Bio-inspired Dynamics for Multi-Agent Decision-Making

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Multi-Agent Decision-Making in Design

How to enable a network of distributed agents to decide as a group?

Which alternative is true?
Which action to take?
Which direction to follow?
Has a change been detected?

WHOI

TRADR

Ted S. Warren, Associated Press

Midlands Power Network

Multi-Agent Decision-Making in Nature

Alaska-in-Pictures.com

Good time to migrate?

Eikenaar, Klinner, Szostek, Bairlein, *Biol. Lett. 2014*

Nathan Stone

Which way to go?

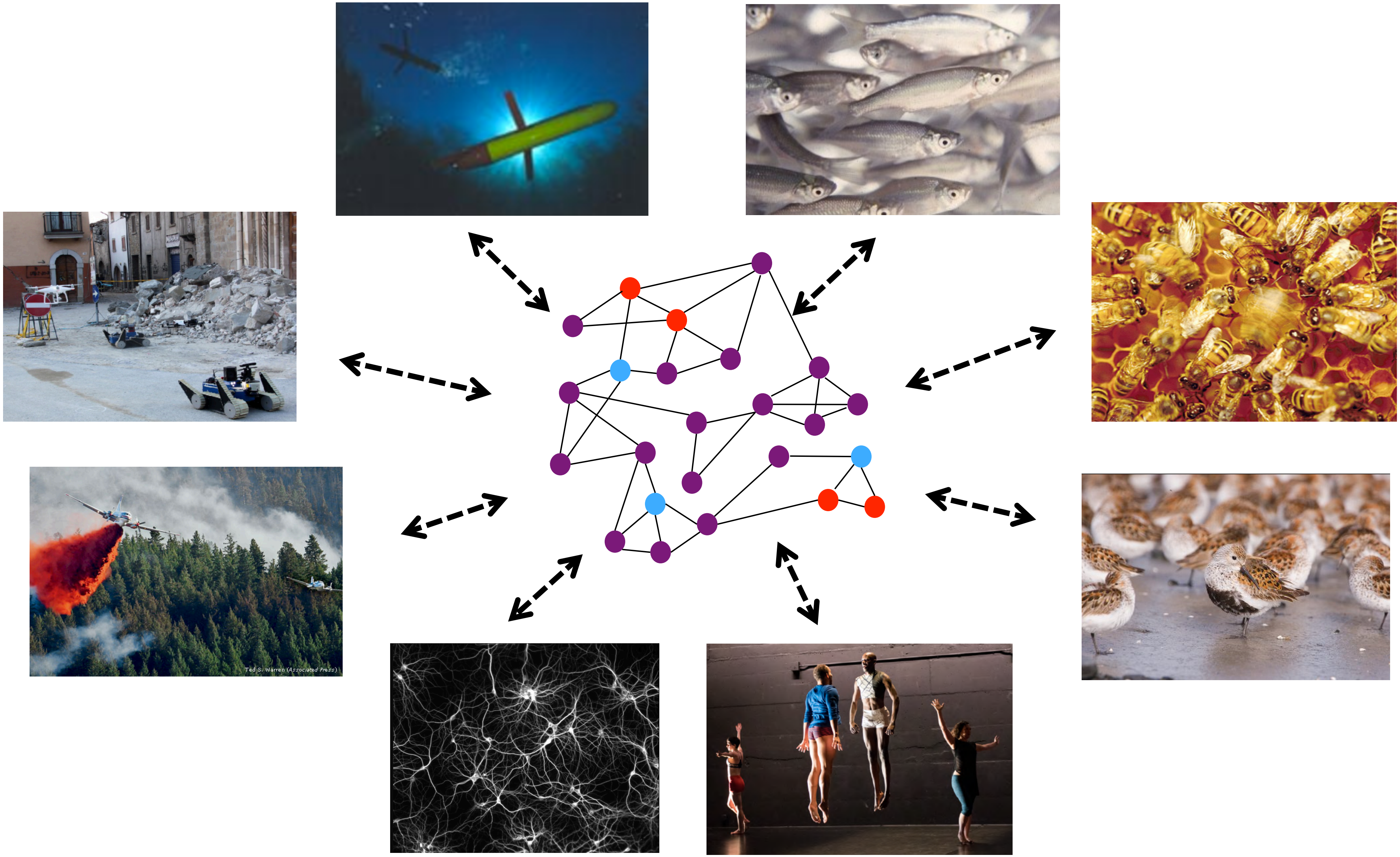
Couzin, Krause, Franks, Levin, *Nature,* 2005

Scott Camazine

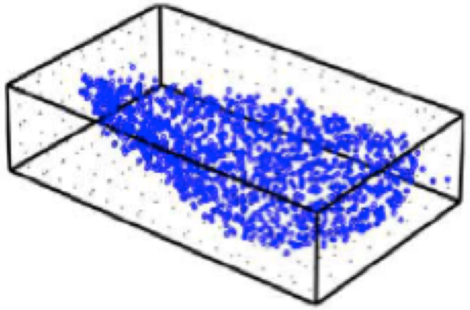
Which new nest site?

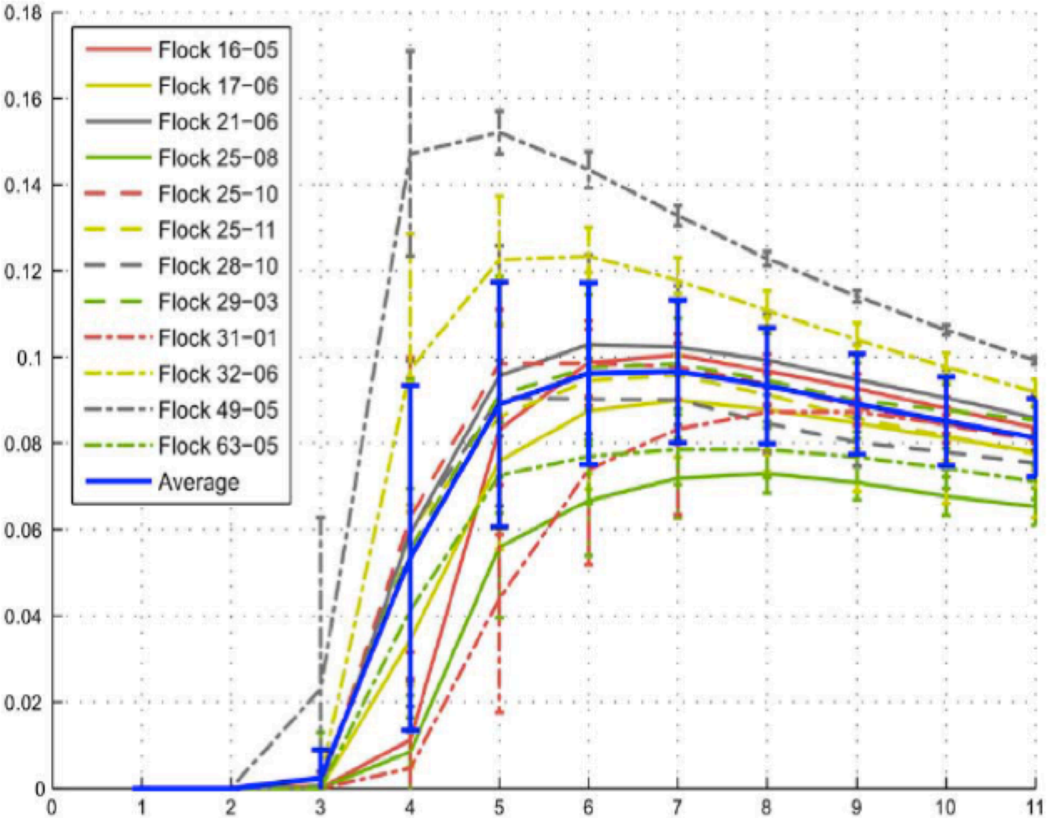
Seeley, Visscher, Schlegel, Hogan, Franks, Marshall, *Science, 2012*

