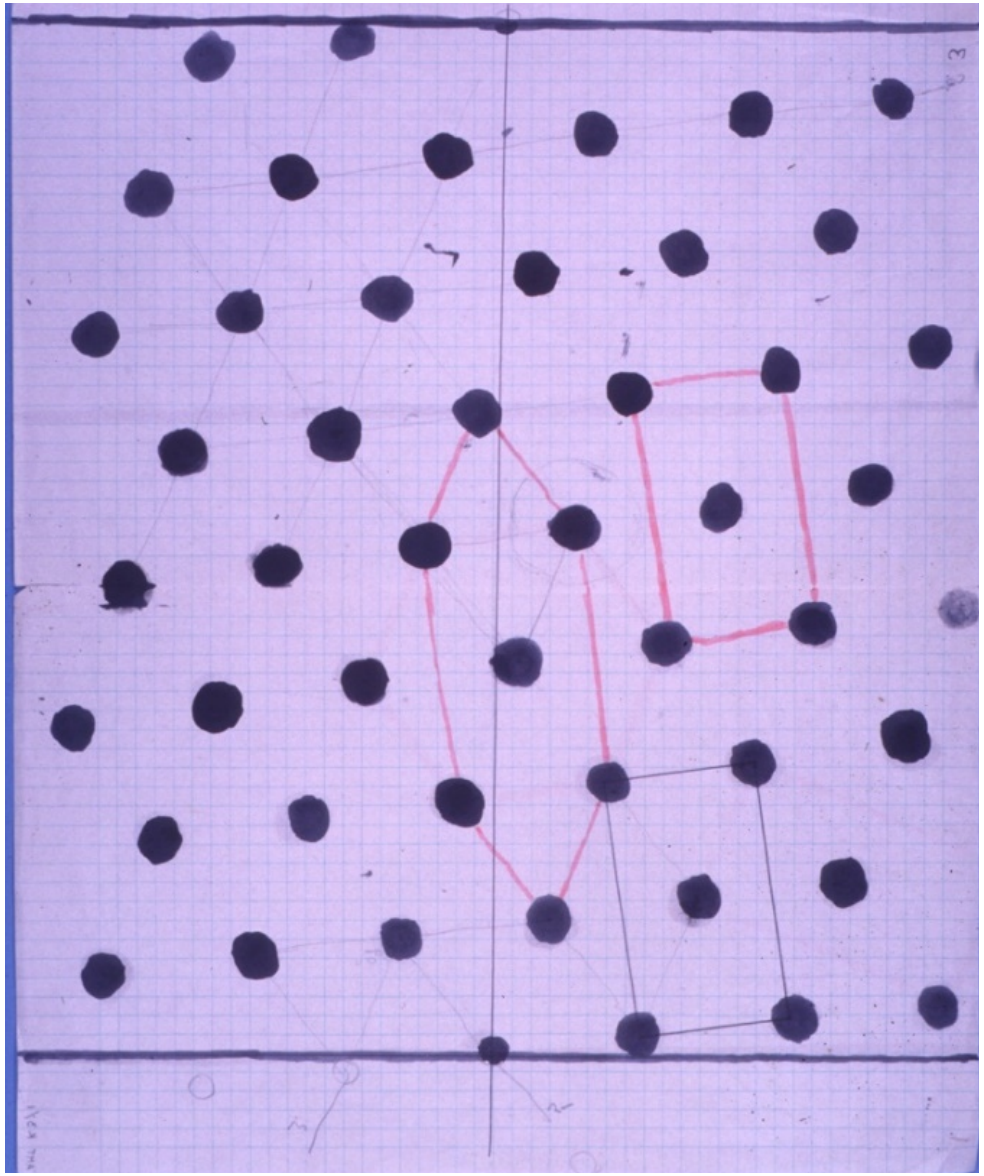


Outline of development of the Daisy.

~~Ey~~ The theory developed in this paper is limited by a number of assumptions which are by no means always satisfied. Two are of special importance







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King's College Cambridge

Dynamic node placement models

- Airy (1873): spheres glued to an elastic band
- Hofmeister's rule: new primordia appears 'periodically' in largest available space
- Snows' rule: new primordia whenever 'enough' space
- Veen & Lindenmayer (1977): Turing-like RD model
- Douady and Couder (1992-) repulsive particles (soft disks)
- Levitov (1991), Kunz (1995) – soft disks
- Mitchison (1977) , Atela et al – coin-dropping models
- All obey the HofGP – why?

