Genetic consequences of range expansion under climate change

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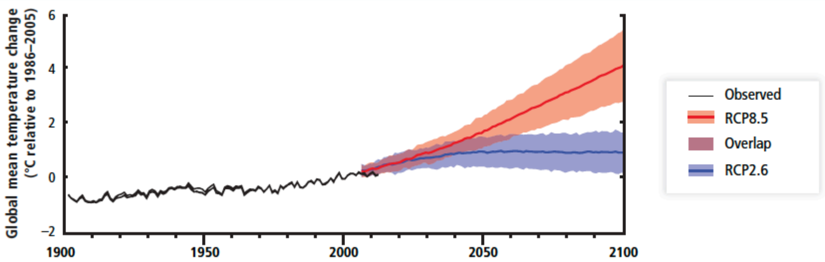
Climate change predictions



- Prediction for a low-emission mitigation scenario is given by Representative Concentration Pathways (RCP) 2.6
- Prediction for a high-emission scenario is given by RCP 8.5

IPCC (2014)

Shifting range boundaries

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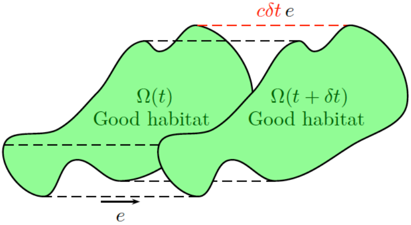

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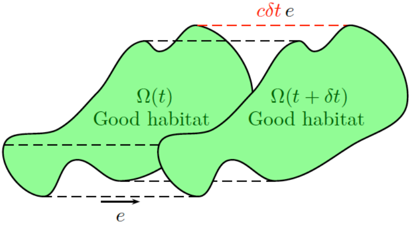

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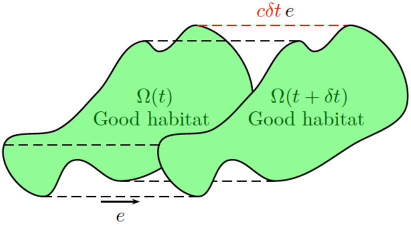
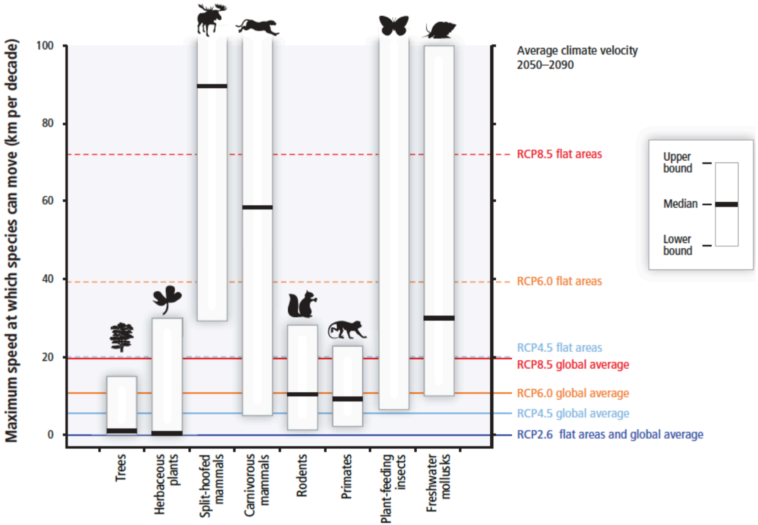


- The climate envelope is a patch of good habitat surrounded by bad habitat
- The climate speed is c

Can species keep up with the climate speed?



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- Range expansions are thought to lead to a loss in diversity due to successive founder effects (Mayr, 1942)
- Are expanding species able to maintain sufficient genetic diversity at the leading edge of the migration?

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- If a population does persist during range shift, how does the genetic diversity change over time? Are there significant founder effects?
- In terms of genetic diversity, what is the difference between range shift due to climate change and expansion in a homogeneous environment?