Dynamics of Decision-Making Networks



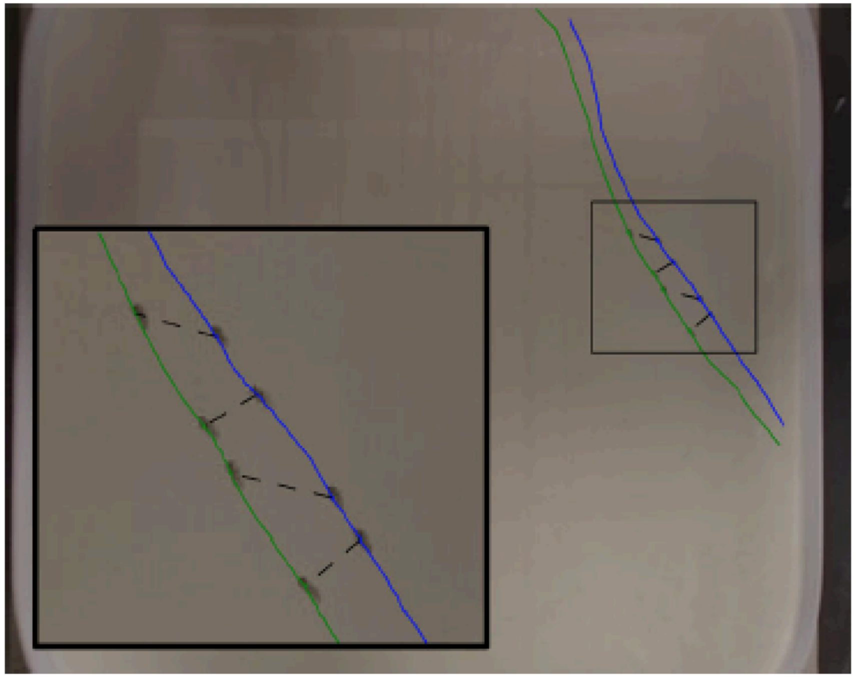
Robustness in starlings

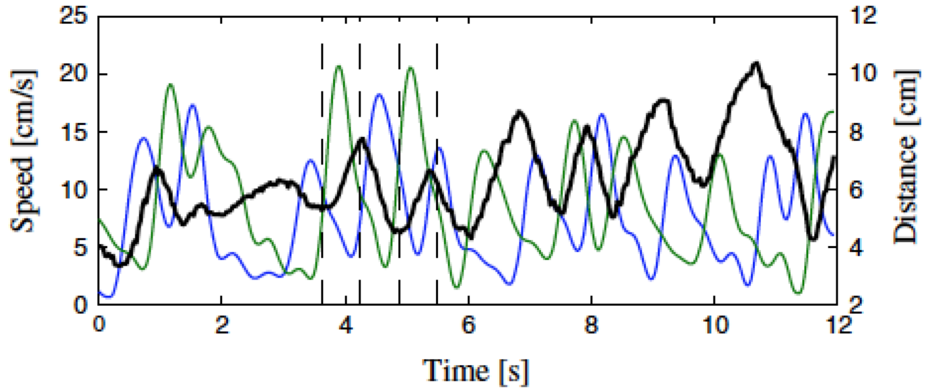




Young, Scardovi, Cavagna, Giardina, Leonard, *PLoS Comp. Bio*, 2013

Speed in killifish

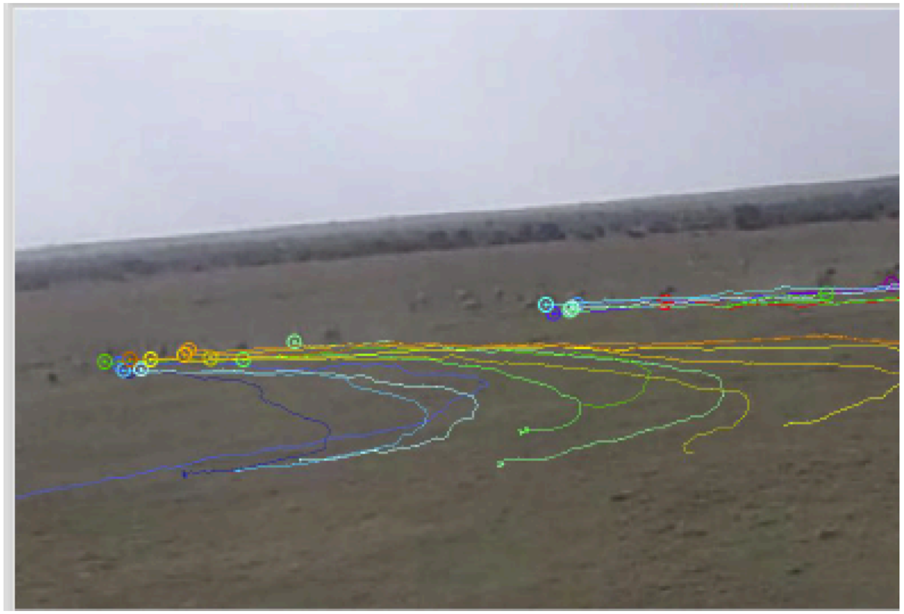




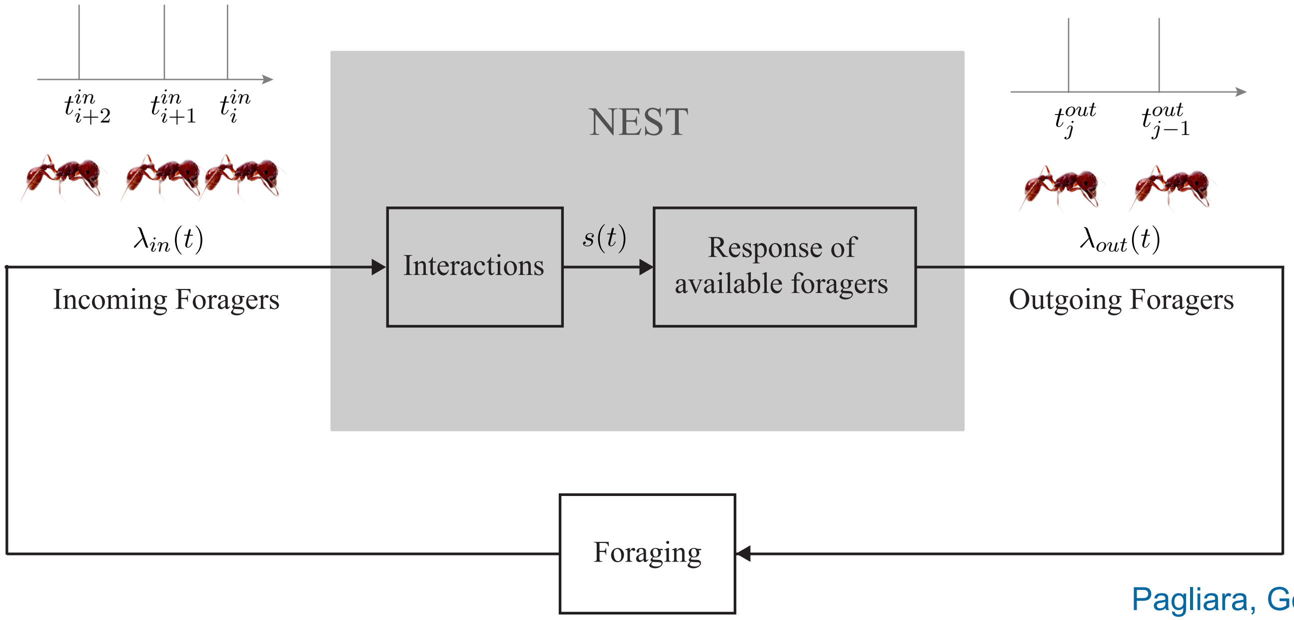
Swain, Couzin, Leonard, *J. Nonlinear Science*, 2015

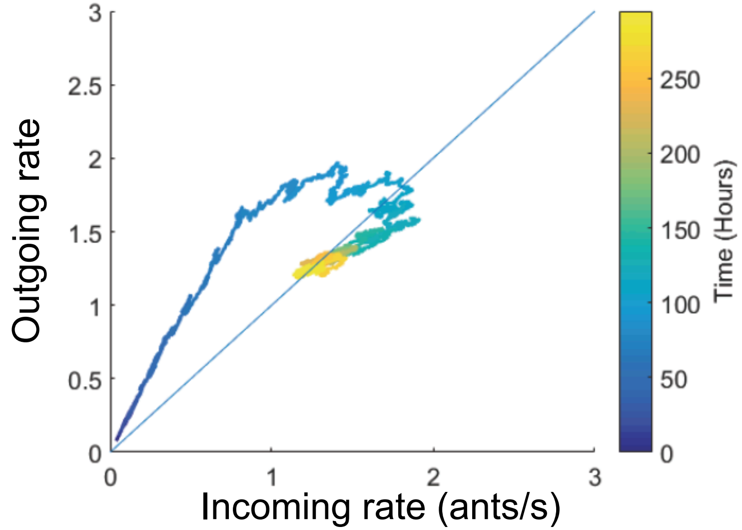
Efficiency of zebra herd evasion

Scott, Dey, Rubenstein, Leonard



Resilience of harvester ant foraging to temperature and humidity





Pagliara, Gordon, Leonard

Modeling Multi-Agent Decision-Making

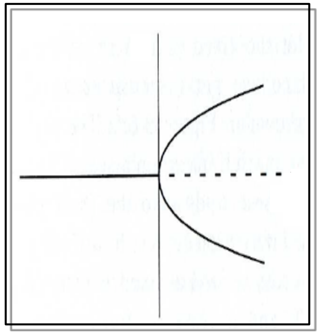
Animals groups adapt decision-making to environment

