Ephemeral Transport Boundaries in Geophysical Flows

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GULF OF MEXICO RESEARCH INITIATIVE

Professor Daniel Karrasch
Professor George Haller

Principle of Objectivity

- Concept applied by Noll (1955, 1958, 1959) to viscous processes in non-Newtonian fluids
- Speziale et al. (1981, 1991, 1996) applied concept to turbulence closure schemes
- Haller et al. (2005, 2013, 2015, 2016, 2017) applied concept to identification of LCS

Unifying theme: Reconcile theory and observations for nonlinear behavior of fluids

Exordium

Material properties are independent of the observer. – *W. Noll*

As far as the laws of mathematics refer to reality, they are uncertain, and as far as far as they are uncertain, they do not refer to reality. – A. Einstein

Some results are just days old!

Goals

1. Oceanographers should worry about Objectivity. I shall illustration why.
2. Study the impact of traveling internal waves on mesoscale transport pathways.

Procedure

Flow fields from solutions to linear, stratified, 3D Euler equations on f-plane.

Stratified 3D Euler Equations on f-plane

Euler Equations

$$\frac{\partial u}{\partial t} - fv = -\rho_0^{-1}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + fu = -\rho_0^{-1}\frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} = \rho_0^{-1}\left(\rho g - \frac{\partial p}{\partial z}\right)$$

$$\frac{\partial \rho}{\partial t} = \left(\frac{\rho_0 N^2}{g}\right) w$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Solution

$$u = -lA_g(z)\sin kx \cos ly - \sum_j \left(A_j(z)\frac{\partial \Psi_j(x,y,t)}{\partial y} - \frac{dB_j(z)}{dz}\frac{\partial \Phi_j(x,y,t)}{\partial x} \right)$$

$$v = kA_g(z)\cos kx \sin ly + \sum_j \left(A_j(z)\frac{\partial \Psi_j(x,y,t)}{\partial x} + \frac{dB_j(z)}{dz}\frac{\partial \Phi_j(x,y,t)}{\partial y} \right)$$

$$w = -\sum_j B_j(z)\nabla_h^2 \Phi_j(x,y,t) = \sum_j \left(k_j^2 + l_j^2\right) B_j(z)\Phi_j(x,y,t)$$

$$p = p_0(z) + p_g(z)\sin kx \sin ly + \sum_j p_j(z)\Psi_j(x,y,t)$$

$$\rho = \rho_0(z) + \rho_g(z)\sin kx \sin ly + \sum_j \rho_j(z)\Psi_j(x,y,t)$$

Eigen Relations

$$p(x, y, z, t) = p_0 + p_g \sin kx \sin ly + p_j \Psi_j$$

$$\left[\frac{d^2}{dz^2} + \left(\frac{N^2}{g} \right) \frac{d}{dz} + \left(\frac{N^2}{gh_j} \right) \right] p_j = 0$$

$$\left\{ \left[\nabla_h^2 - \left(\frac{\partial^2}{\partial t^2} + f^2 \right) (gh_j)^{-1} \right] + \beta \mathcal{L} \right\} \Psi_j = 0$$

$\beta = 0$

Dynamic Constraints

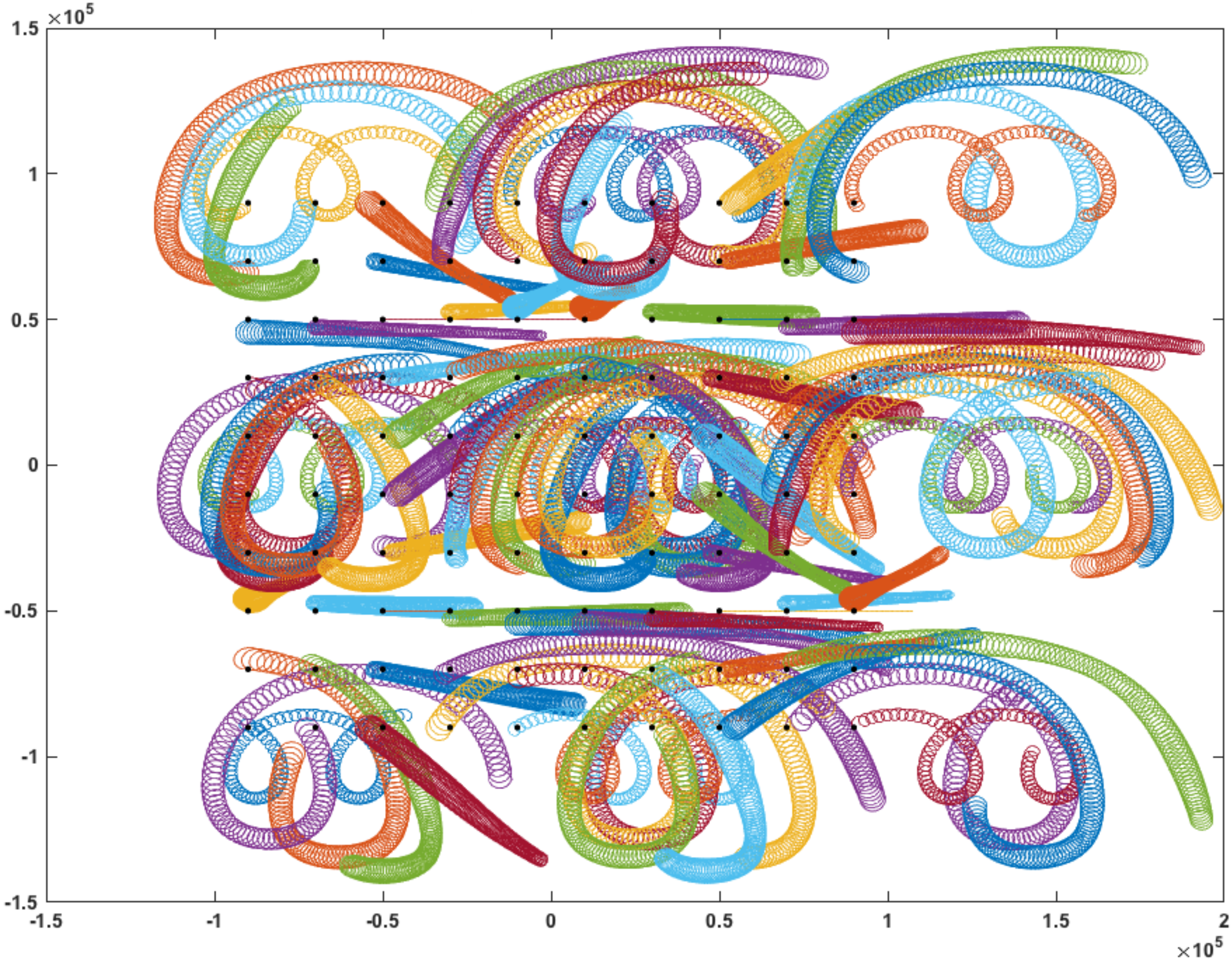
$$\omega_j^2 = f^2 + \frac{N^2 H^2}{L_j^2}$$

$$\frac{\partial \Psi_j}{\partial t} = -\omega_j \Phi_j$$

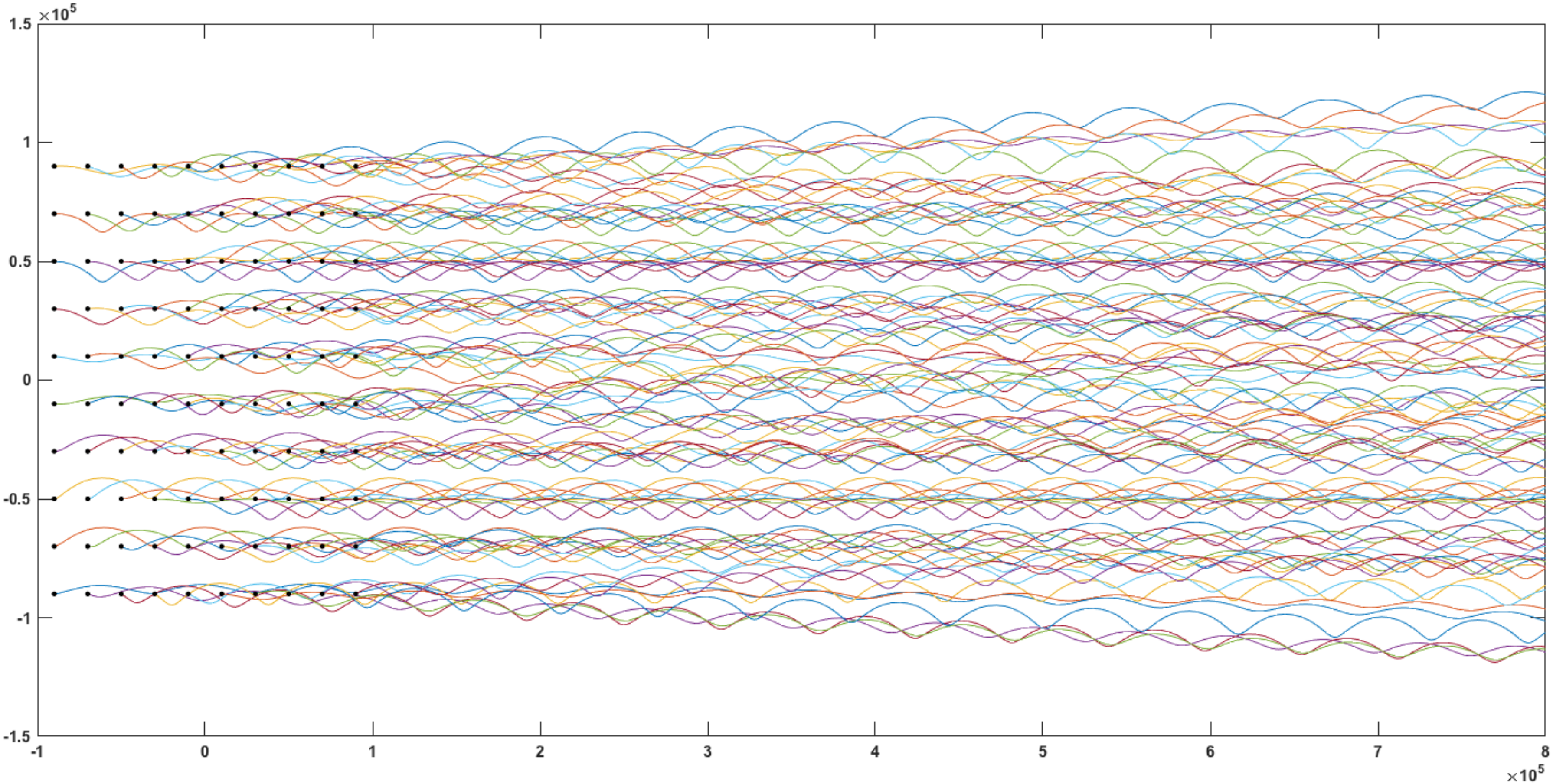
$$\frac{\partial \Phi_j}{\partial t} = \omega_j \Psi_j$$