HofGP-type models say:

- Placement in older parts of the plant influences placement in younger (later) parts but not vice versa
- Increases in parastichy number in later parts of the plant
- Parastichy numbers remain adjacent members of (usually) the Fibonacci sequence
- If not strict Fibonacci, then adjacent members of one particular Fibonacci-structure sequence
- Ratio of parastichy numbers close to the golden ratio
- Specific models will predict how development responds to noise

- Hard to imagine any other model structure that could predict high Fibonacci numbers such as the sunflower head
- Intrinsically mathematical (or at least computational) – by contrast with eg current developmental biology

New generations of models with noise/ defects

58 M.F. Pennybacker et al. / P

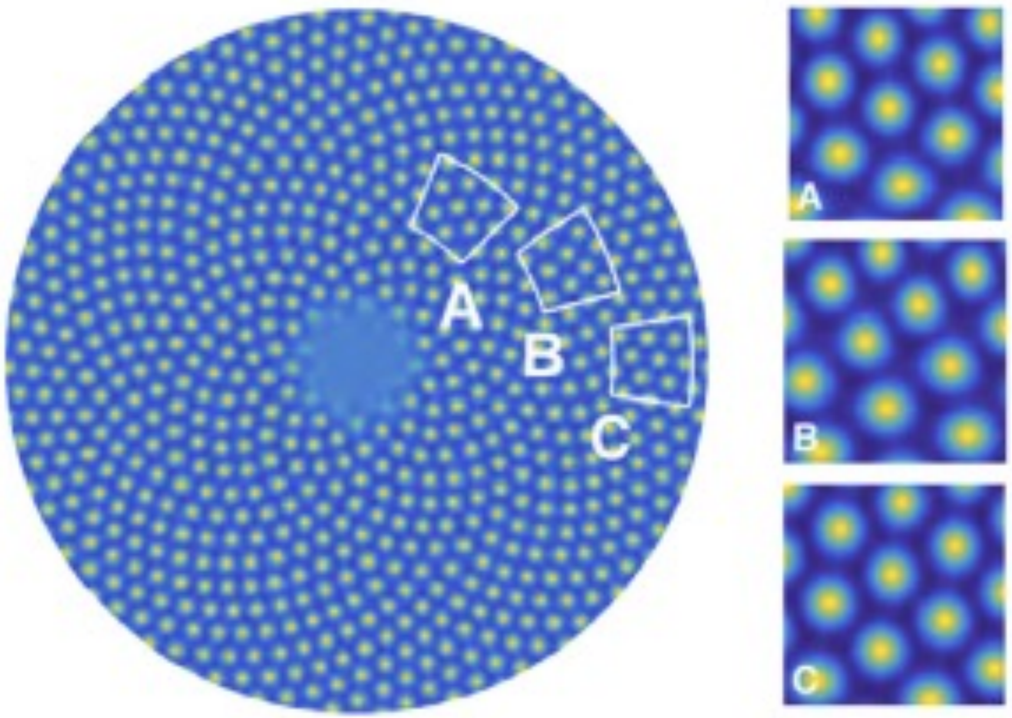


Fig. 13. Simulation of (1) showing a phyllotactic pattern that has propagated as a front from the outer edge towards the center of a disk, a model of pattern formation on a sunflower inflorescence meristem. The simulation is described in detail in Section 3. Cutouts of the pattern centered at A $r = 75/\phi$, B $r = 75/\sqrt{\phi}$, and C $r = 75$, where ϕ is the golden number, transformed as described in the text, appear as panels A–C.

Pennybacker 2015

Literature before 2012

Table 7.7. *Patterns for 319* Helianthus *(sunflower) capituli*

Sequence	Frequency	%
$\langle 1, 2, 3, 5, 8, \ldots \rangle$	262	82.13
$2\langle 1, 2, 3, 5, 8, \ldots \rangle$	9	2.82
$\langle 1, 3, 4, 7, 11, \ldots \rangle$	46	14.42
$\langle 1, 4, 5, 9, 14, \ldots \rangle$	2	0.63

Source: Schoute (1938).

Table 7.8. *Data for 140* Helianthus *capituli*

Sequence	Frequency	%
$\langle 1, 2, 3, 5, 8, \ldots \rangle$	133	95.0
$2\langle 1, 2, 3, 5, 8, \ldots \rangle$	1	0.71
$\langle 1, 3, 4, 7, 11, \ldots \rangle$	6	4.29
$\langle 1, 4, 5, 9, 14, \ldots \rangle$	0	0