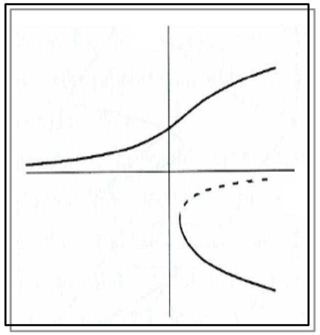
Case study: House hunting honey bees

Wild About Britain

Singularities organize group behaviors

Singularity theory: *Robust bifurcation theory*





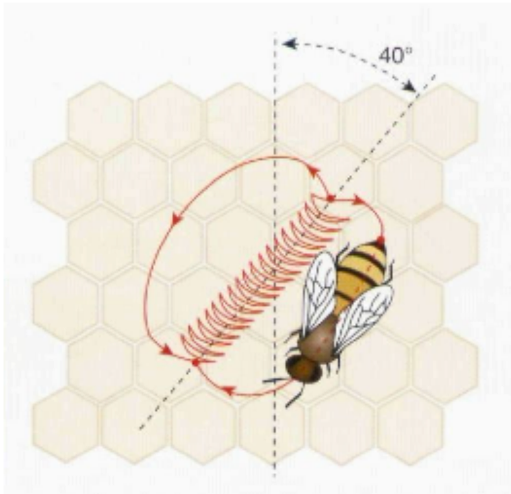
Proposed agent-based dynamic model provides realization

Equivalence: connects collective decision making in nature and design

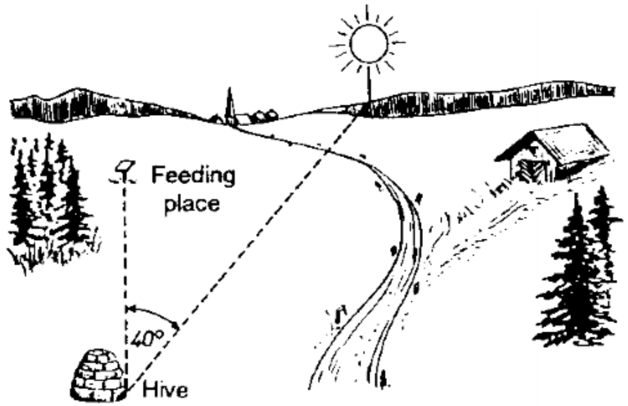
House Hunting Honey Bees and the “Waggle Dance”

K. von Frisch, *Bees: their vision, chemical senses, and language,* 1956.
M. Landauer, *Communication among social bees,* 1961
T.Seeley and S.Buhrman, Group decision making in swarms of honey bees, *Behav Ecol Sociobiol,* 1999

Scott Camazine



Seeley, Visscher, Passino, Am. Sci., 2006

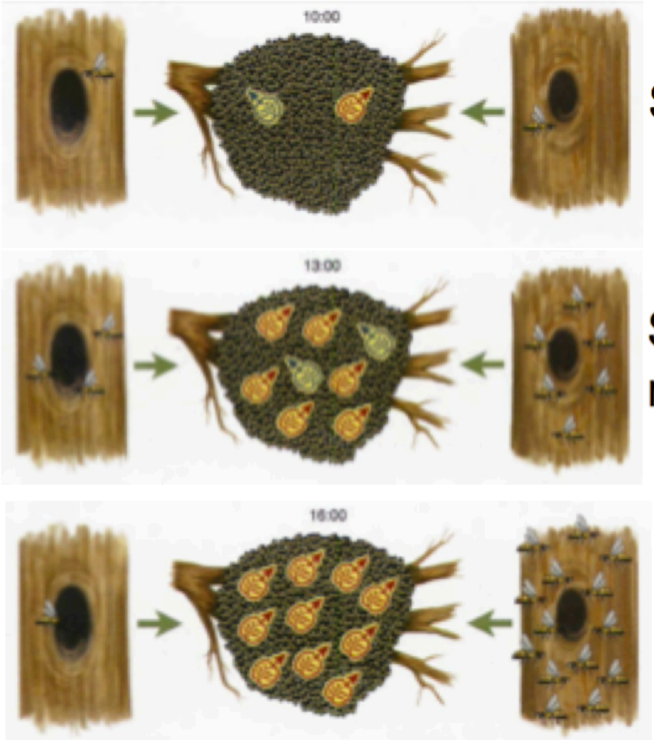


K. von Frisch, Nobel Lecture, 1973

Scout communicates: direction, distance, and ***quality*** v_i of visited site i

House-hunting Honey Bee Decision-Making

$A \quad v_A < v_B \quad B$



Scout, commit, recruit

Scout, commit,
recruit, lose interest

Signal decision
when quorum reached

Seeley et al., Am. Scientist, 2006

James Nieh

Scouts apply “stop signal” with head butt to dancers for alternative sites.

Seeley, Visscher, Schlegel, Hogan, Franks, Marshall, Stop signals provide cross inhibition in collective decision-making by honeybee swarms, *Science, 2012.*

Dynamic Model

Seeley et al, *Science, 2012.*

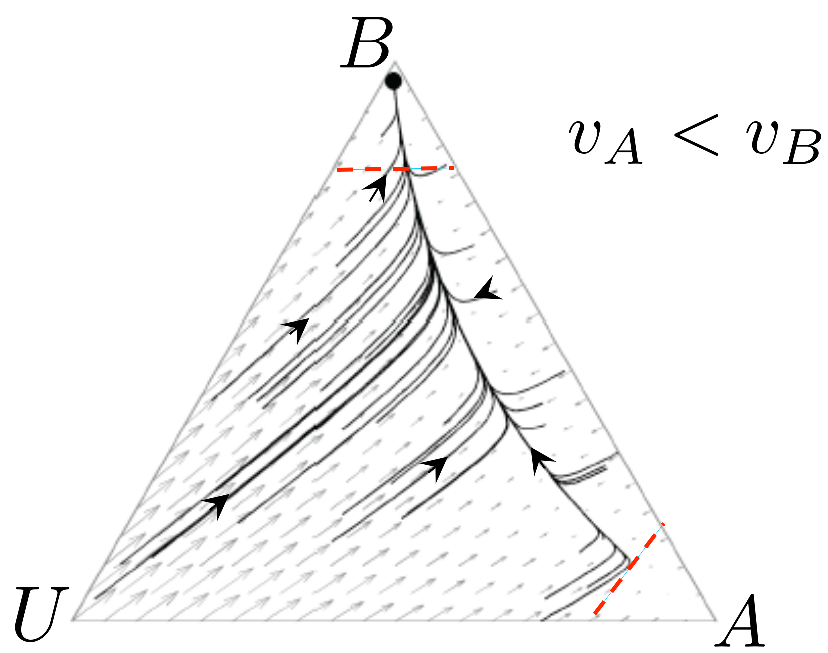
$$\frac{dy_A}{dt} = -\frac{y_A}{v_A} + v_A y_U + v_A y_U y_A - \sigma y_A y_B$$

$$\frac{dy_B}{dt} = -\frac{y_B}{v_B} + v_B y_U + v_B y_U y_B - \sigma y_A y_B$$

Decay   Commitment   Recruitment   Stop signal inhibition

y_A is fraction of population committed to A
y_B is fraction of population committed to B
$y_U = 1 - y_A - y_B$

$v_A > 1, \quad v_B > 1$

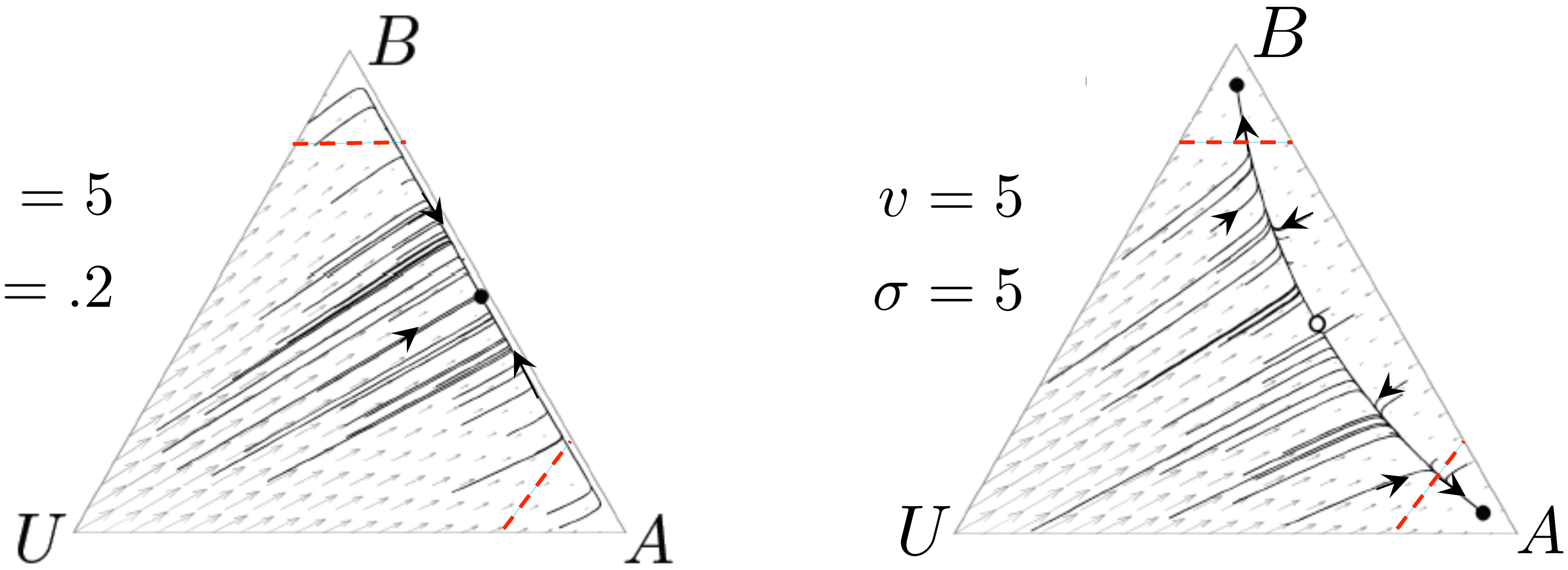


Equal Alternatives $v_A = v_B = v$

$$\frac{dy_A}{dt} = -\frac{y_A}{v} + v(1 - y_A - y_B)(1 + y_A) - \sigma y_A y_B$$
$$\frac{dy_B}{dt} = -\frac{y_B}{v} + v(1 - y_A - y_B)(1 + y_B) - \sigma y_A y_B$$