Outline

1. Intrinsic spreading speed c^*
2. Population persistence during range shift
3. Tracking neutral fractions and inside dynamics
4. Measuring diversity for neutral fractions
5. Conclusions

Fisher's model (1937)

$$\begin{array}{ccccc} \text{Rate of change of density} & = & \text{Growth} & + & \text{Dispersal} \\ \frac{\partial u}{\partial t} & = & ru(1-u) & + & D\frac{\partial^2 u}{\partial x^2} \end{array}$$

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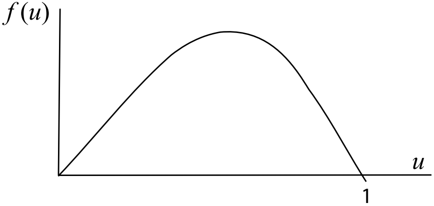

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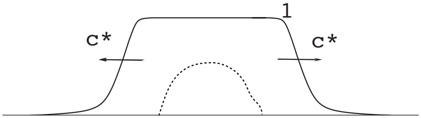

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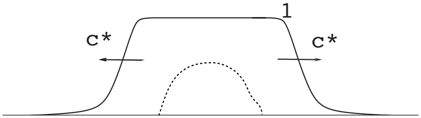
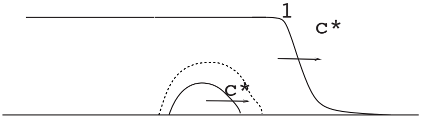


- Proof uses a comparison theorem (solutions that are initially ordered remain ordered for all time) plus super- and sub-solutions with speeds c^* as $t \to \infty$



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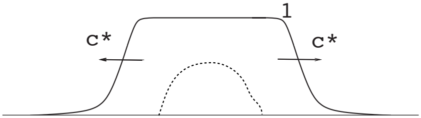

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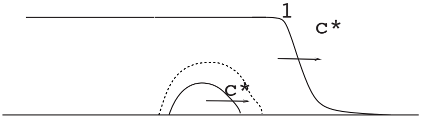

- Luther (1906) argued speed of a related chemical reaction was $c^* \propto \sqrt{rD}$ using dimensional arguments.

Travelling wave

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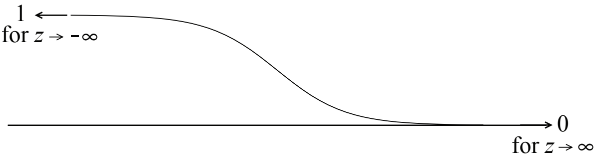

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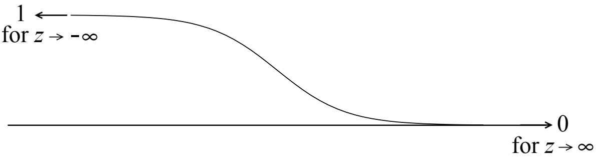
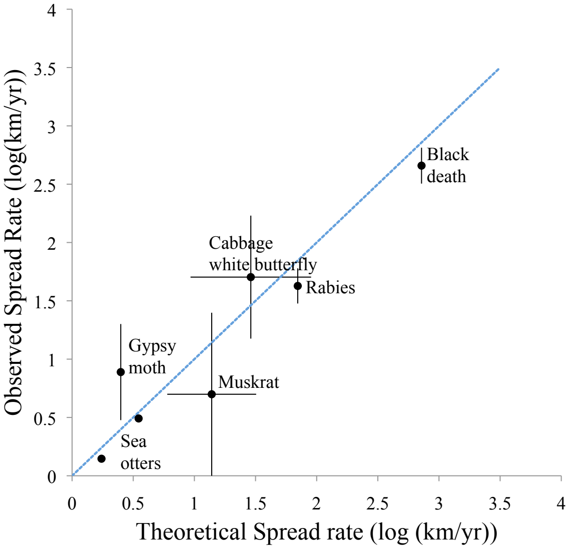


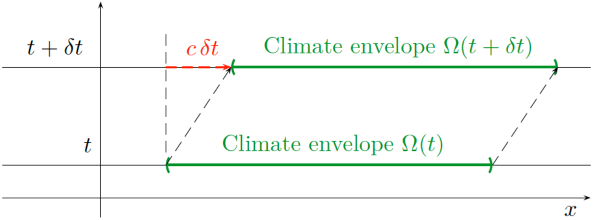
There is a family of travelling wave solutions. A solution exists for each $c \geq c^*$. Hence the spreading speed coincides with the minimal travelling wave speed.