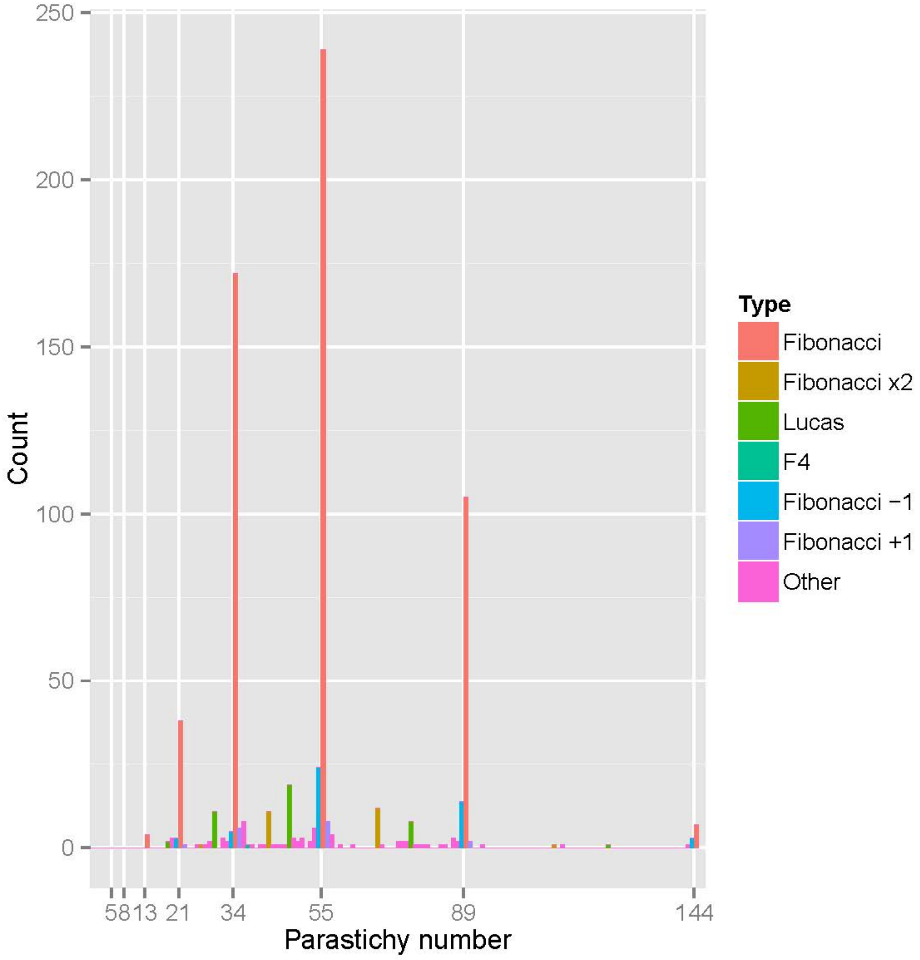
Source: Weisse (1897).