Objectivity – Comparison of 2 Flow Realizations

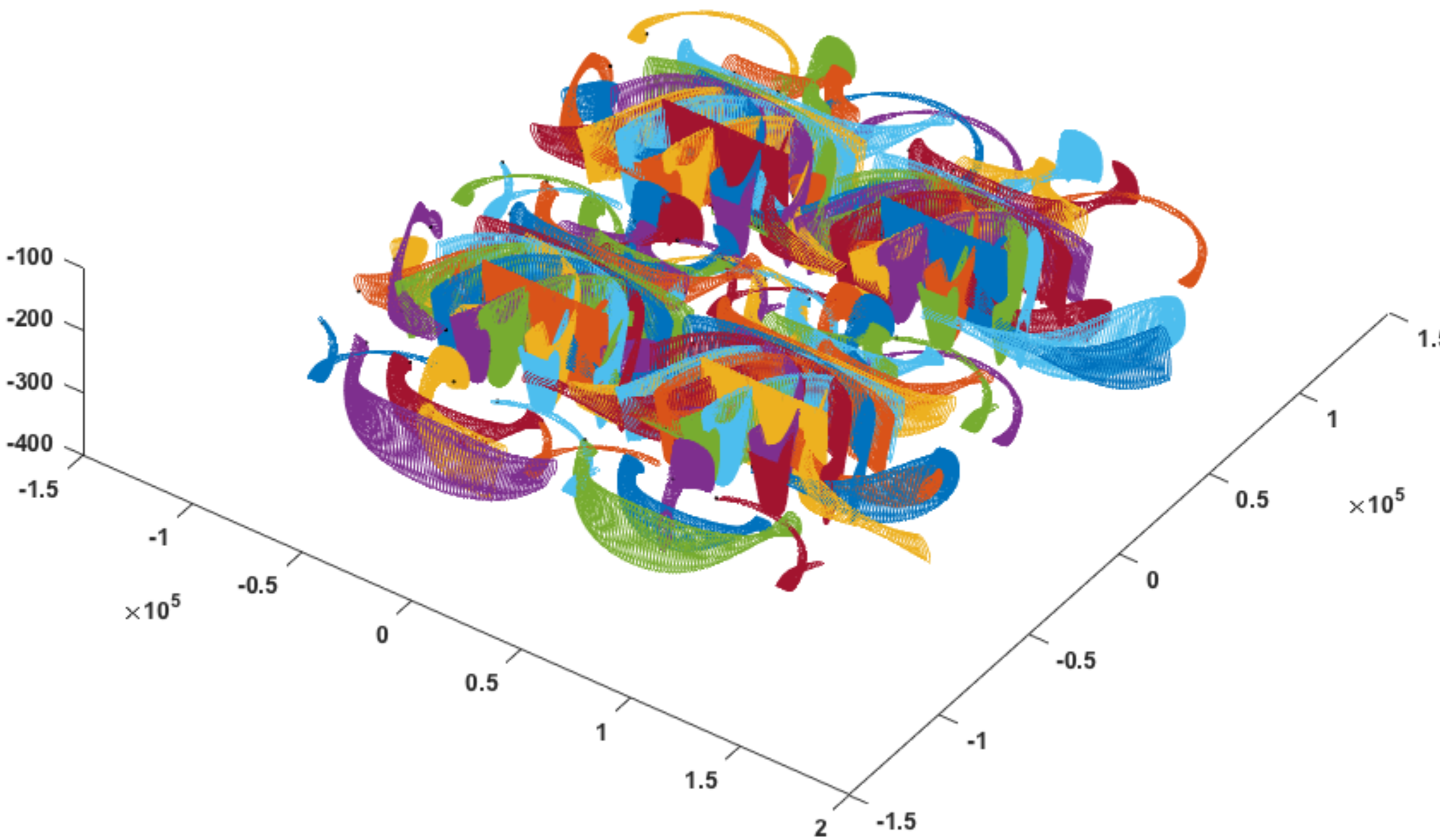
Surface Realization 1



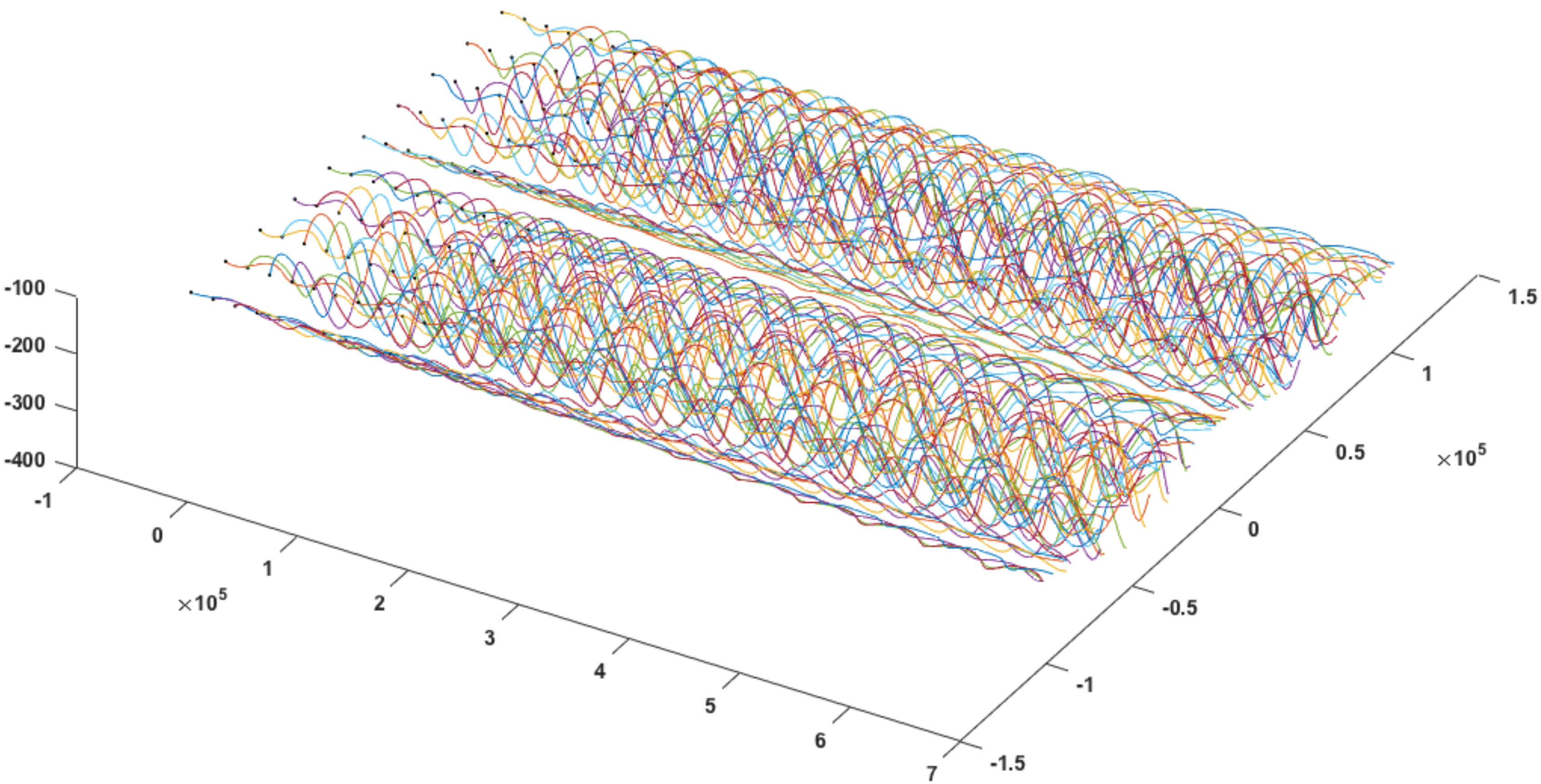
Surface Realization 2



Subsurface Realization 1

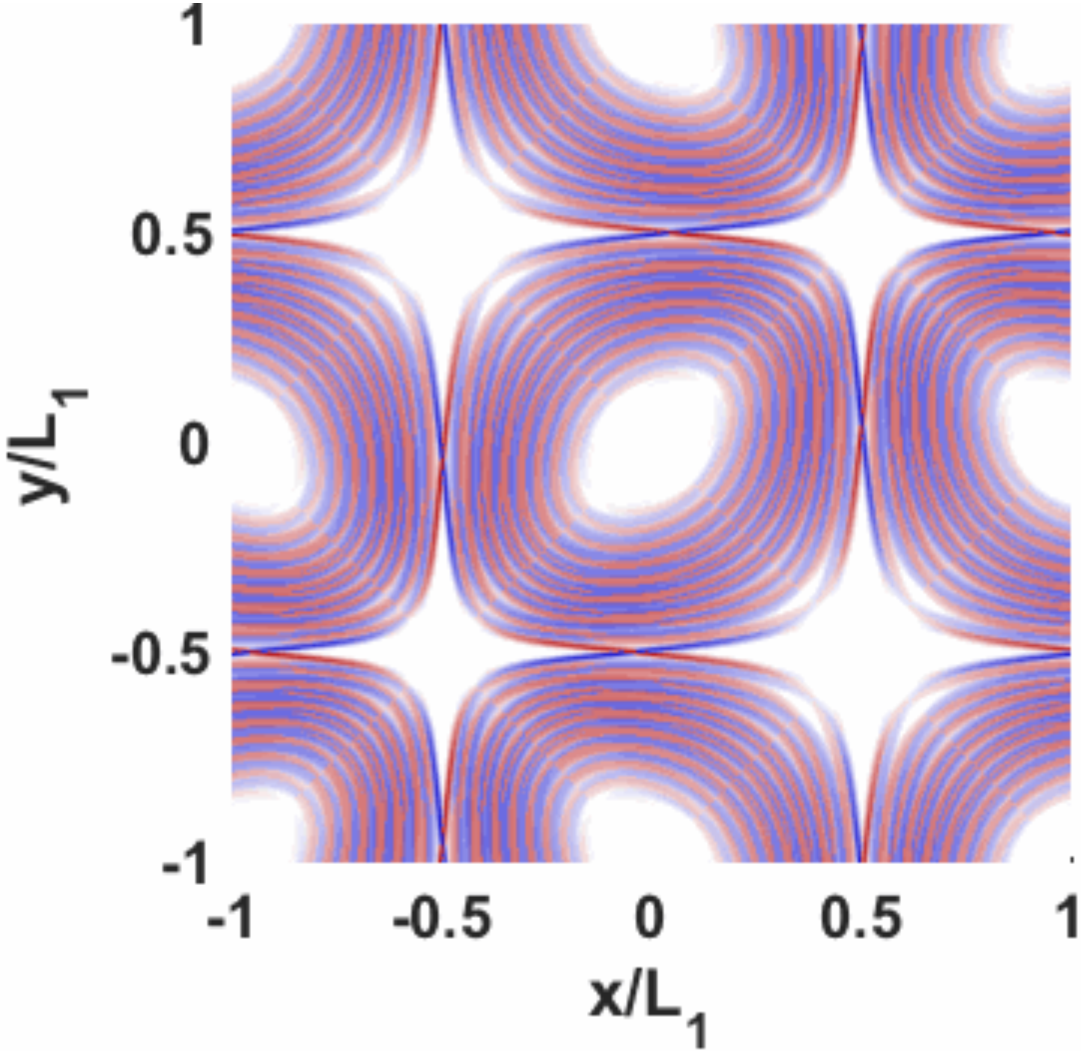


Subsurface Realization 2

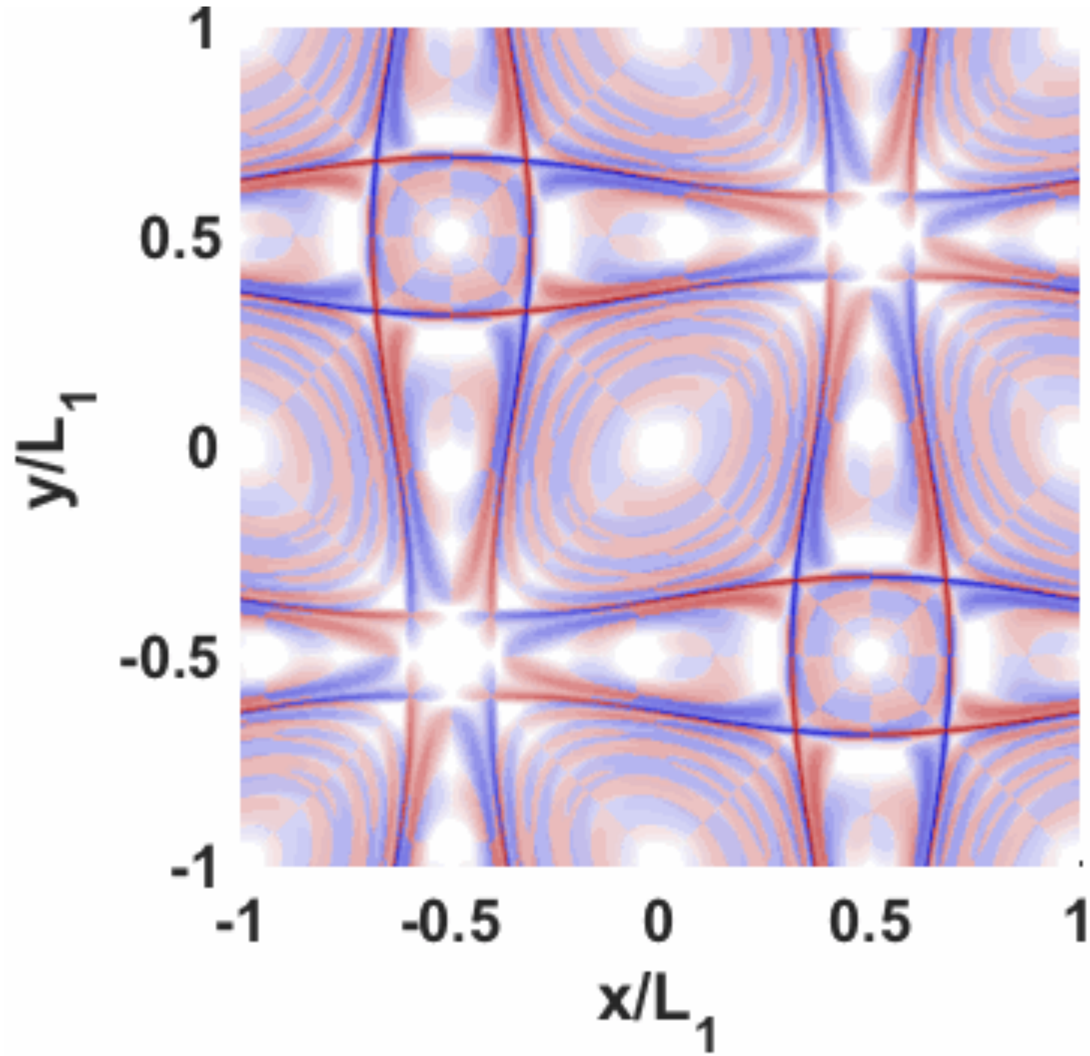


Objective Diagnostics – FTLE