$v = 5$

$\sigma = .2$



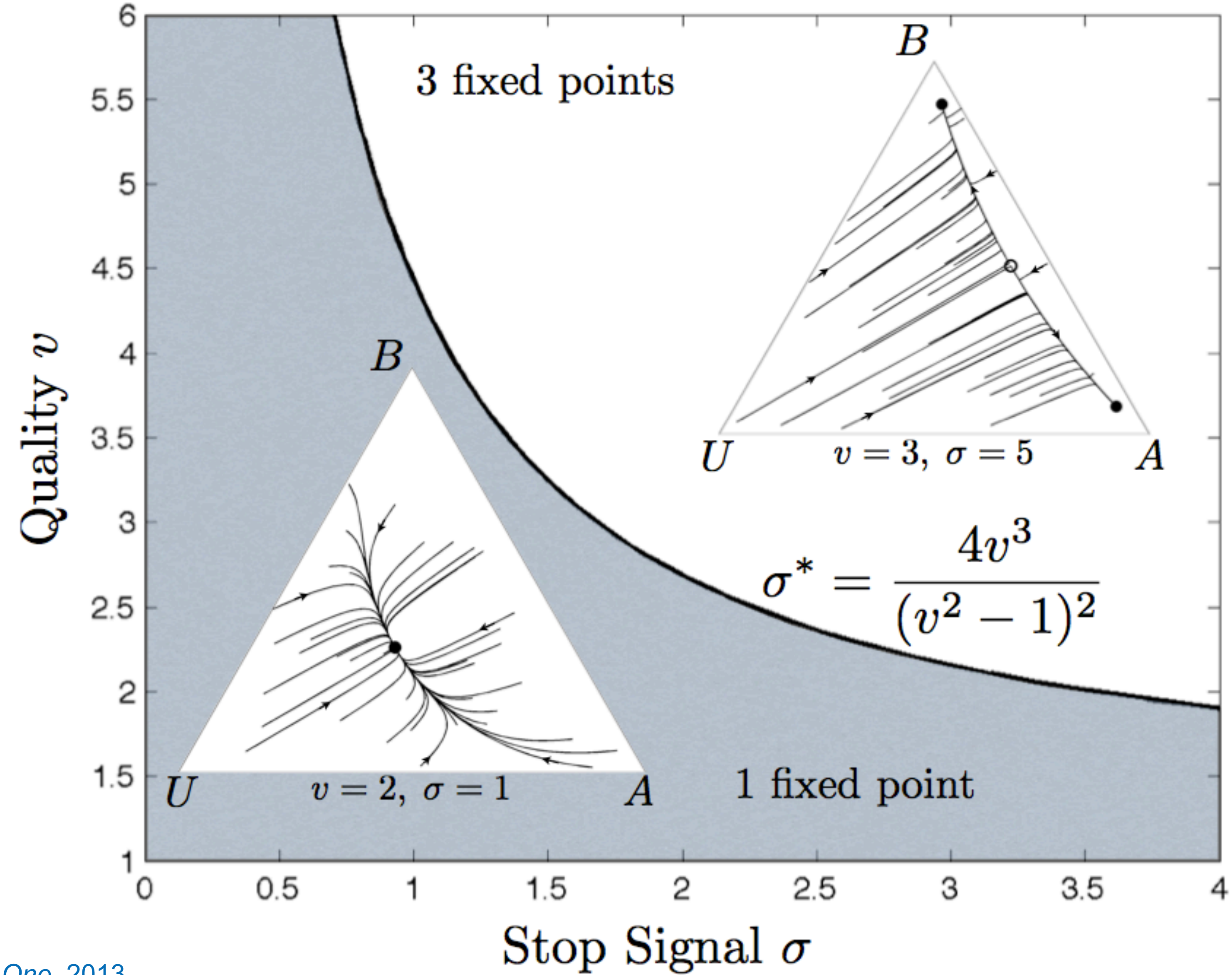
$v = 5$

$\sigma = 5$

D. Pais, P.M. Hogan, T. Schlegel, N.R. Franks, N.E. Leonard, J.A.R. Marshall, A mechanism for value-sensitive decision-making, *PLoS One,* 2013.

Value-Sensitive Decision-Making

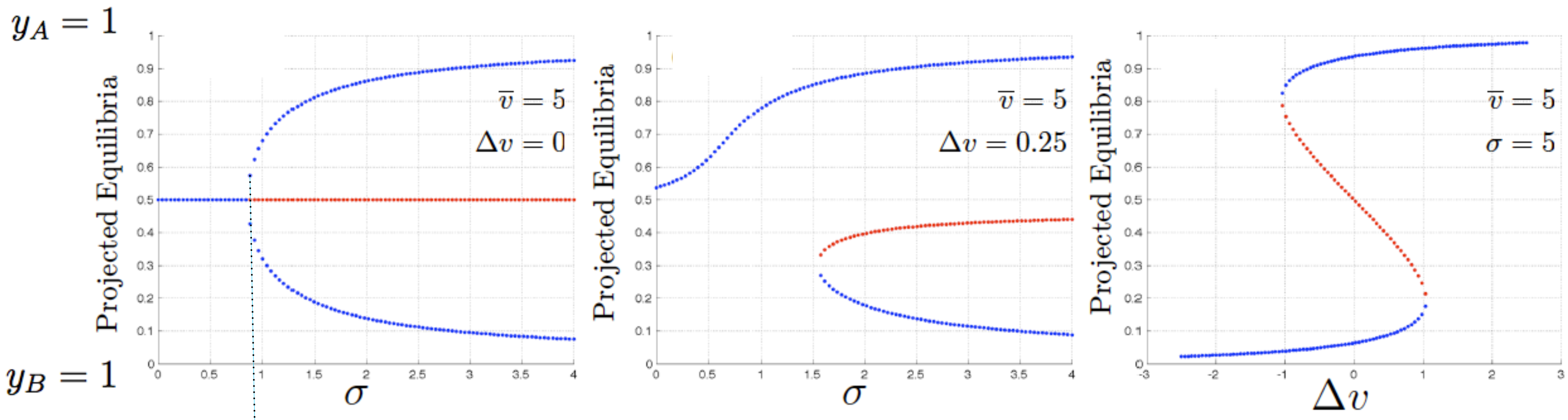
$v_A = v_B = v$



Pitchfork bifurcation

Pais et al, *PLoS One*, 2013

Robustness of Value-Sensitive Decision-Making



$$\sigma^* = \frac{4v^3}{(v^2 - 1)^2}$$

Pais et al, *PLoS One,* 2013.

What is influence of heterogeneity across group?
What if σ were a control input?
How to translate to design?

Singularities in Bifurcation Theory

Golubitsky and Schaeffer, Springer, 1985

Lyapunov-Schmidt reduction: $g(y, \lambda) = 0, \quad y, \lambda \in \mathbb{R}$

(y^*, λ^*) is a *singularity* if $g(y^*, \lambda^*) = g_y(y^*, \lambda^*) = 0$

1. Recognition of qualitative type: $h(y, \lambda) = 0 \ \sim \ g(y, \lambda) = 0$
2. Classification of qualitative types $\Longrightarrow$ organizing centers
3. Enumeration of *all* perturbations of g:

Universal unfolding of g: k-parameter family of functions $G(y, \lambda, \alpha_1, \ldots, \alpha_k)$

a) $G(y, \lambda, 0, \ldots, 0) = g(y, \lambda)$

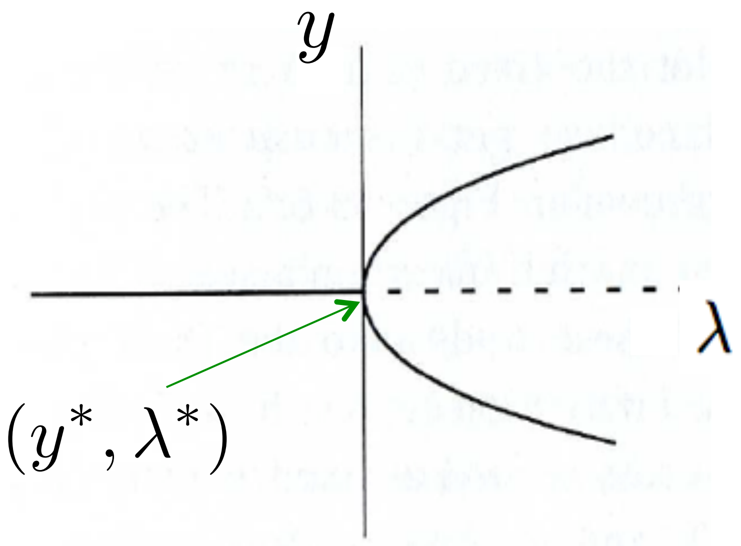
b) $\forall$ small perturbation p, $\exists\, \boldsymbol{\alpha} = \alpha_1, \ldots, \alpha_k \quad \Longrightarrow \quad g + p \ \sim \ G(\cdot, \cdot, \boldsymbol{\alpha})$