Comparison with data



Outline

Shifting range boundaries



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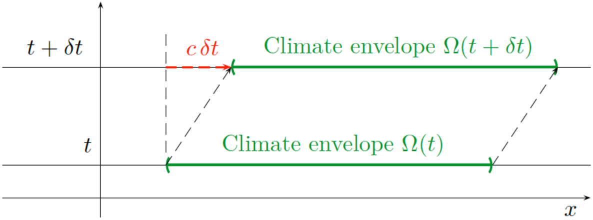

- The length of the climate envelope Ω is L

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c: speed of climate change
L: length of the climate envelope;
d: death rate in the unsuitable regions;
r: per capita growth rate in the good habitat;
D: diffusion coefficient;
$c^* = 2\sqrt{rD}$: spreading speed.

Critical length depends on climate and spread speeds

Theorem (extinction, travelling pulse and convergence)

- *Extinction: Whenever* $0 < c < c^* = 2\sqrt{rD}$ *there exists a critical length*

$$L^*(c^*, c) := \frac{\frac{c^*}{r}}{\sqrt{1 - \left(\frac{c}{c^*}\right)^2}} \arctan\left(\frac{\sqrt{\frac{d}{r} + \left(\frac{c}{c^*}\right)^2}}{\sqrt{1 - \left(\frac{c}{c^*}\right)^2}}\right)$$

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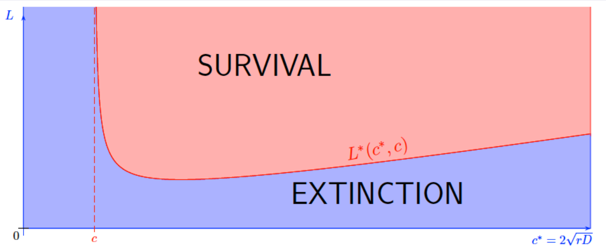

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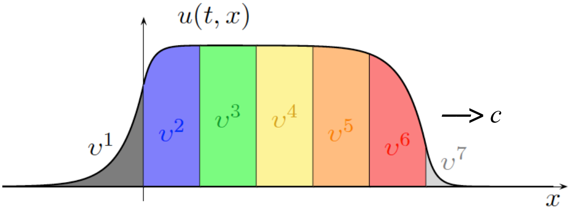
- *Convergence: If u_0 is nontrivial and compactly supported, the solution converges to a traveling pulse for large time.*

Potapov and Lewis (2004); Berestycki and Rossi (2007); Berestycki et al. (2009)

Traveling pulse



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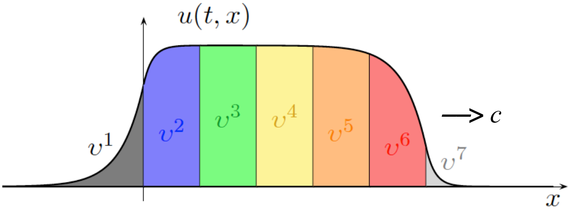

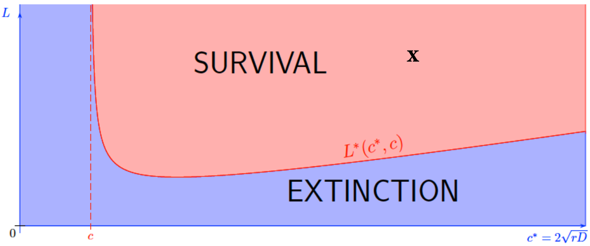

Outline

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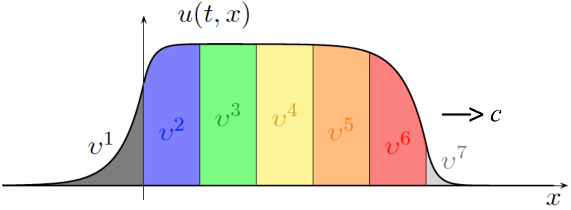