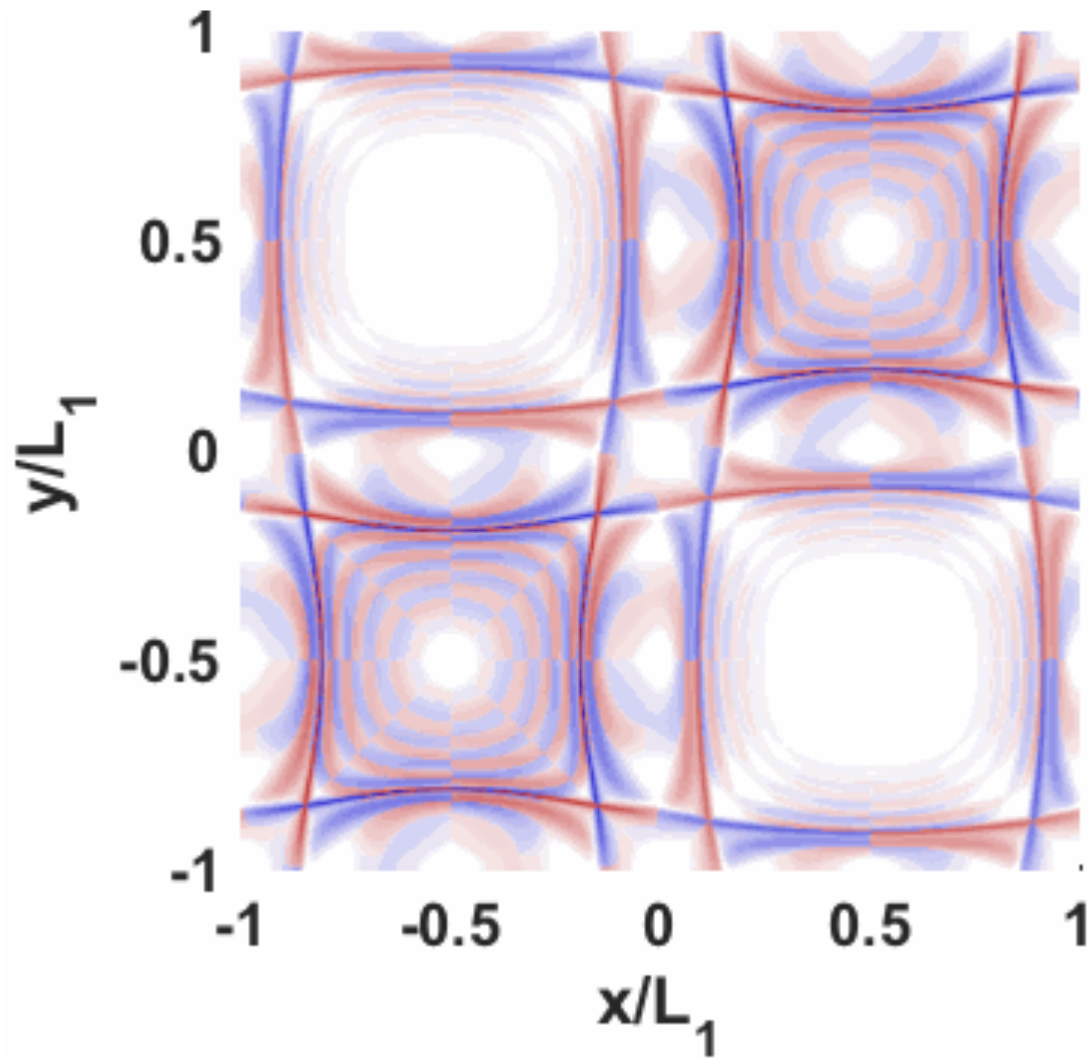
FTLE at z = 0



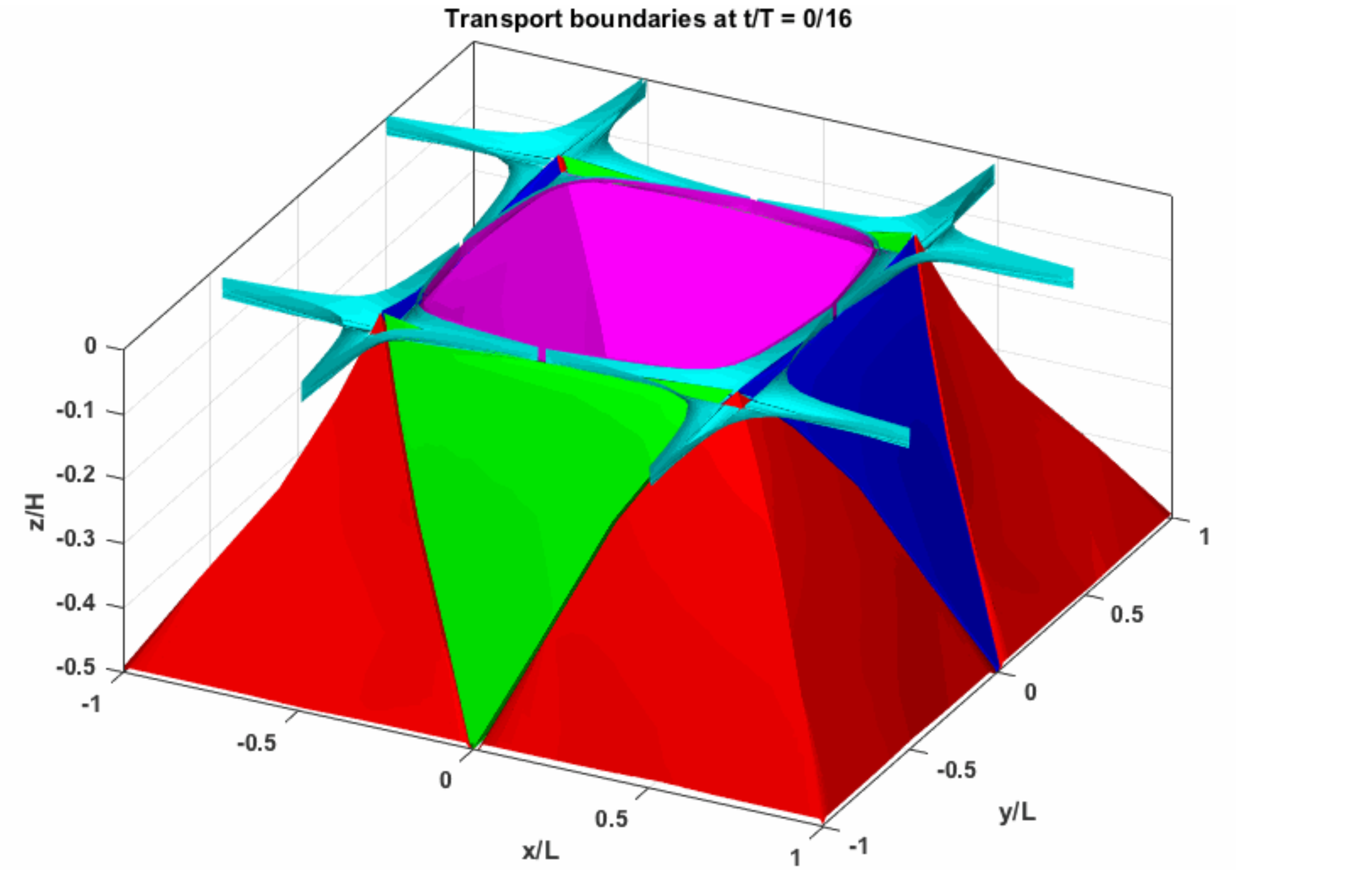
FTLE at z = H/8



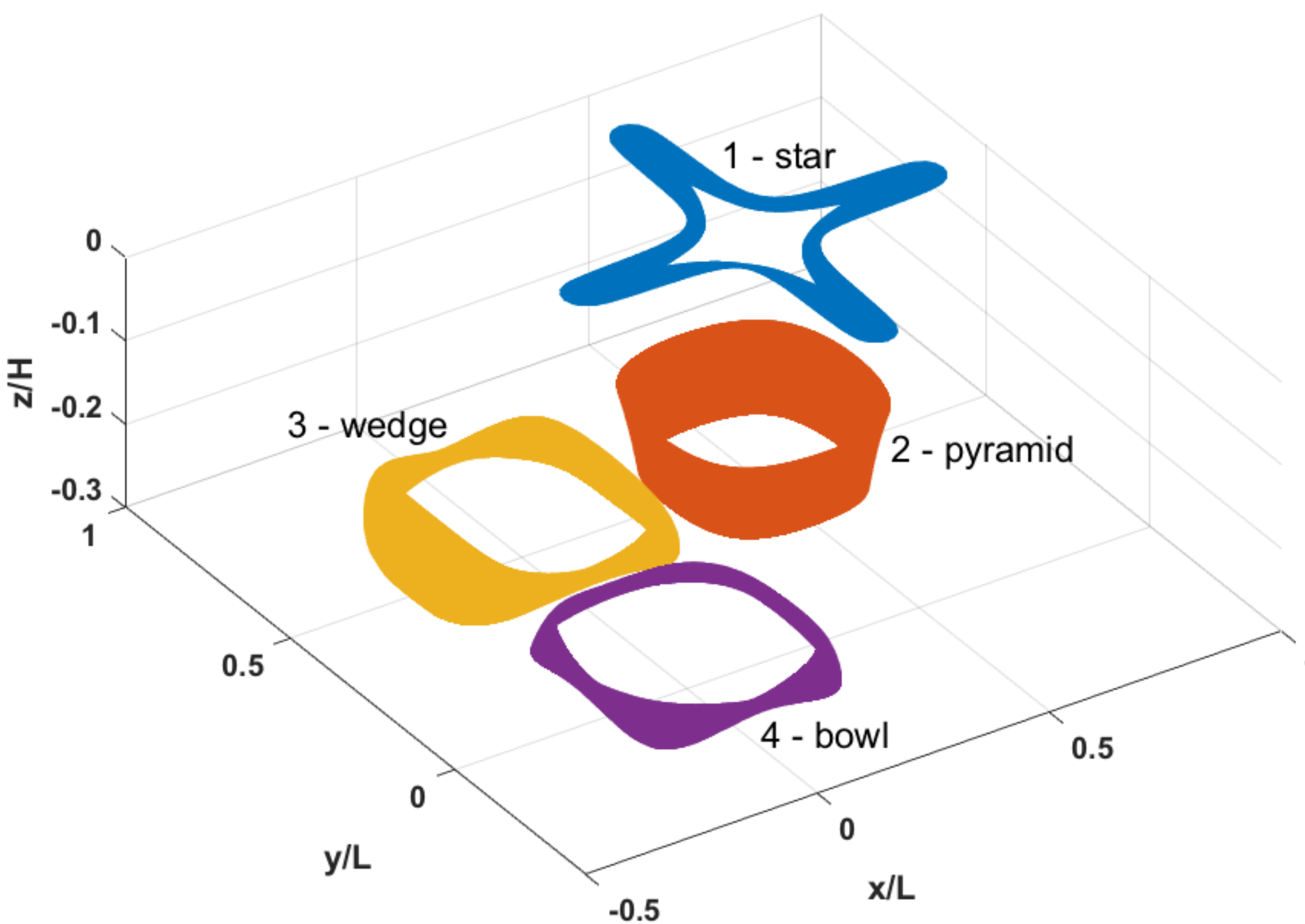
FTLE at z = 3 H/8



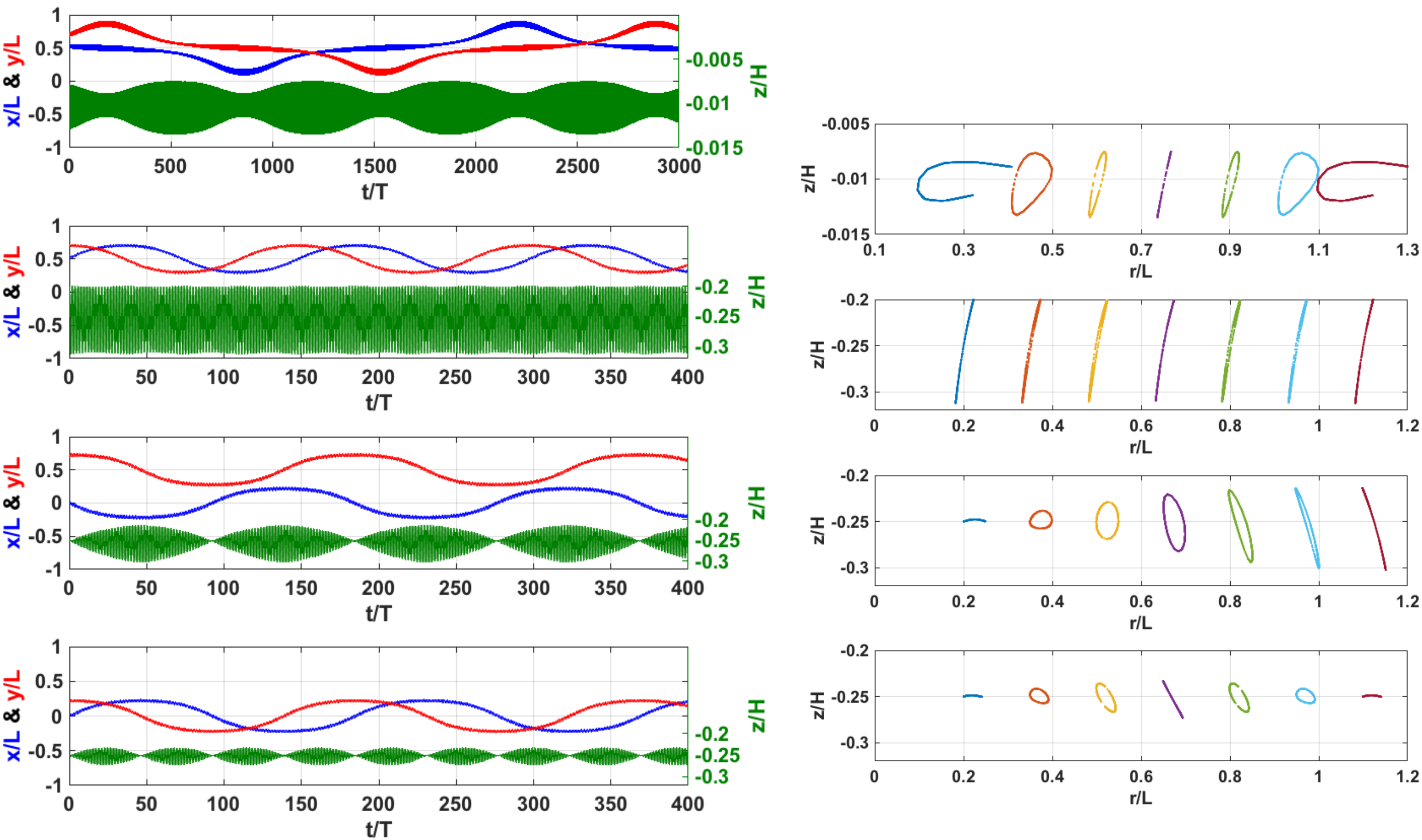
Transport Boundaries



Representative 3D Trajectories



Time Series & Poincare Sections