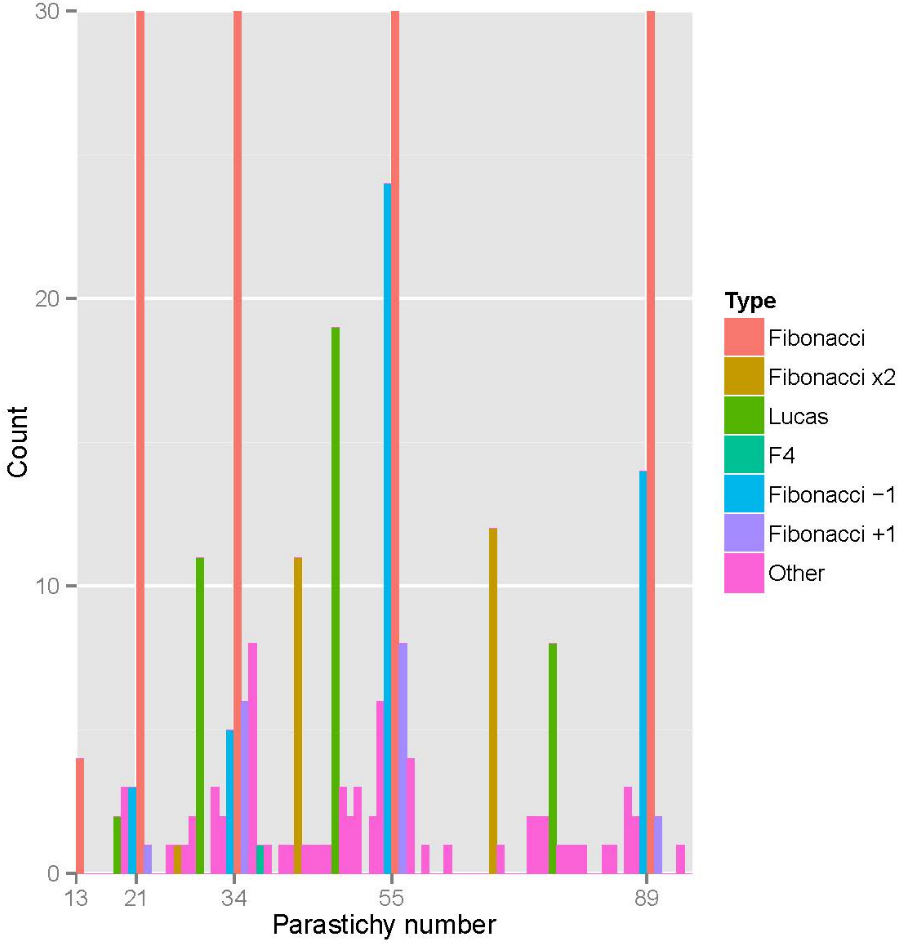
Jean 1994

Fibonacci structure

- Fibonacci rule $F_n = F_{n-1}+F_{n-2}$
  - Any sequence obeying this rule has $F_n / F_{n-1} \to \tau$
  - $\tau \simeq 1.618$ is the golden ratio $\tau^2 = \tau + 1$
- Fibonacci sequence 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 …
- Lucas sequence 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, …
- F4: 1, 4, 5, 9, 14, 23, 37, 60, 97, …
- F5: 1, 5, 6, 11, 17, 28, 45, 73, …
- Double Fibonacci 2, 2 , 4, 6, 10, 16, 26, 42, 68, 110, …