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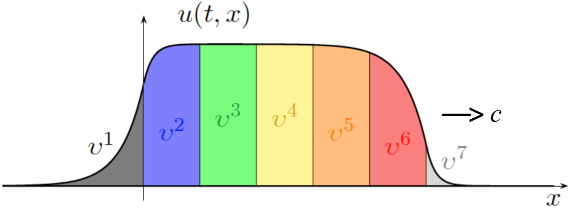

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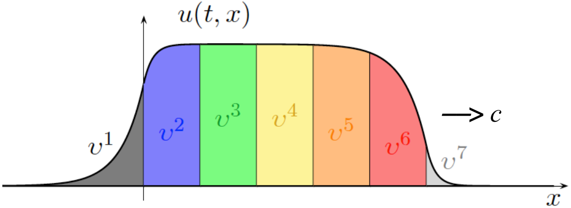

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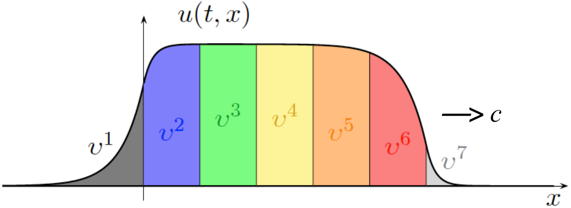

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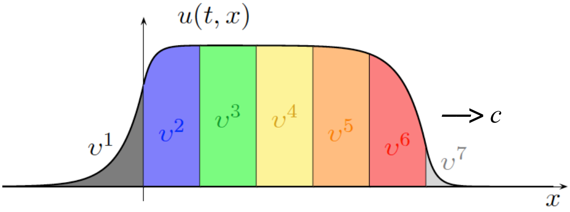
- Changes are governed by the "inside dynamics" of the wave (Roques et al. 2012)

Inside dynamics of the traveling pulse

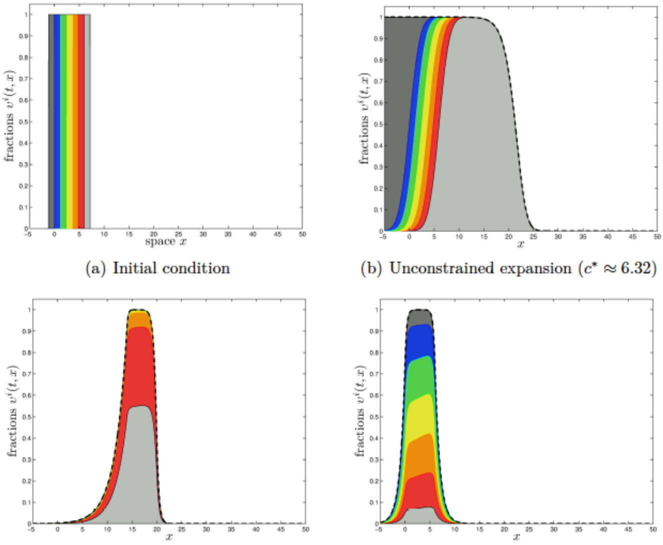


- The *inside dynamics* of each neutral fraction v^i satisfy

$$\begin{cases} \partial_t v^i = D\partial_{xx} v^i + g(x-ct,u)v^i, & t>0,\ x\in(-\infty,\infty), \\ v^i(0,x) = v_0^i(x), & x\in\mathbb{R}. \end{cases}$$

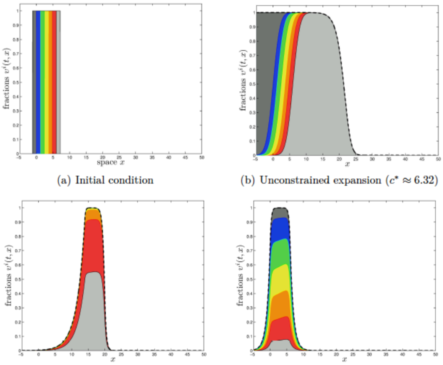
  where $g(x,u) = f(x,u)/u$ is the per capita growth rate of each fraction of the total population u.
- It follows that the sum of the fraction densities v^i satisfies the equation for the population density u.