Pitchfork Singularity and Its Unfolding

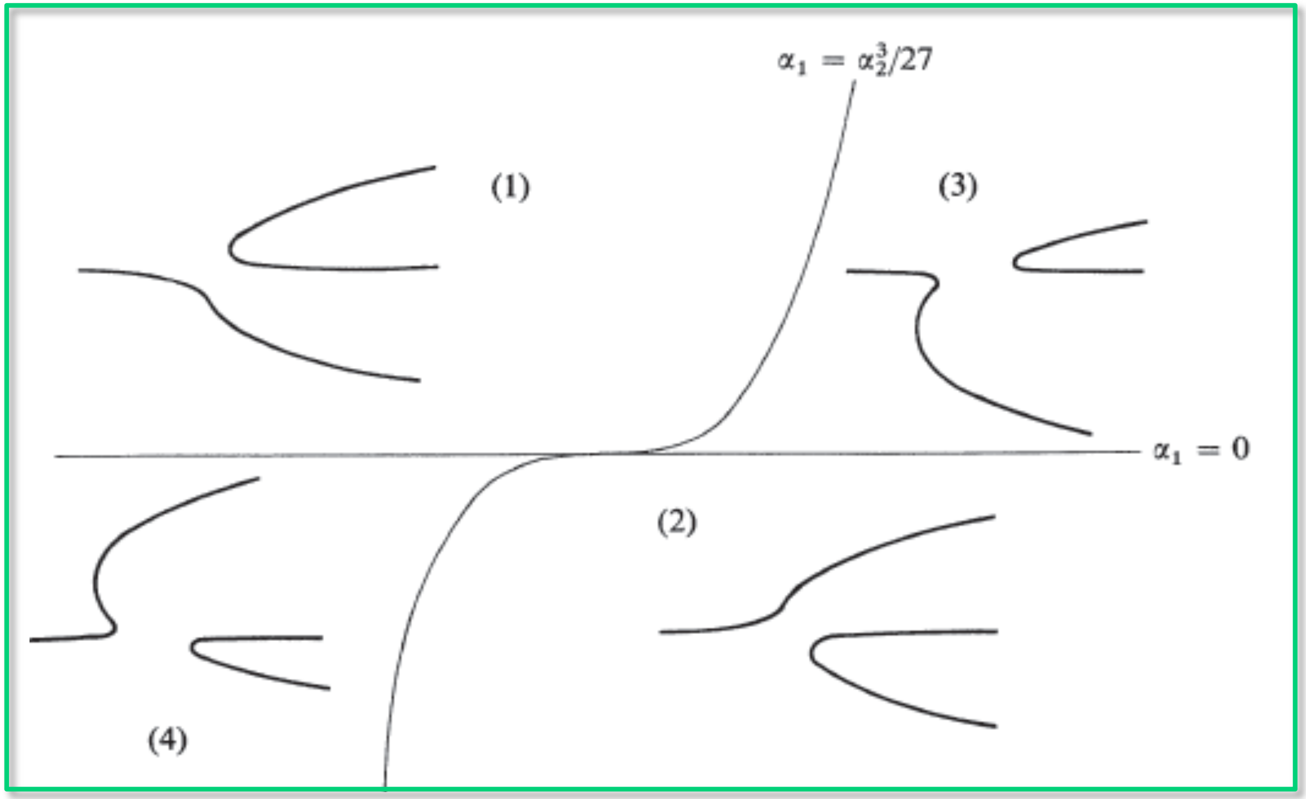
$$g = g_y = g_{yy} = g_\lambda = 0; \quad g_{yyy} > 0, \;\; g_{\lambda y} < 0$$

$$g(y,\lambda) = \lambda y - y^3$$

$$G(y,\lambda,\alpha_1,\alpha_2) = \lambda y - y^3 + \alpha_1 + \alpha_2 y^2$$



$\alpha_1 = \alpha_2 = 0$



Golubitsky and Schaeffer, Springer, 1985

Honey Bee Model: Organized by Pitchfork

$$\phi = y_A - y_B, \quad \psi = y_A + y_B$$

Solutions satisfy:

$$v_A = v_B = v$$

$$\boxed{\begin{aligned} g(\phi,\sigma) &= -\frac{\phi}{v} + v(1-\psi^*(\phi,\sigma,v))\phi \\ \psi^*(\phi,\sigma,v) &= \frac{\sqrt{\phi^2\sigma^2v^2 + 2\phi^2\sigma v^3 + 4\sigma v^3 + 9v^4 + 2v^2 + 1} - v^2 - 1}{2v^2 + \sigma v} \end{aligned}}$$

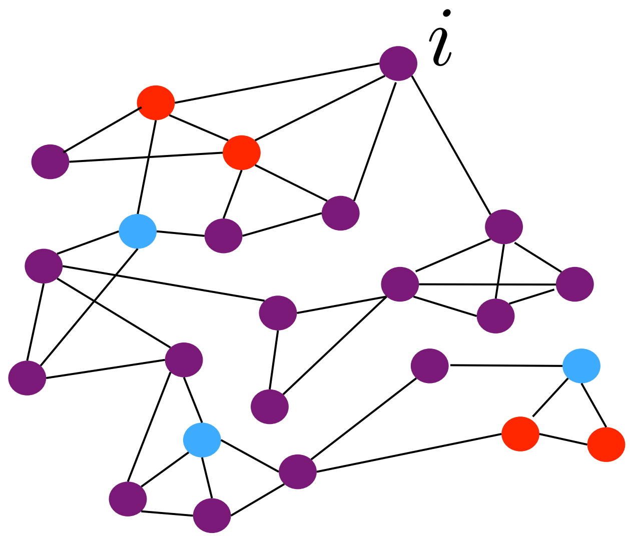
Recognition of pitchfork singularity: $(\phi^*, \sigma^*) = \left(0, \dfrac{4v^3}{(v^2-1)^2}\right)$

$g = g_\phi = g_{\phi\phi} = g_\sigma = 0, \quad g_{\phi\phi\phi} < 0, \quad g_{\sigma\phi} > 0$

Unfolding: $\boxed{G(\phi, \sigma, \Delta v) = 0}$ $\quad \Delta v := v_A - v_B$

Abstract Model

Agent state: $x_i(t) \in \mathbb{R}, \quad 1, \ldots, N$



$$\begin{array}{ll} x_i(t) > 0 & \text{for alternative A} \\ x_i(t) = 0 & \text{uncommitted} \\ x_i(t) < 0 & \text{for alternative B} \end{array}$$

Collective decision:

$$y(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t)$$

$$\begin{array}{ll} y_{ss} > +\eta & \text{for alternative A} \\ y_{ss} = 0 & \text{deadlock} \\ y_{ss} < -\eta & \text{for alternative B} \end{array}$$

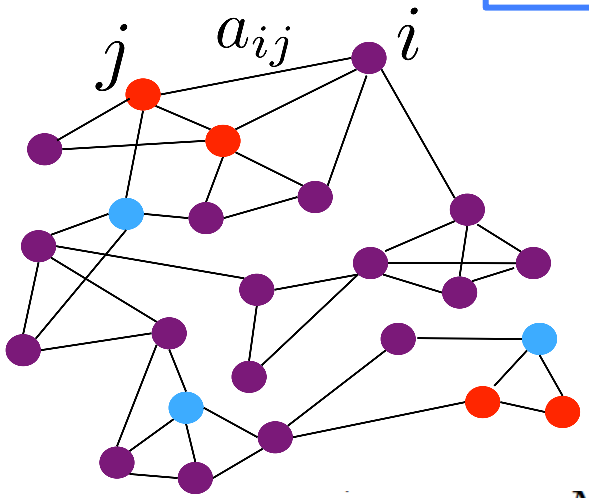
Abstract Model

negative self feedback

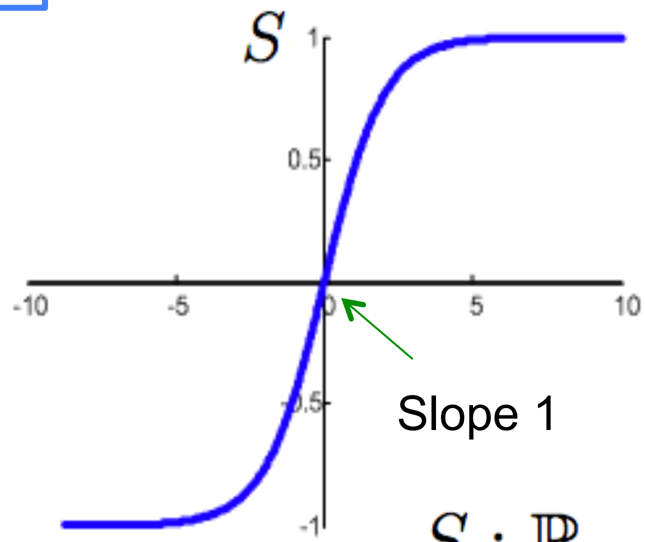
positive social feedback

$$\dot{x}_i = \beta_i - \sum_{j=1}^{N} a_{ij}(x_i - uS(x_j)) = \beta_i - d_i x_i + \sum_{j=1}^{N} u a_{ij} S(x_j)$$

$\boxed{u > 0 \text{ social effort}}$



$$d_i := \sum_{j=1}^{N} a_{ij}$$



$$S : \mathbb{R} \to (-1, 1)$$

Franci, Srivastava, Leonard (2015) A realization theory for bio-inspired collective decision-making, arXiv:1503.08526v1

Gray, Franci, Srivastava, Leonard (2017a) An agent-based framework for bio-inspired value-sensitive decision-making, *IFAC WC*.