Manchester experiment





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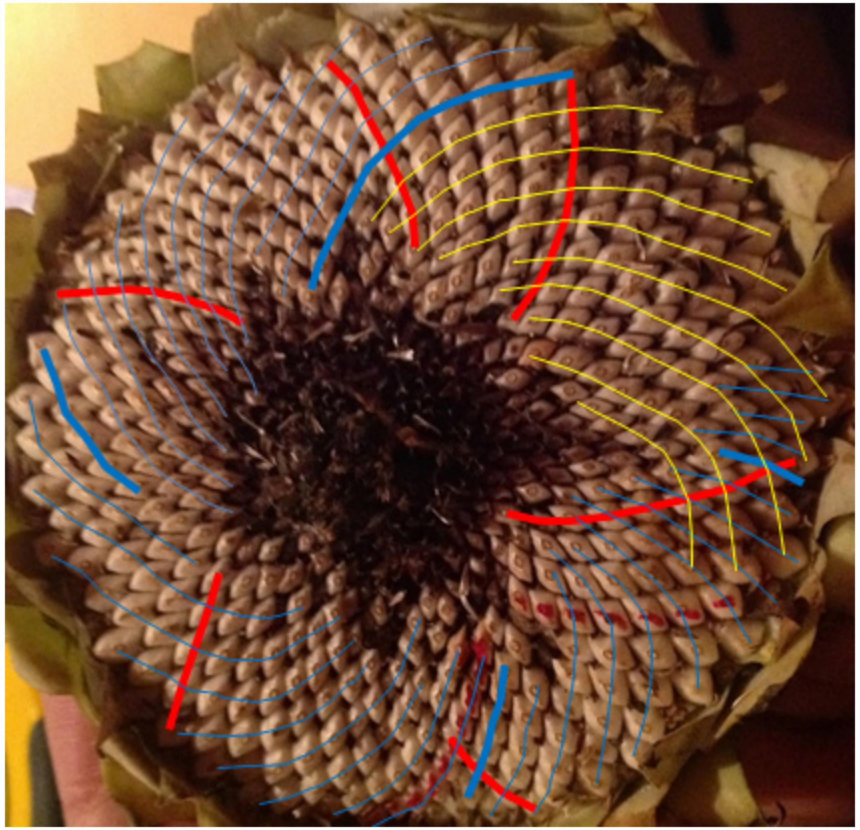