Numerical solutions show spatial structure



(a) Initial condition
(b) Unconstrained expansion ($c^* \approx 6.32$)
(c) Range shift ($c = 2$)
(d) Range boundaries ($c = 0$)

Spatial structure of solutions



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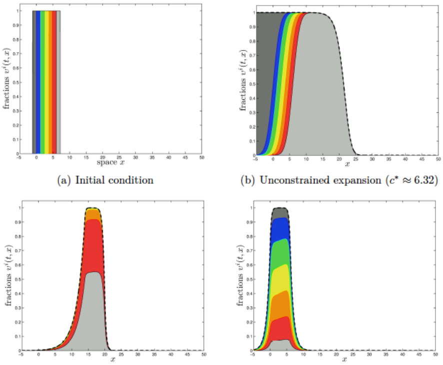

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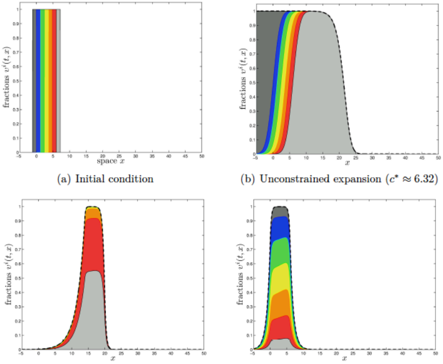

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- The founder effect is least pronounced with stationary range boundaries

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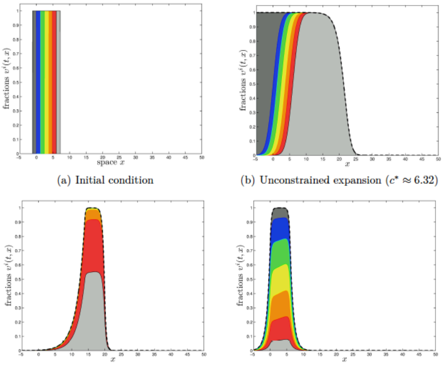

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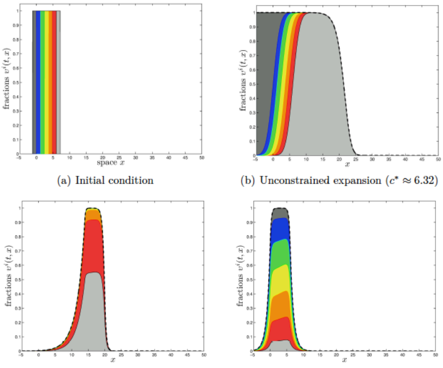

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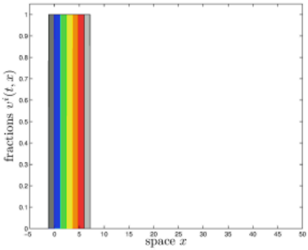
- as c increases, greater weight is given to fractions at the leading edge

Numerical solutions show spatial structure



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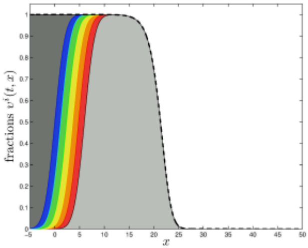

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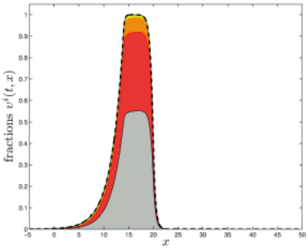

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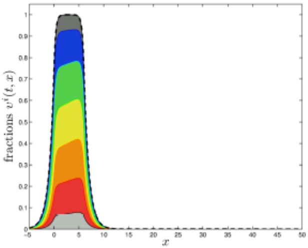