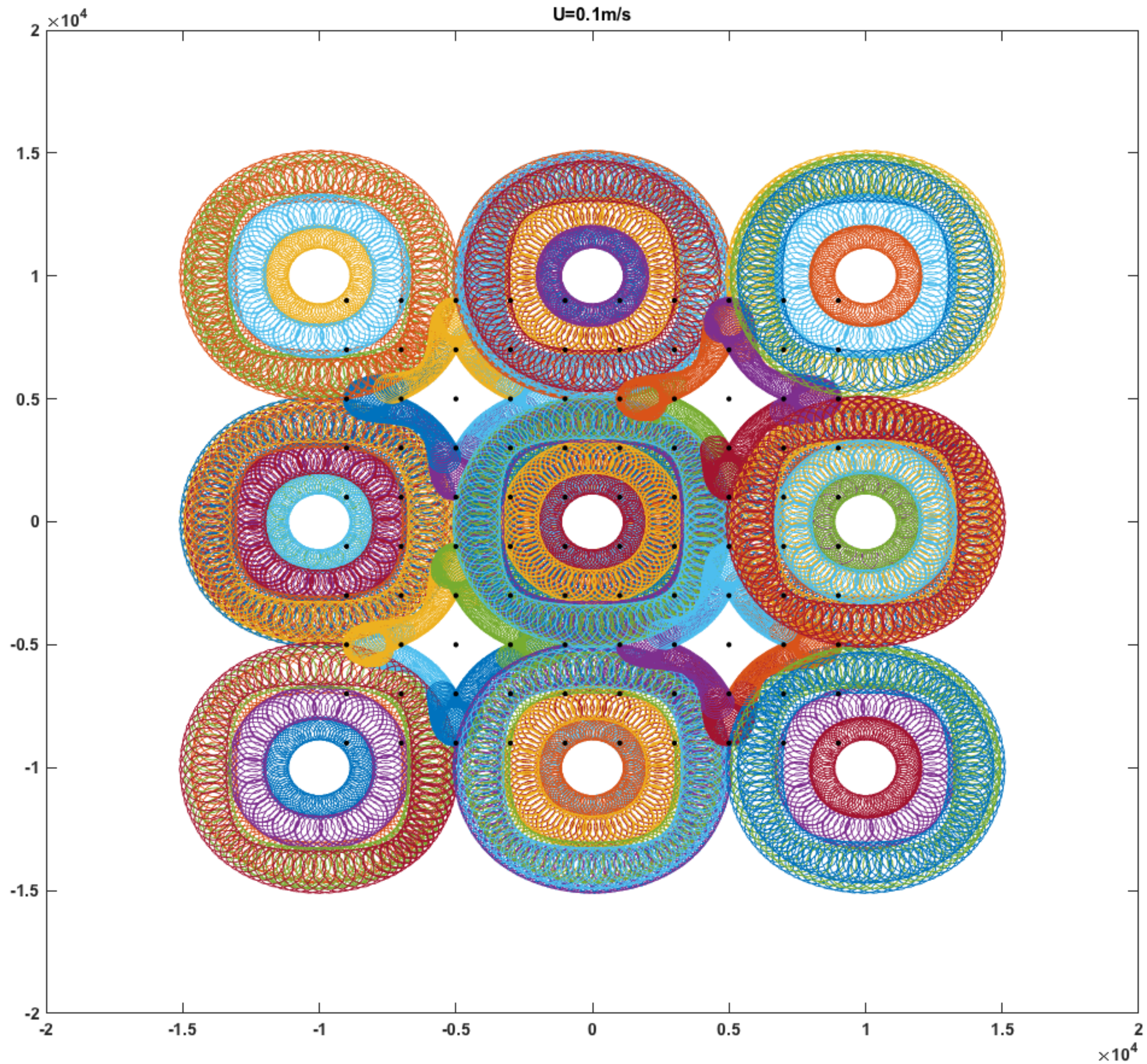
Recap

- One flow field as seen by 2 observers
- Internal wave solution with 4 well-defined transport boundaries
- Boundaries organized by critical trajectories that oscillate at internal wave frequency
- All other trajectories live on ‘strange’ tori with highly variable return periods