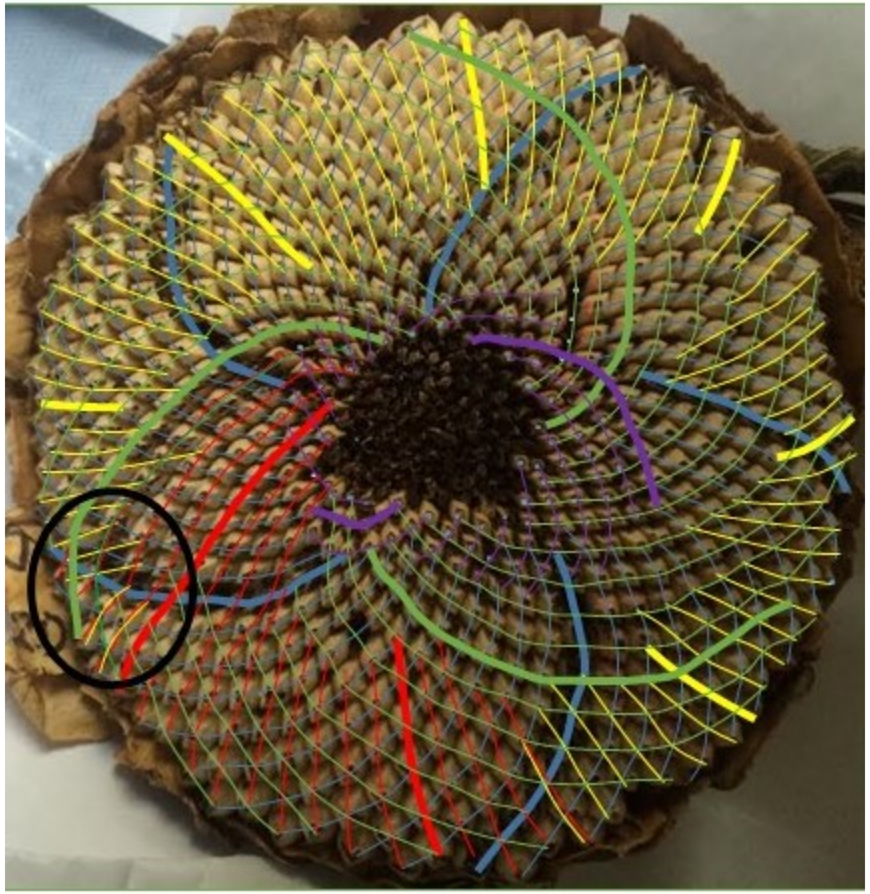
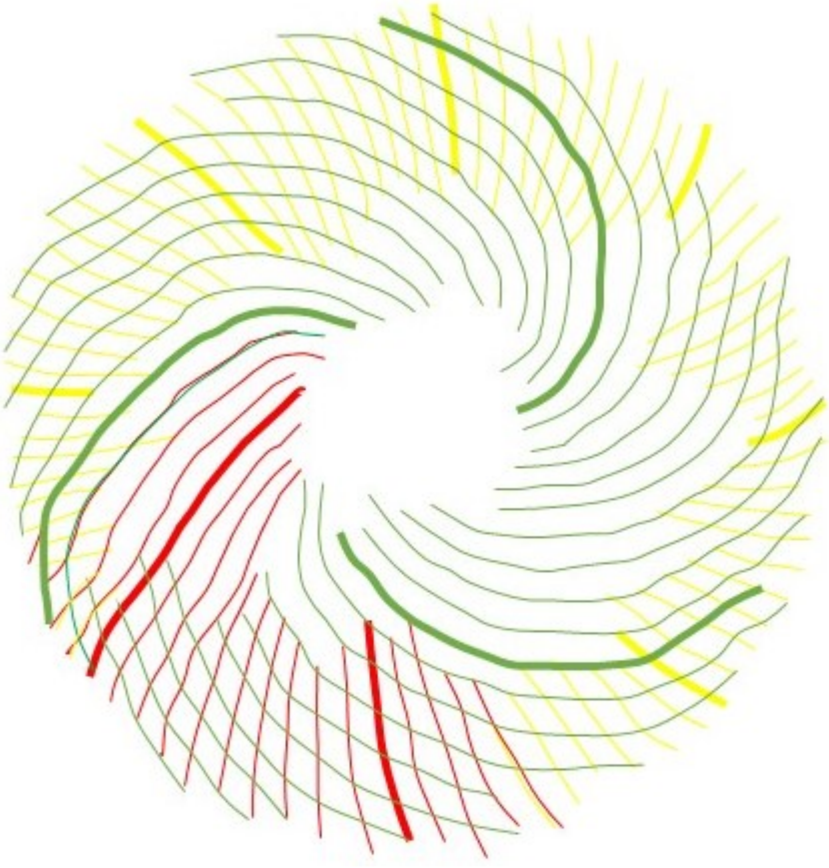
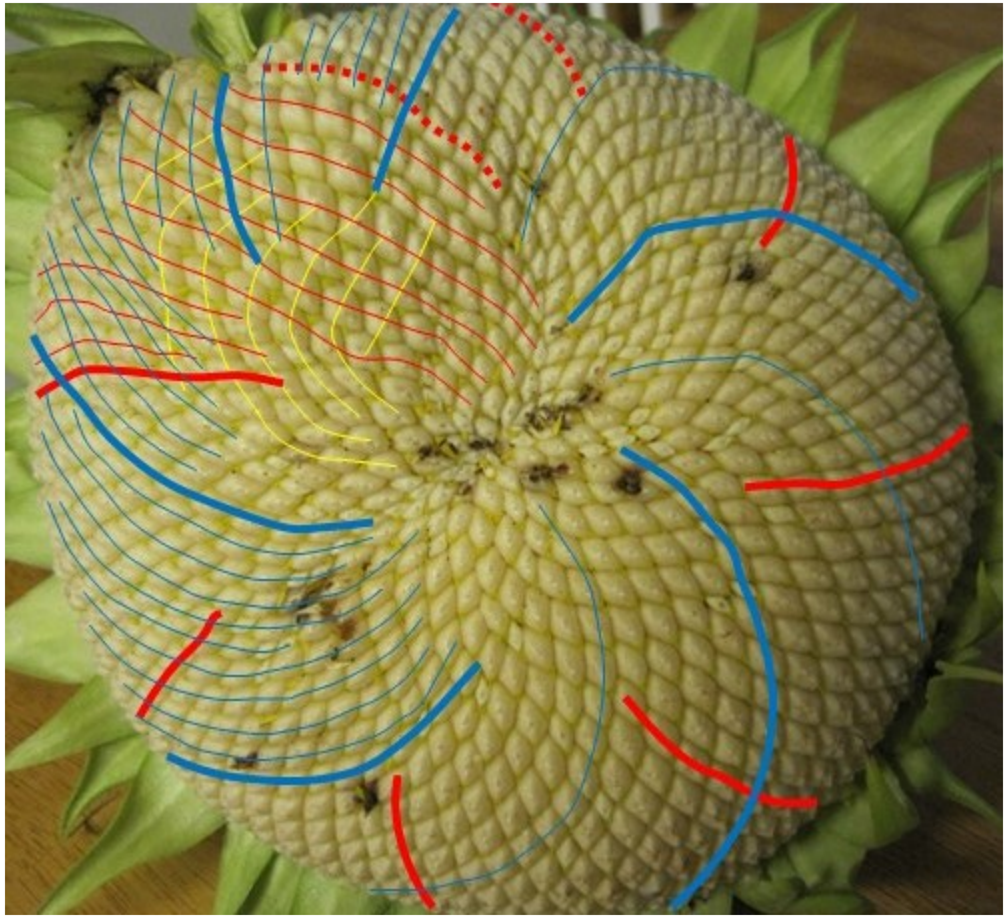