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Outline

1. Intrinsic spreading speed c^*
2. Population persistence during range shift
3. Tracking neutral fractions and inside dynamics
4. Measuring diversity for neutral fractions
5. Conclusions

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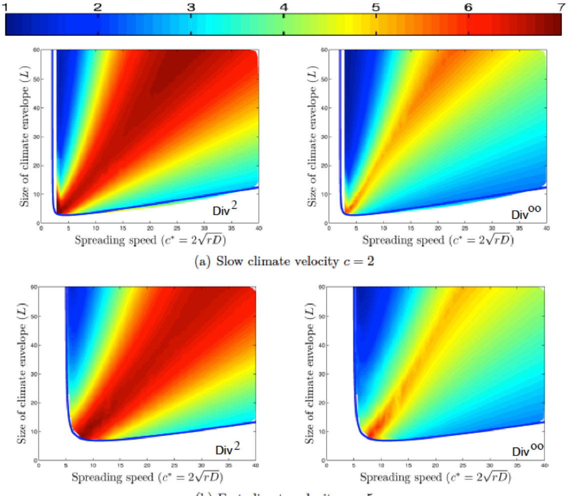
- Extension to the case $\mathrm{Div}^1 = \prod_{i=1}^{I}\frac{1}{(p^i)^{p^i}}$ yields the exponential of the Shannon index of diversity: $-\sum_{i=1}^{I} p^i \ln(p^i)$.

Diversity as a function of spreading speed c^* and size of climate envelope L.



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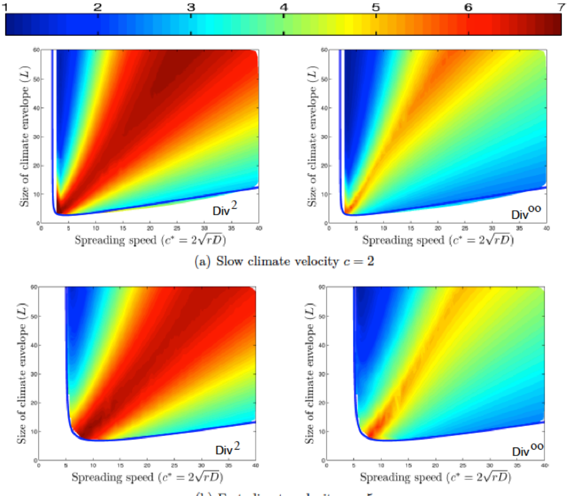

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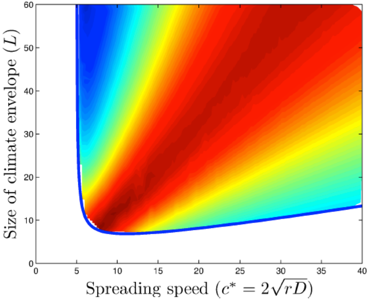

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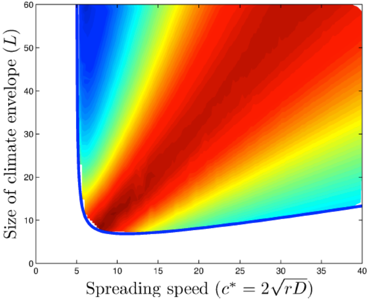

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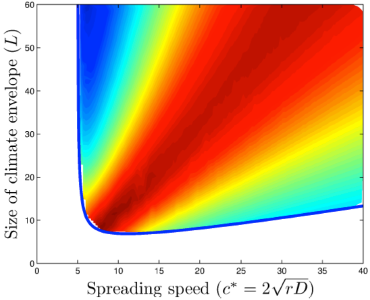

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- Diversity is highest at intermediate spreading speeds. High spreading speed leads to loss at the front of the envelope. Low spreading speed leads to loss at the back of the envelope.

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Thanks

- Thank you to friends and colleagues who gave us ideas and feedback on this work
- MAL acknowledges support from a Canada Research Chair, a Killam Research Fellowship and Discovery and Accelerator grants from the Canadian Natural Sciences and Engineering Research Council
- JG acknowledges support from the French National Research Agency