- Consequence: aperiodic mixing within each transport region — distinct from traditional IW diapycnal mixing

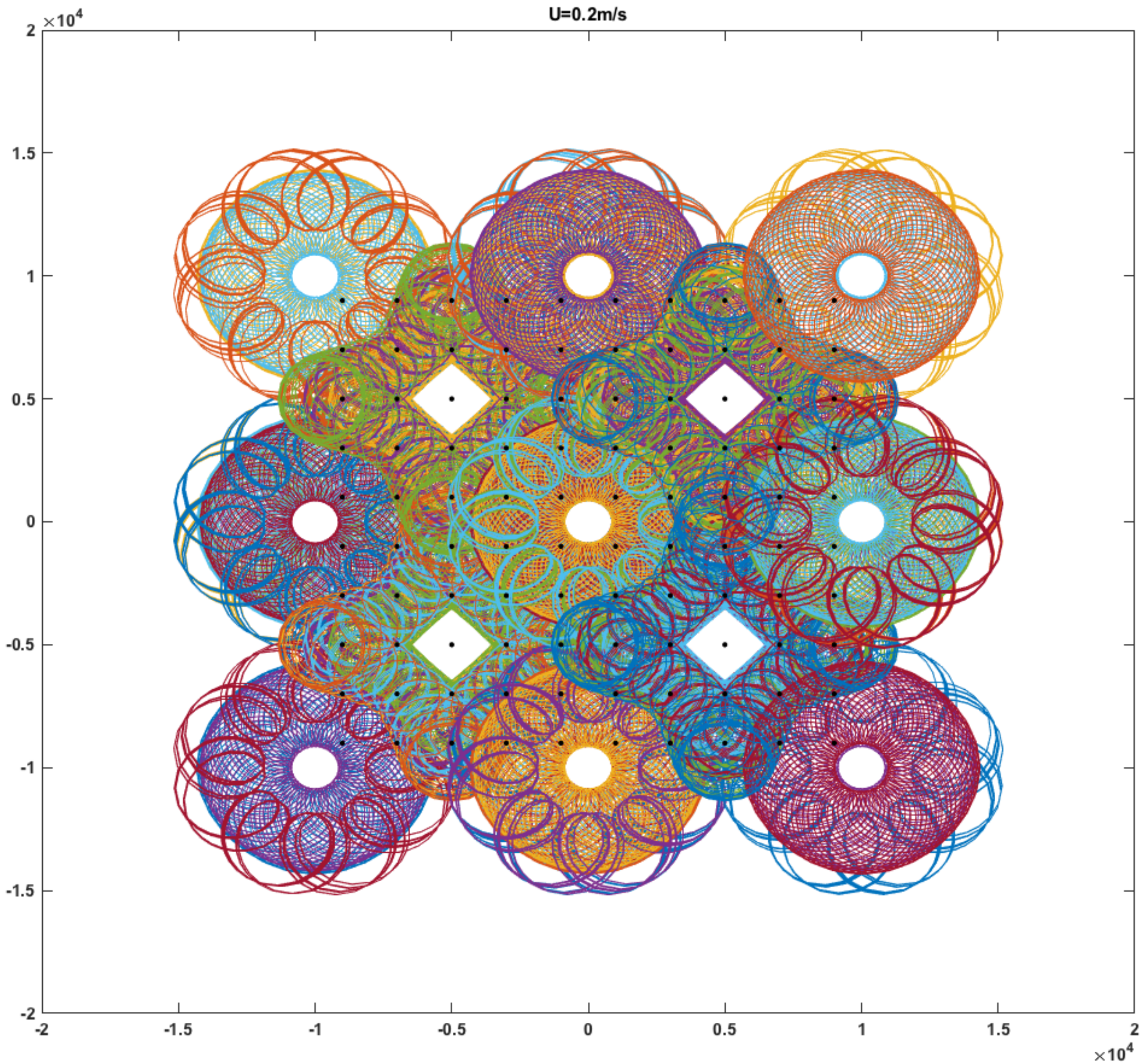
Sidebar – Another Solution

- Waves occur at many different scales in oceanography
- Breaking waves are common in oceanography
- What happens when particle velocity exceeds phase velocity of disturbance?