Gray, Franci, Srivastava, Leonard (2017b) Honey-bee inspired dynamics for multi-agent decision-making, arXiv:1503.08526v2.

Bifurcation and Unanimity by Design

$$\boxed{\dot{\boldsymbol{x}} = -D\boldsymbol{x} + uAS(\boldsymbol{x}) + \boldsymbol{\beta}}$$

For $\boldsymbol{\beta} = \mathbf{0}_N$, linearization at $\boldsymbol{x} = \mathbf{0}_N$ is $\boxed{\dot{\boldsymbol{x}} = -(D - uA)\boldsymbol{x}}$

- for $u = 1$, linearization $\dot{\boldsymbol{x}} = -L\boldsymbol{x}$ has $\lambda_1 = 0$ and $\lambda_p < 0$
- for $u > 1$ and $u - 1$ small, $\lambda_1 > 0$ and $\boldsymbol{x} = \mathbf{0}_N$ is unstable

$\mathbf{1}_N$ is the consensus manifold: $x_i = x_j$

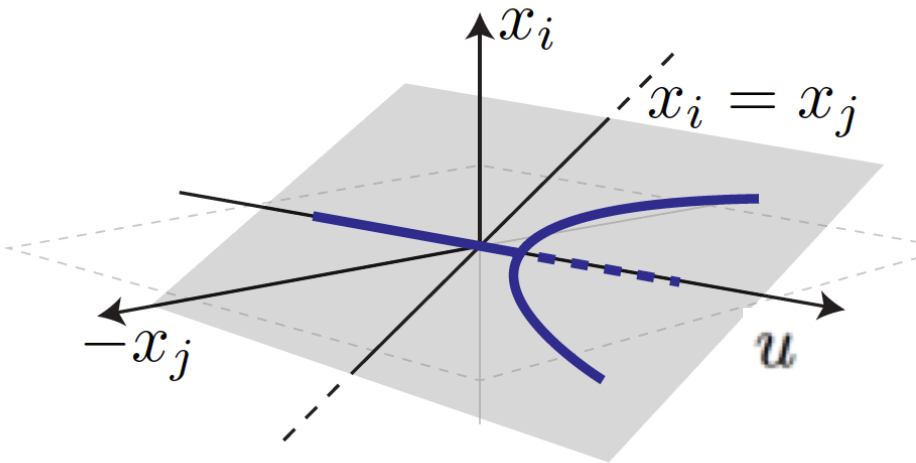
Zero eigenvalue → can create bifurcation by design
Center manifold is tangent to consensus manifold → get unanimity by design

Franci, Srivastava, Leonard (2015), Gray, Franci, Srivastava, Leonard (2017a, 2017b)

Intuition: All-to-all and $\boldsymbol{\beta} = \mathbf{0}_N$

Theorem:

i) the consensus manifold is globally exponentially stable for each $u \in \mathbb{R}_{\geq 0}$;
ii) the origin is globally exponentially stable for $u \in [0, 1)$ and globally asymptotically stable for $u = 1$;
iii) the origin is unstable and there exist two stable equilibrium points on the consensus manifold for $u > 1$.



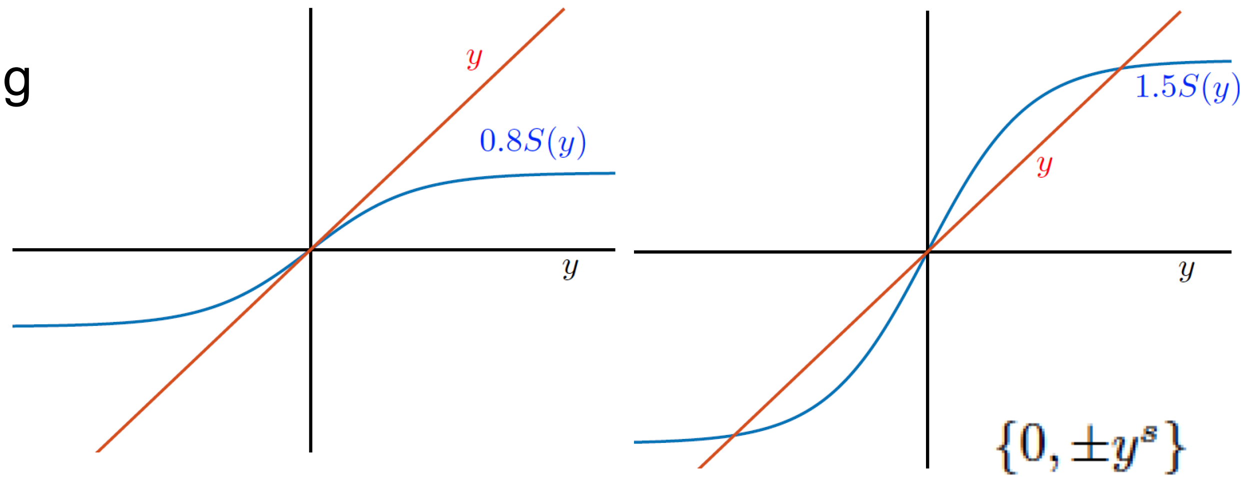
Proof:

Consensus manifold globally stable using

$$V(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_j)^2/2.$$

On consensus manifold, reduces to

$$\dot{y} = -y + uS(y)$$



Bifurcation Depends on Graph and $\beta \neq \mathbf{0}_N$

Lyapunov-Schmidt reduction:

Equation for equilibria

Golubitsky & Shaeffer, 1985

For nonlinear equation $\boldsymbol{\zeta}(\boldsymbol{x}) = \mathbf{0}$, singular point is where $\frac{d\boldsymbol{\zeta}}{d\boldsymbol{x}} = \mathbf{0}$

$P = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^\top$ projects onto space orthogonal to $\mathbf{1}_N$

$$\bar{d} := (L - (u-1)PA)^+ Pd$$

$$\bar{\beta} := (L - (u-1)PA)^+ P\beta$$

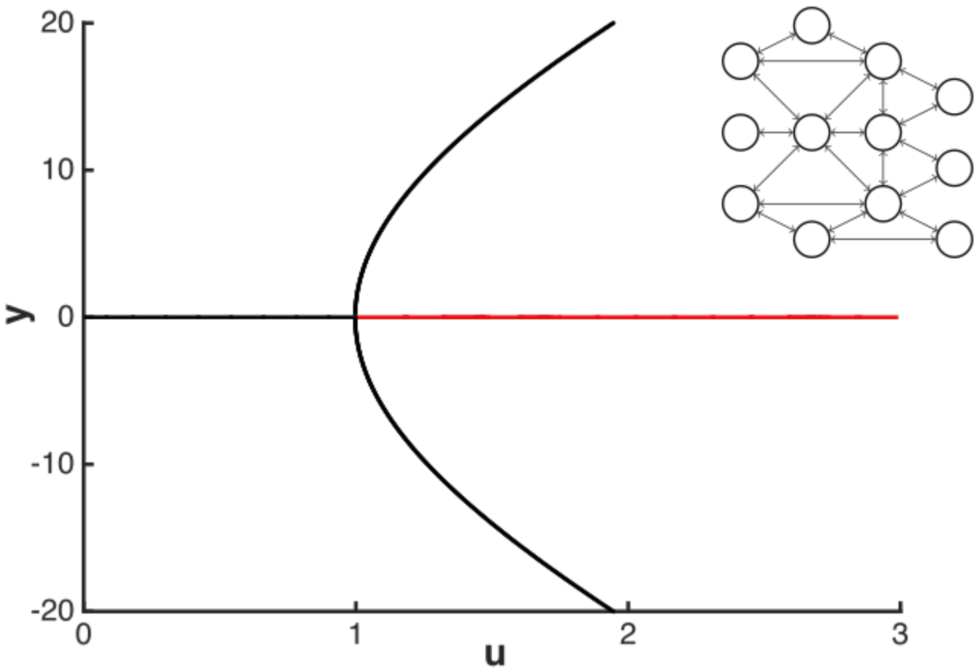
$$\epsilon = (u-1)\bar{d} + \mathbf{1}_N$$