Overlapping parastichies (50,81/31/20)





? (77,56)

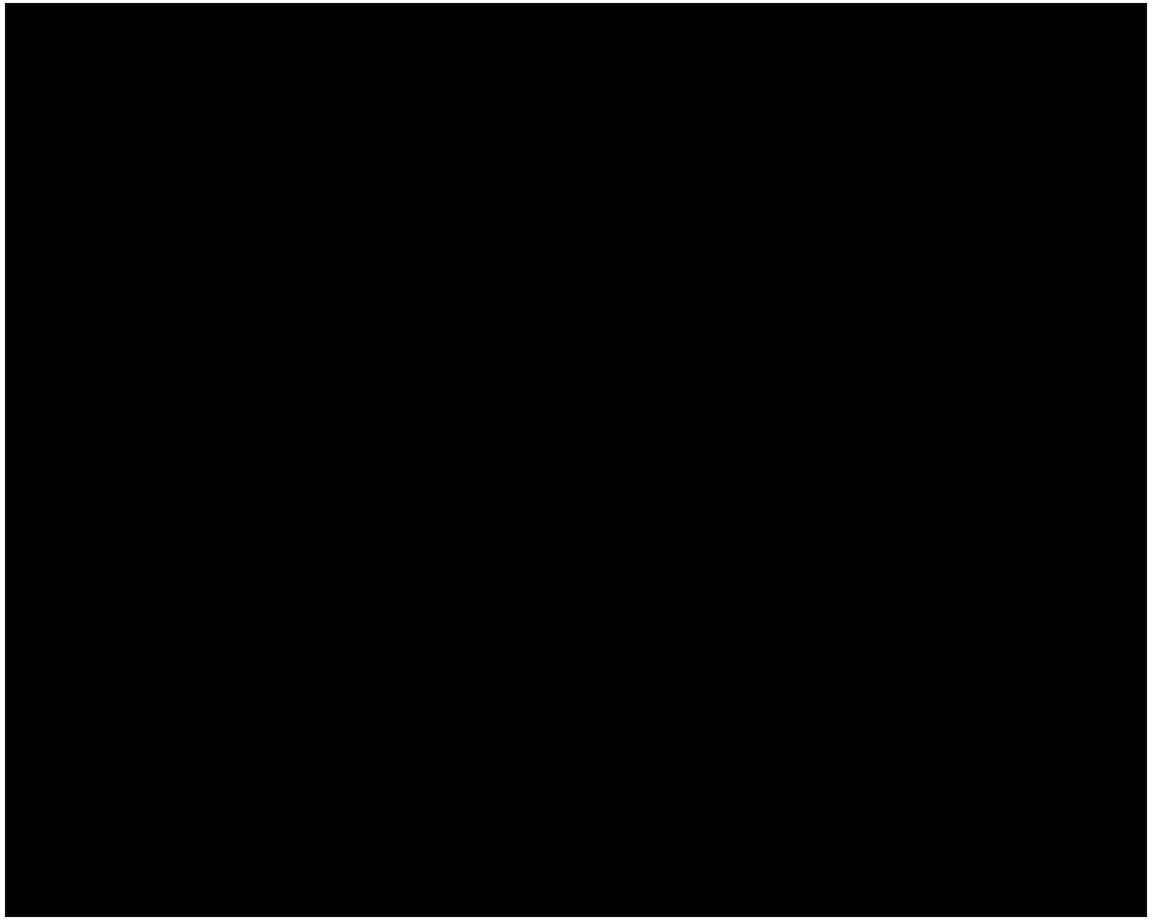


? (62,31)

Manchester results summary

- Fibonacci is commonest; Lucas more common than double Fibonacci
- Approximately Fibonacci is common (54 more common than Lucas 47) mainly F-1, F+1
- F-1 (significantly) more common than F+1
- Common departures from rotational symmetry making parastichies uncountable
- Count in one direction often much more ordered than in the other
- Some seedheads possess completely disordered regions

Turing's sunflowers: a citizen science experiment



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turingssunflowers.org

Summary

- Fibonacci phyllotaxis is a real phenomenon whose observation deserved replication
- We have plausible mathematical reasons why it should exist – although some more work on why the HofGP is often true would be worthwhile
- The assumptions and predictions of these models are not well connected to empirical biology
- If they were, they would likely make a powerful argument for systems biology
- The Manchester dataset should be useful for this – let's make some more!