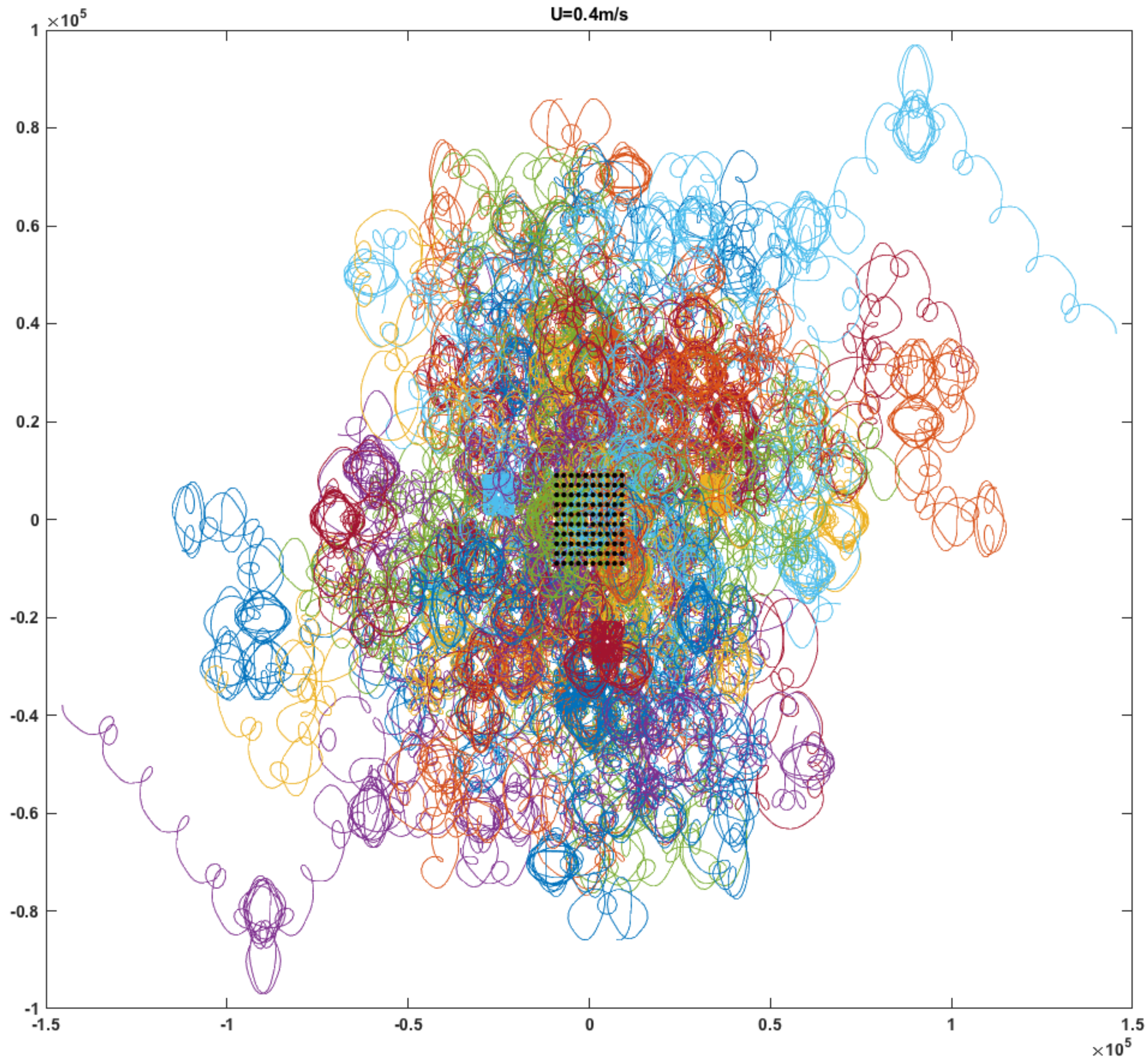
C_p = 35 cm/s, U_W = 10 cm/s



$C_p = 35$ cm/s, $U_W = 20$ cm/s



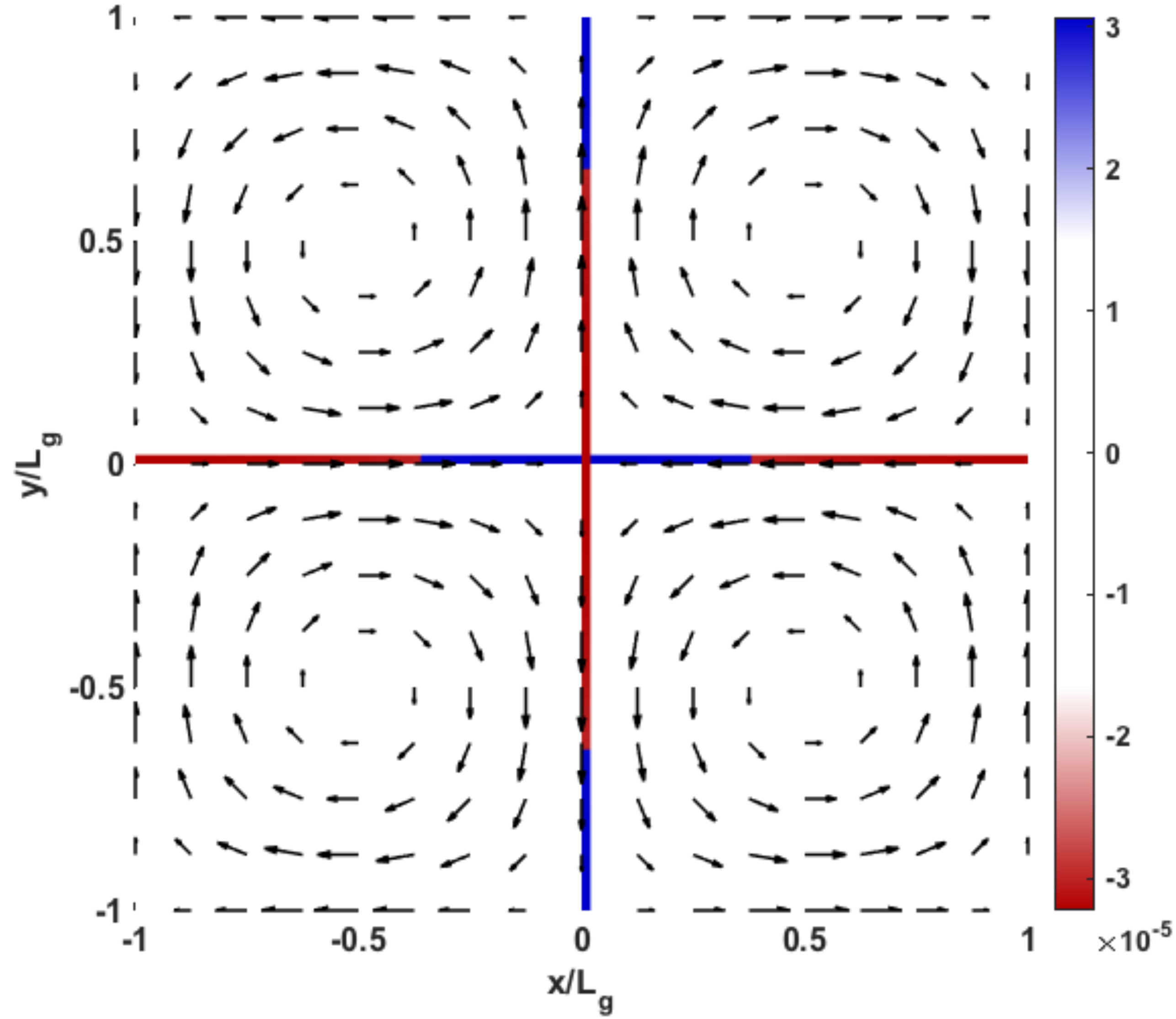
Incoherent Mixing: C_p = 35 cm/s, U_W = 40 cm/s