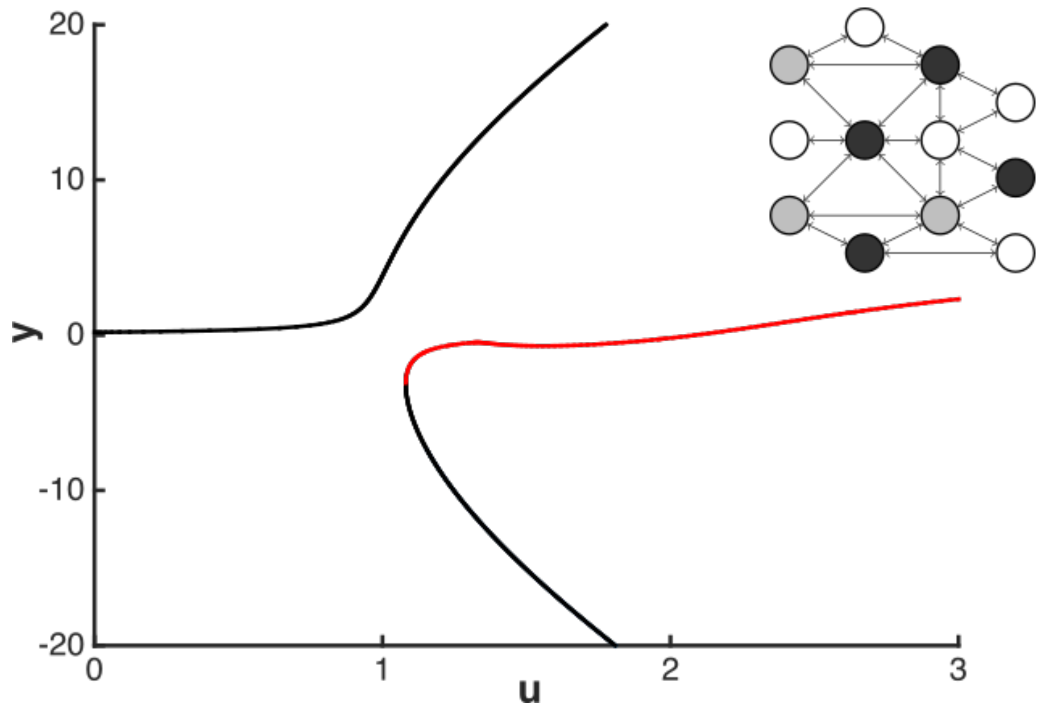
Reduced equation for equilibria along center manifold at $(\boldsymbol{x}^*, u, \beta) = (0, 1, 0)$

$$g(y, u, \beta) := \sum_{i=1}^{N} d_i \left(u \tanh(\epsilon_i y + \bar{\beta}_i) - (\epsilon_i y + \bar{\beta}_i)\right) + \sum_{i=1}^{N} \beta_i + O(\beta^2, (u-1)^2, y^4)$$

Pitchfork bifurcation and its unfolding depend on graph and β

Balanced graphs: Information asymmetry yields unfolding of the pitchfork

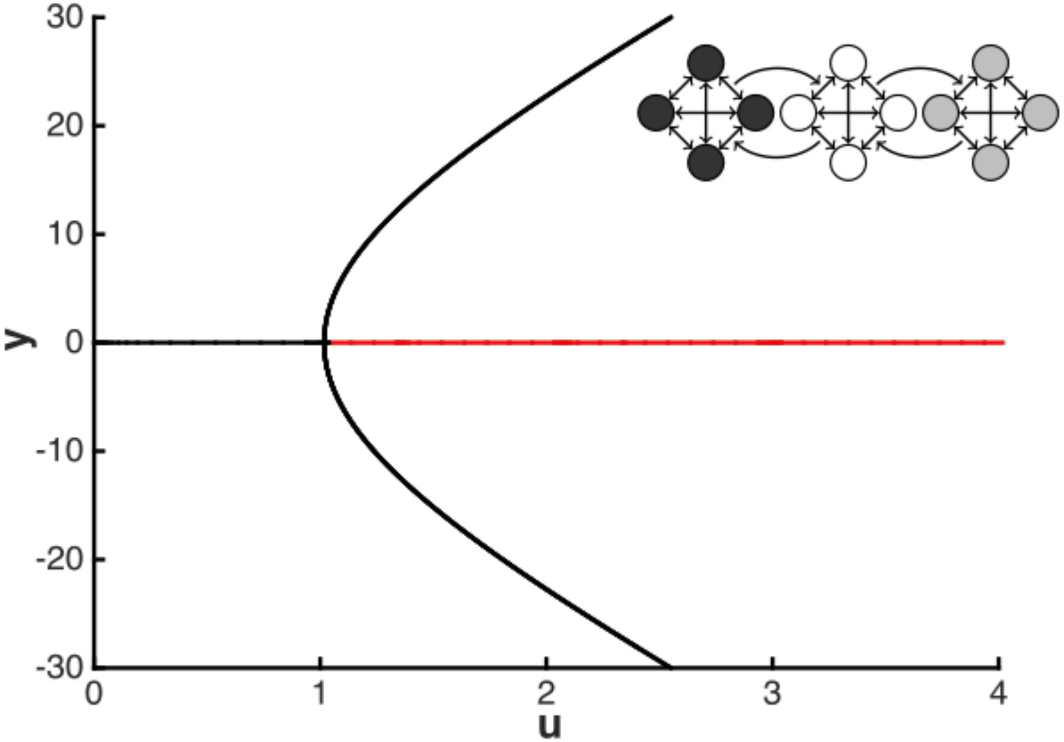




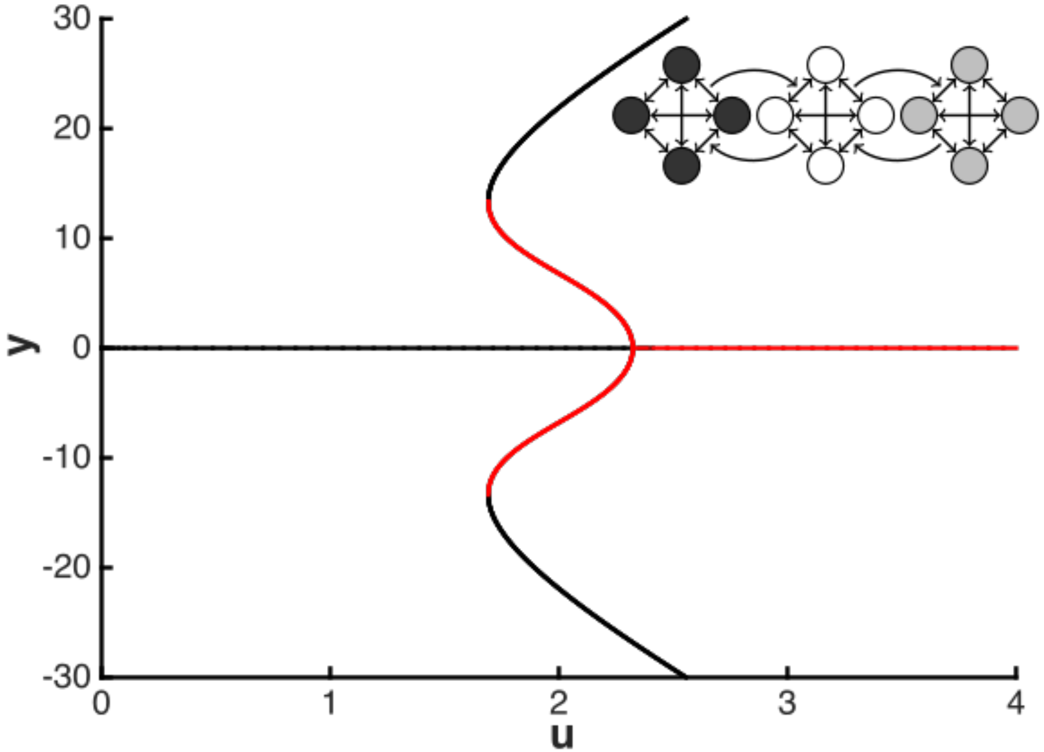
$$\boldsymbol{\beta} = \mathbf{0}_N \qquad \beta_1 = -\beta_2 = \beta$$