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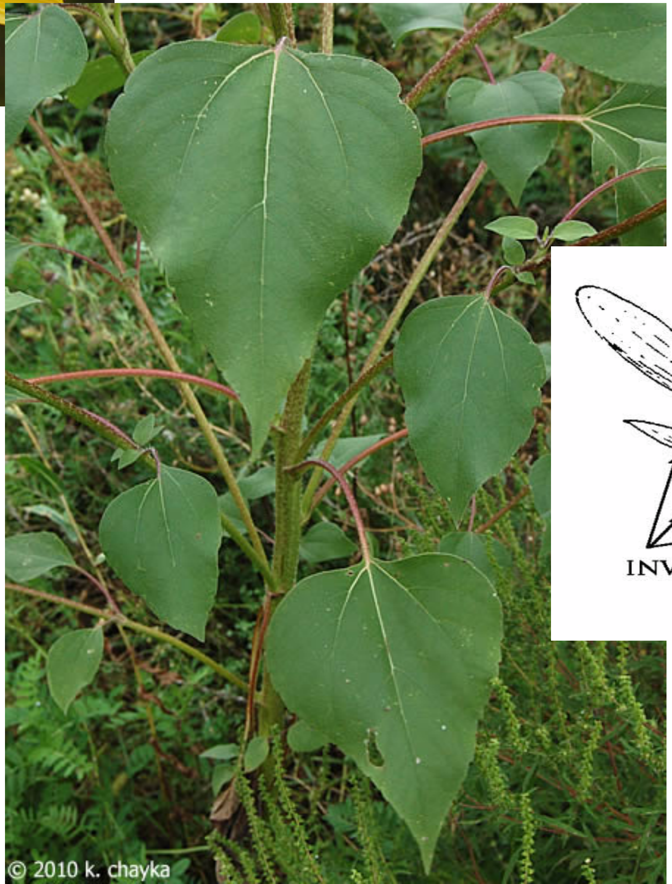
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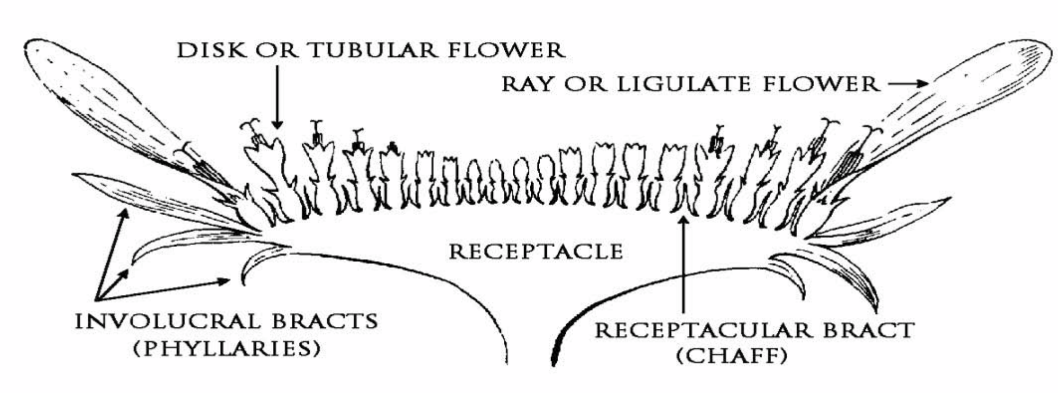
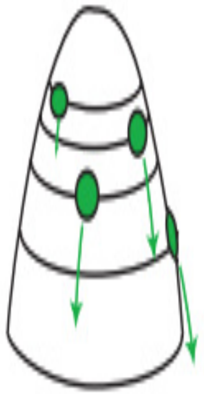
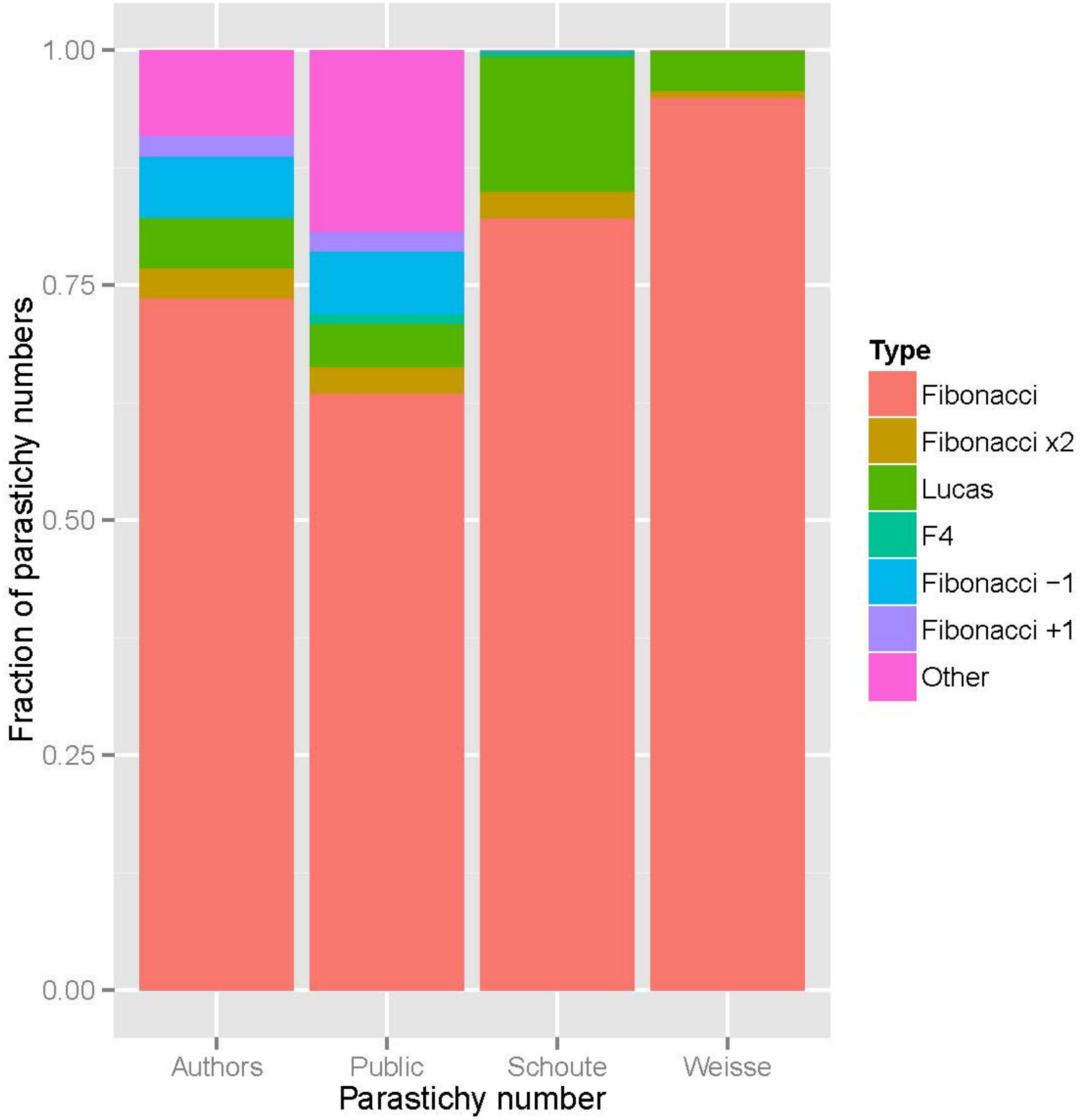

DISK OR TUBULAR FLOWER
RAY OR LIGULATE FLOWER
RECEPTACLE
INVOLUCRAL BRACTS
(PHYLLARIES)
RECEPTACULAR BRACT
(CHAFF)

garden.org



Atela 2002

Comparison with previous studies



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