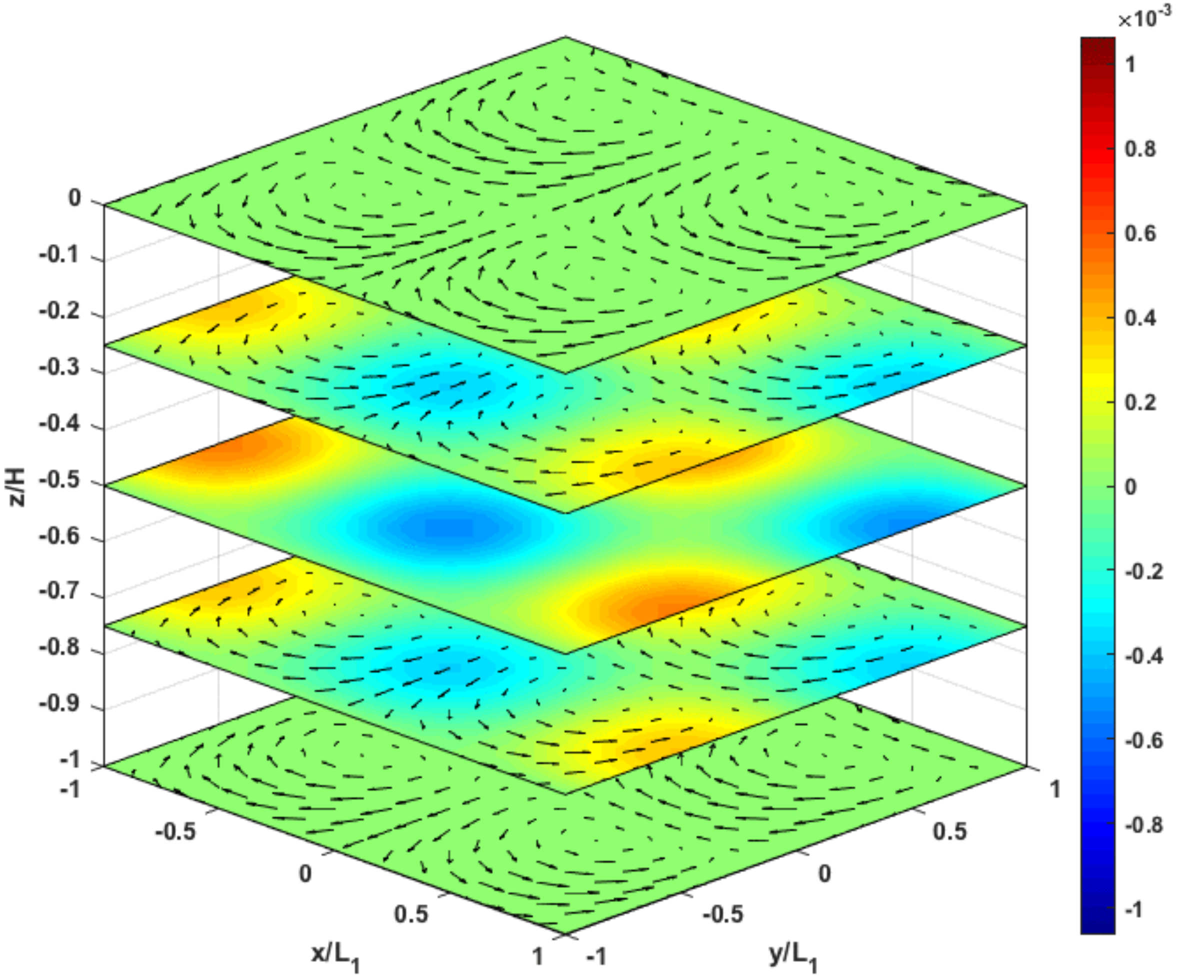
Can Internal Waves Disrupt Mesoscale Transport Pathways?*

*Chang et al., MS 162 Superior B 2:15 today: A Stratified 3D Model for Ocean Flows with Internal Waves