Symmetric network: super versus subcritical pitchfork

$$\beta_1 = -\beta_2 = \beta,\ n_1 = n_2$$



$\beta = 1$



$\beta = 5$

Value-sensitive decision-making

$$\dot{x}(t) = -u_1 D x(t) + u_2 A S(x) + v$$

$v_i = v$ for n_1 agents who prefer alternative A
$v_i = -v$ for n_2 agents who prefer alternative B
$v_i = 0$ for n_3 agents who have no preference

$$\tau = u_1 t$$
$$u = u_2 v$$
$$\beta_i \in \{-v^2, 0, v^2\}$$

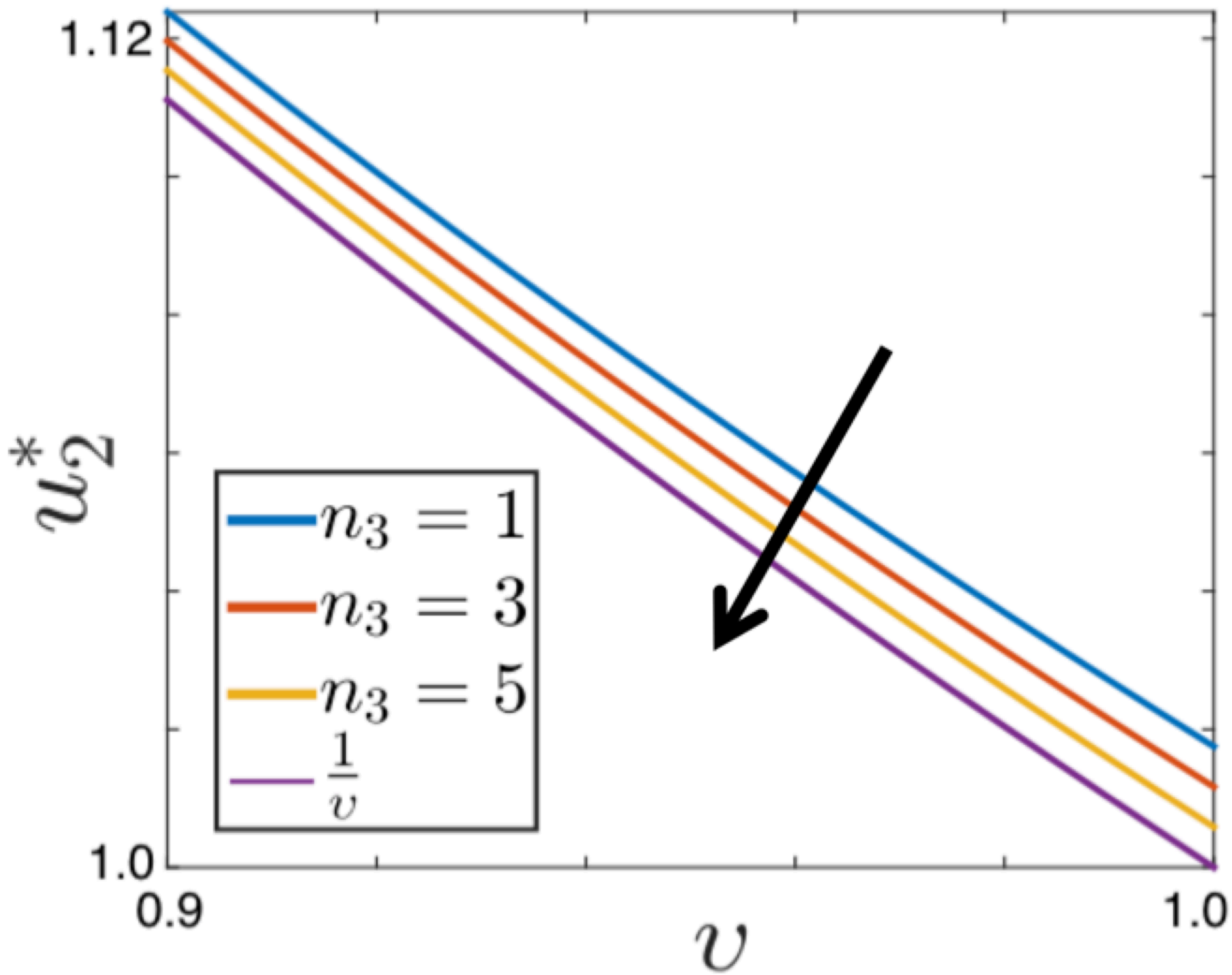
Let $u_1 = 1/v$ and $u_2 > 0$ is social effort

For $n_1 = n_2$, $0 < v < 1$ get pitchfork bifurcation with bifurcation point

$$u_2^* = \frac{1}{v} + \frac{(1+3N^3)^2(N-n_3)}{9N^9} v^3 + O(v^7)$$

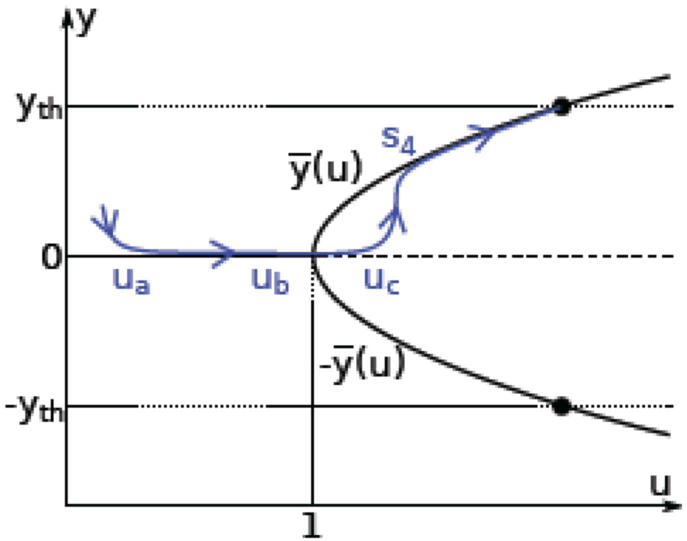
Trends in value-sensitive decision-making

Increasing n_3 for $N = 7$

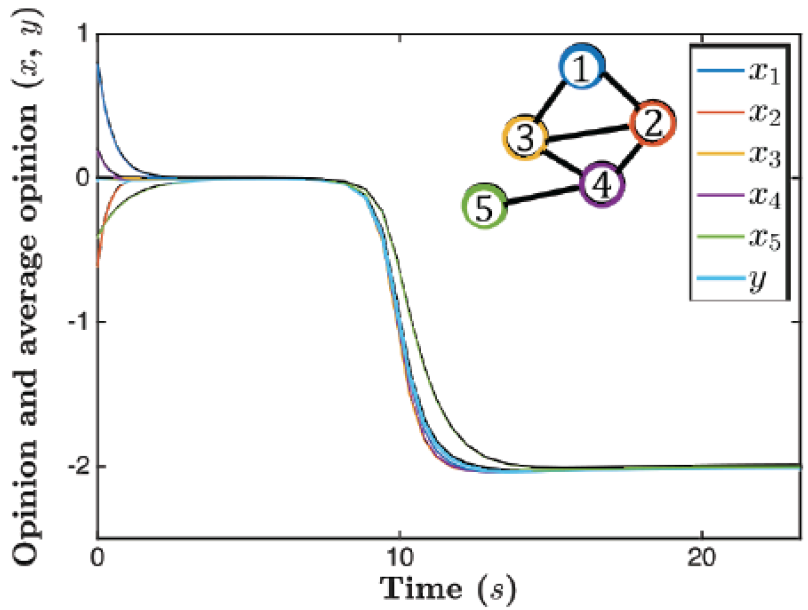


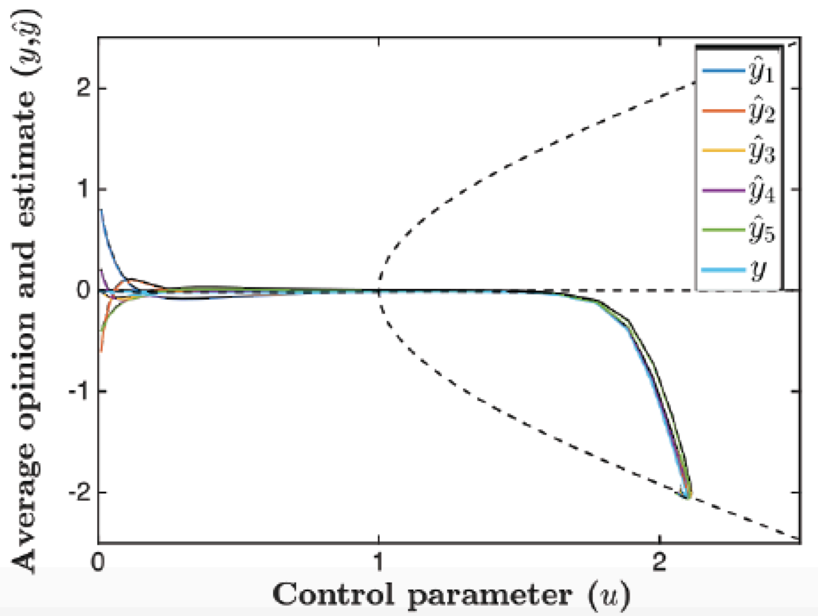
Feedback control of bifurcation parameter

$$\dot{x} = -Dx + \mathrm{diag}(u)AS(x)$$
$$\dot{\hat{y}} = -L\hat{y} + \dot{x}$$
$$\dot{u} = \epsilon\left(y_{th}^2 \mathbf{1}_N - \mathrm{diag}(\hat{y})\hat{y}\right),$$



Berglund and Genz, 2006





Summary and Ongoing

Decision-making model organized by pitchfork singularity rigorously connects analysis of animal groups with multi-agent control design

Generalize singularity theory approach: E.g., $n > 2$ alternatives

Influence of heterogeneity in information and in graph structure

Heterogeneity of leadership

Heterogeneity of social activity

$$d\boldsymbol{x}(t) = \boldsymbol{\beta} dt - (D + K)\boldsymbol{x}(t) + AU S(\boldsymbol{x}) dt + \Sigma d\boldsymbol{W}_N(t)$$

Heterogeneity of information

Heterogeneity of graph location

Heterogeneity in correlation

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