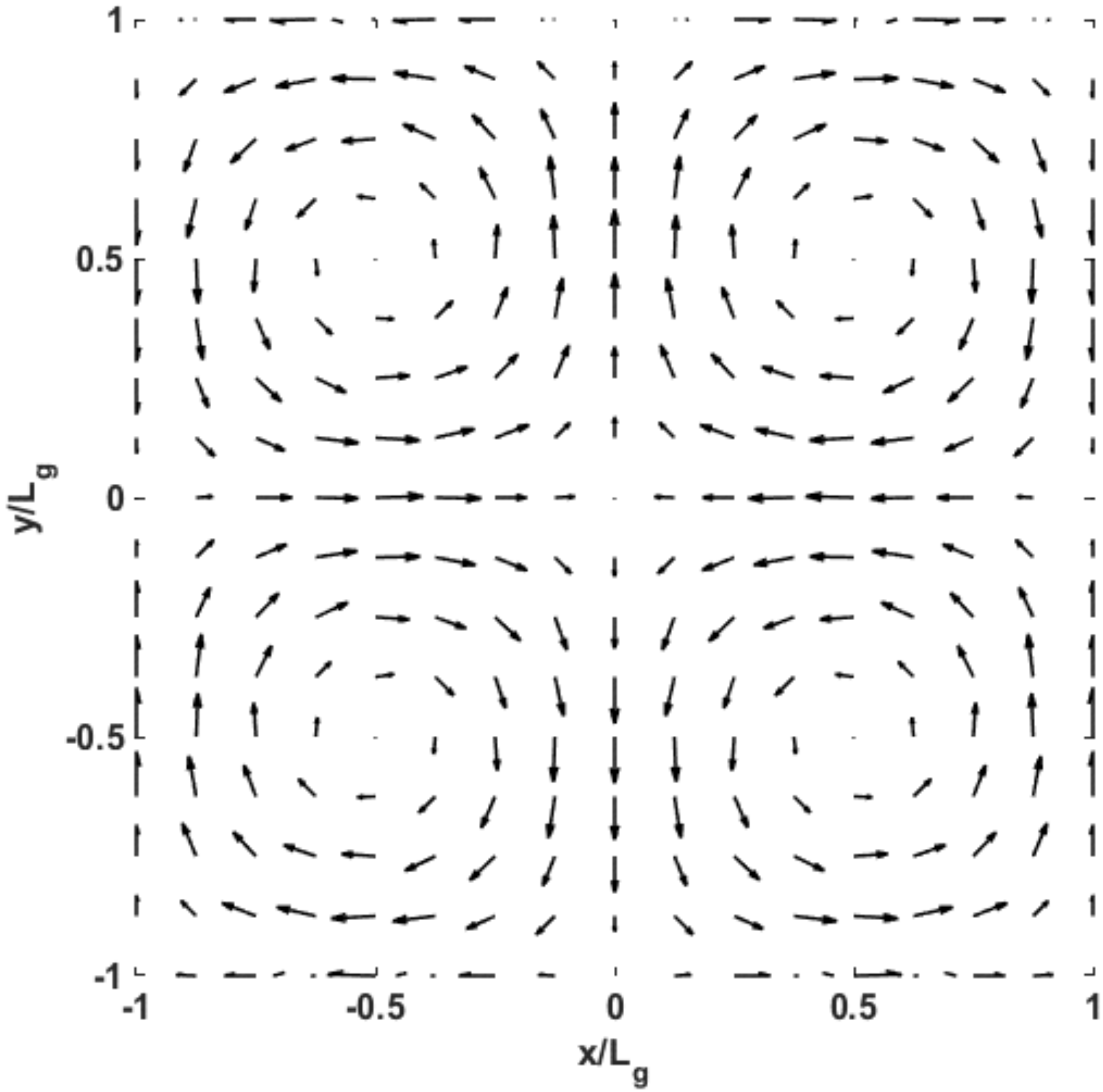
Surface Velocity & FTLE for QP



Traveling Internal Wave Velocity

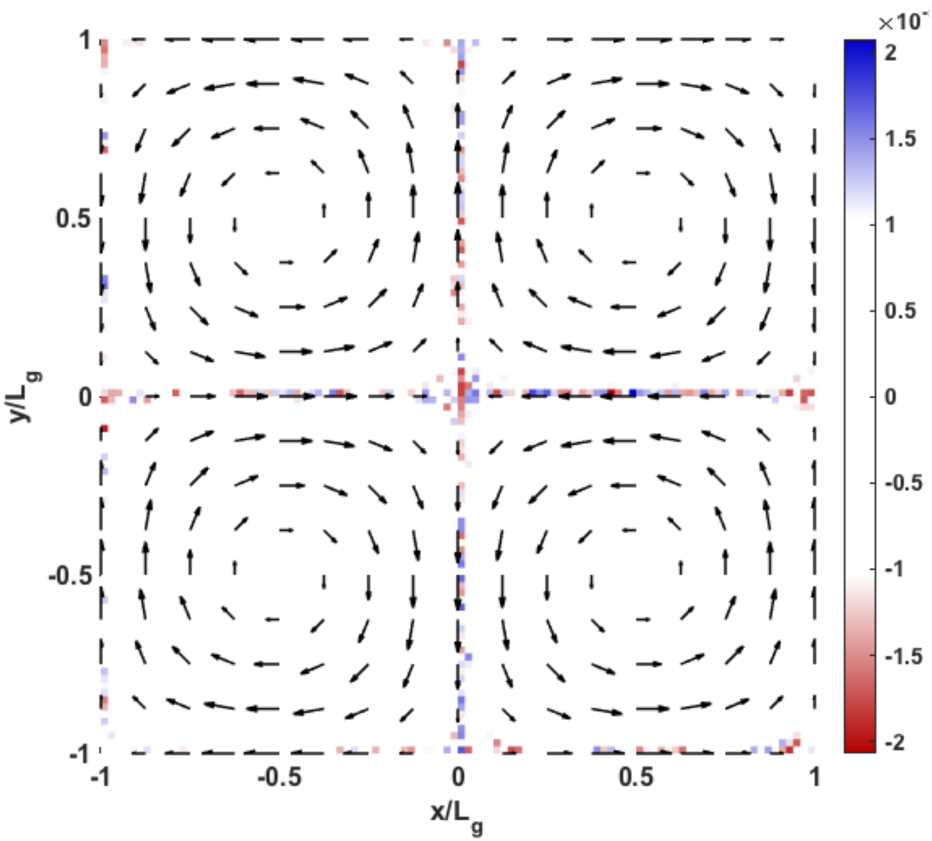


QP + IW Surface Velocity

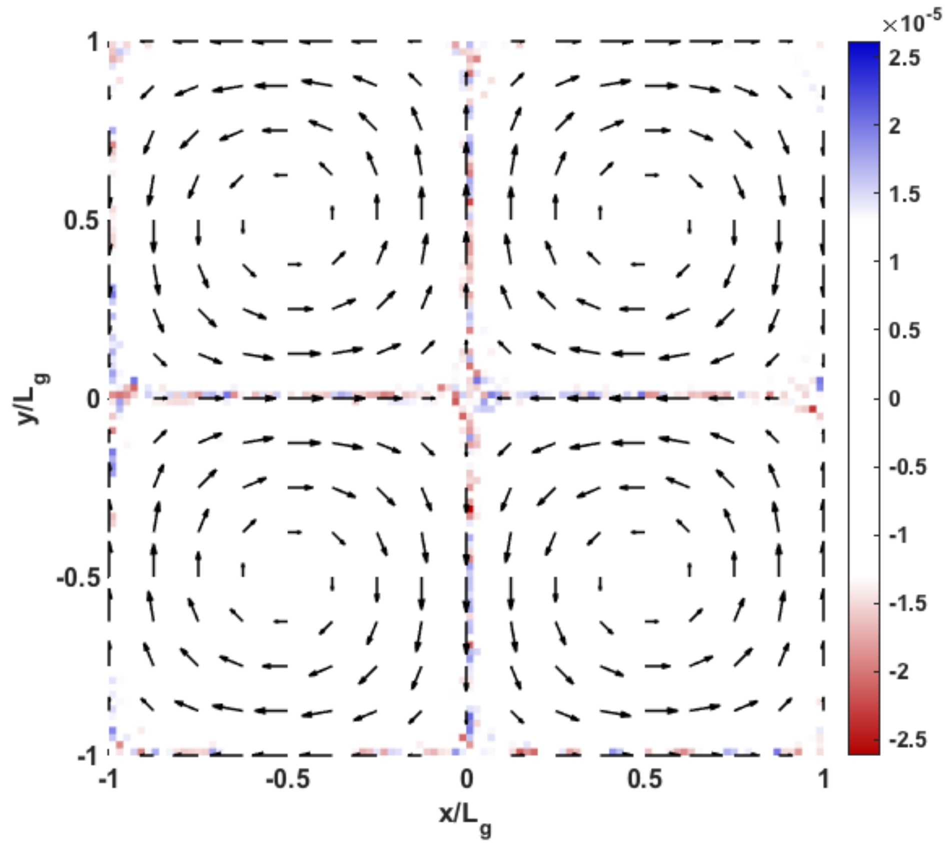


FTLE for QP and Wave

Surface

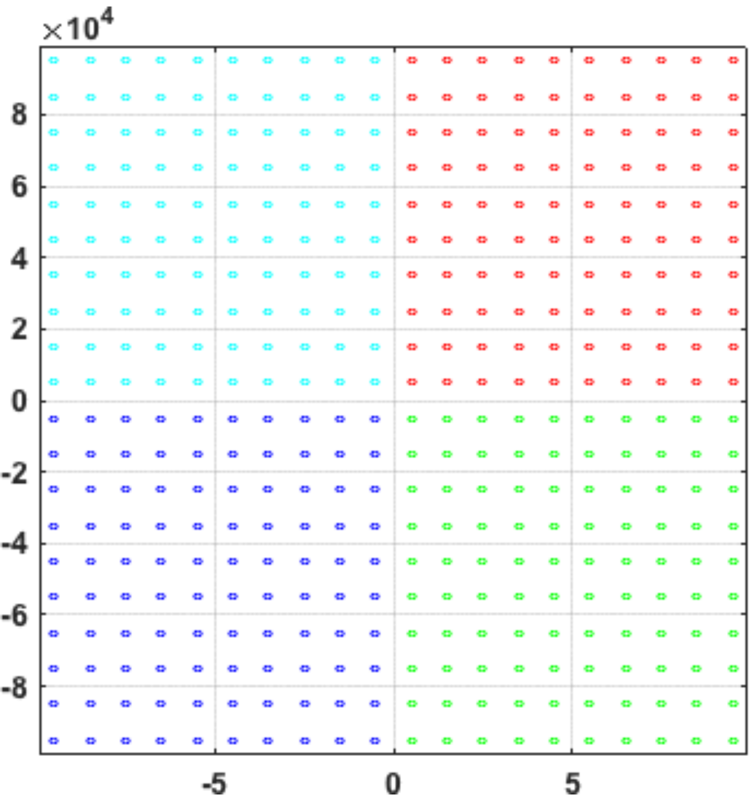


Subsurface

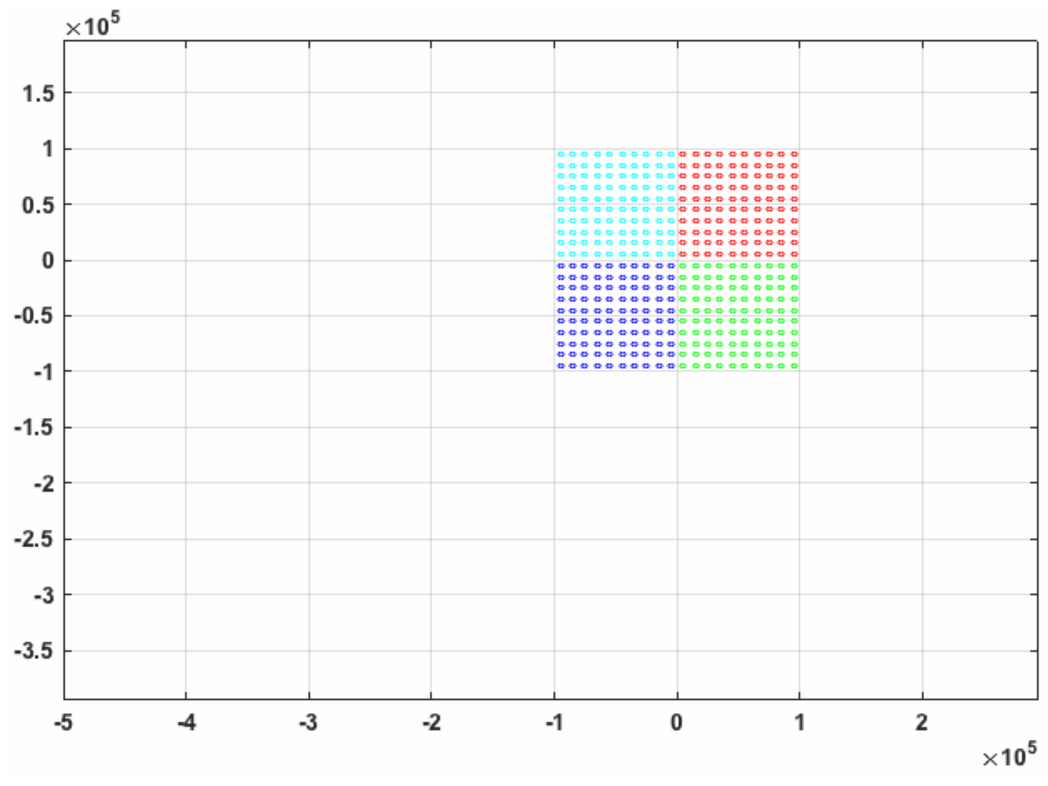


Effect of IW Disturbance on QP

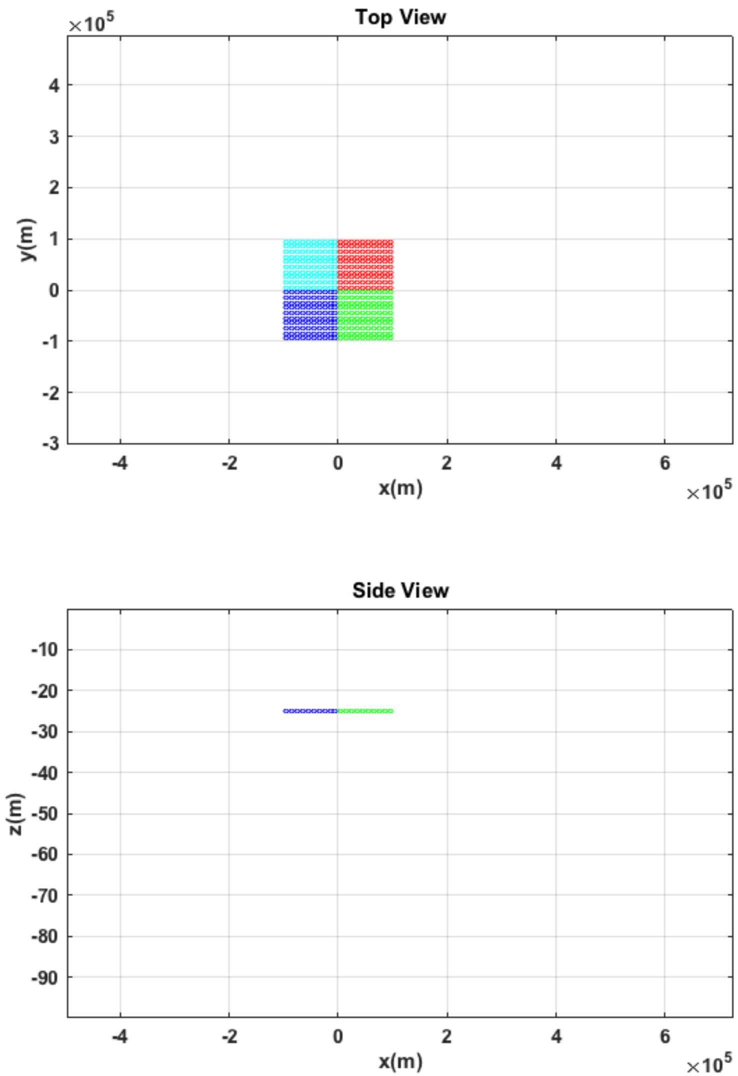
Just QP



QP + IW



QP and IW – Mixing at H/4



Summary

- Example of objective diagnostics in analyzing time dependent transport *surfaces*
- Internal waves may have well-defined ephemeral transport boundaries identified by 3D velocity
- Aperiodic mixing in internal waves distinct from diapycnal mixing
- Incoherent mixing when particle velocity exceeds phase velocity of traveling disturbance
- Internal waves cause leakage across mesoscale transport boundary surfaces

Envoi

Hypothesis: Internal waves cause turnstile exchange across QP boundaries. Since this is a 3D flow field the turnstiles are 2D surfaces. It would be interesting to see what they look like.

Thanks for your time and patience

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