Data Assimilation for the Radiation Belt Environment

Humberto C. Godinez and Michael Henderson

Los Alamos National Laboratory

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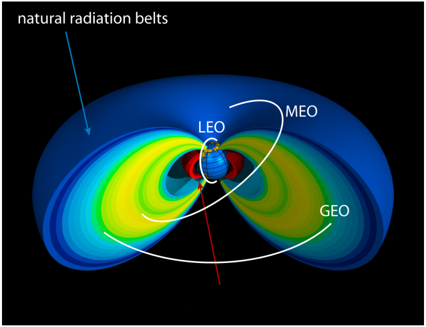
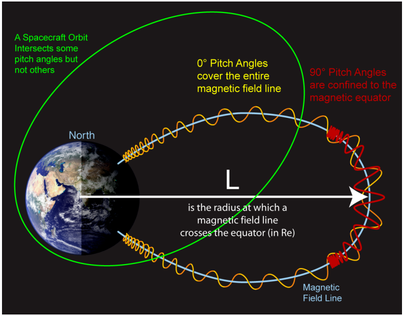
Introduction

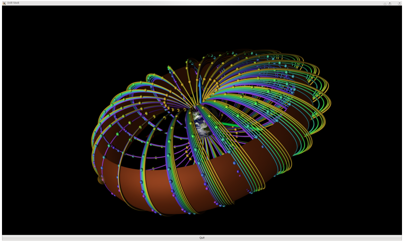


- Important to understand, specify, and predict Earth's radiation belt: high impact on space infrastructure.

- Radiation Belt Models: typically 1-D diffusion, 3-D models on the works or currently being tested.
- Observations for radiation belts gathered by various satellites; essential for our understanding.
- Data Assimilation is key in bringing together models and observations to provide an accurate nowcast. Forecast of radiation belts will depend on improving the models and observations.

Earth Magnetic Fields





Particle Motions:

- Gyration (first adiabatic invariant μ)
- Bounce (second adiabatic invariant K)
- Drift (third adiabatic invariant Φ)
- L*: radial distance in equatorial location where electrons would be found (third adiabatic invariant).

Radial Diffusion Model

Describes the radial evolution in L^* of phase space density (PSD) of electrons in the radiation belts:

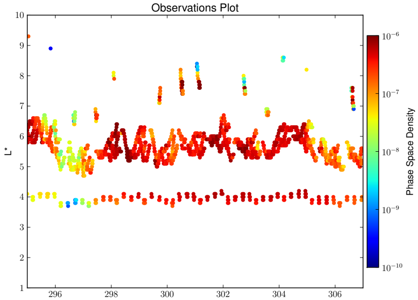
$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L} \left(\frac{D_{LL}}{L^2} \frac{\partial f}{\partial L} \right) + F$$

where F is the forcing term, and D_{LL} is a parametrized diffusion coefficient

$$D_{LL}\left(Kp, L\right) = 10^{(0.506Kp - 9.325)} L^{10}.$$

The radial diffusion model is discretized using finite difference in space, where the Crank-Nicolson scheme is used for time integration.

Radiation Belt Observations



- Observations are in flux form (count rates), equations in PSD.
- The conversion from flux (observations) to PSD (model space) is a critical step in radiation belt modeling.
- This conversion must also be reversed at the end to get predictions in physically-relevant quantities.
- Resulting observation field is very sparse.

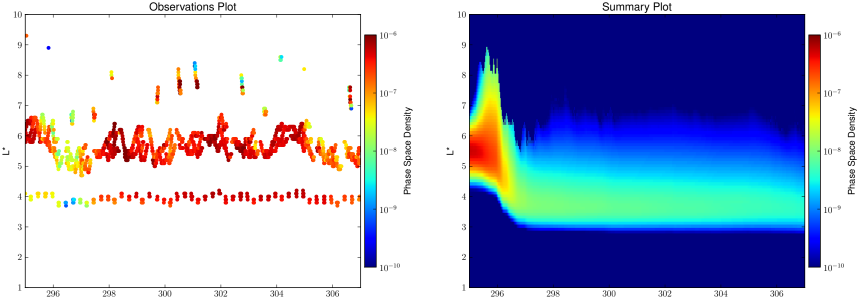


Figure: Left: Observations, right: radiation belt model solution.

Ensemble Kalman Filter

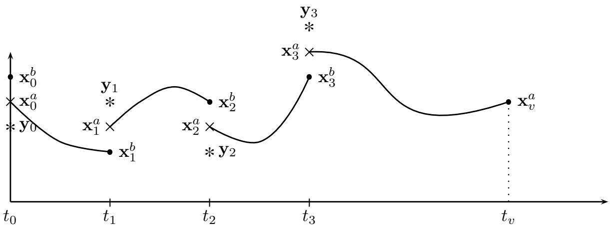


Figure: Schematic representation of sequential data assimilation.

Data Assimilation: Methods to produce an accurate estimate of the state of a model for a given data set (observations).
Ensemble Kalman Filter (EnKF): Sequential data assimilation method that uses an ensemble of model states to calculate the state mean and error covariance matrix needed to compute an improved model state.

EnKF formulation

For a vector of observations $\mathbf{y}^o \in \mathbb{R}^m$ and an ensemble of N forecast $\mathbf{x}_i^f \in \mathbb{R}^n, i = 1, \dots, N$ the EnKF analysis equation are given by:

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}\left(\mathbf{y}_i^o - \mathbf{H}\mathbf{x}_i^f\right), \quad i = 1, \dots, N \tag{1}$$

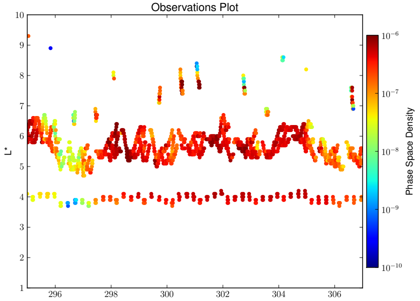
$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R}\right)^{-1}. \tag{2}$$

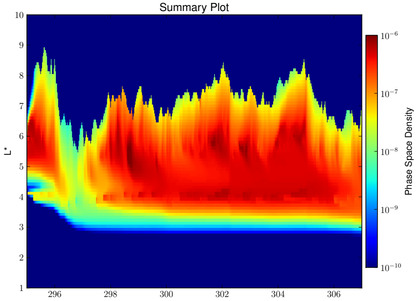
In the EnKF the forecast error covariance matrix is obtained through the ensemble of model forecast, using the relation

$$\mathbf{P}^f = \frac{1}{N-1}\sum_{i=1}^{N}\left(\mathbf{x}_i^f - \overline{\mathbf{x}}^f\right)\left(\mathbf{x}_i^f - \overline{\mathbf{x}}^f\right)^T, \tag{3}$$

where $\overline{\mathbf{x}}^f$ is the forecast ensemble average

EnKF assimilation for Radiation Belt Model





EnKF data assimilation with inflation around observations. The assimilation is smoother but with some irregular patches within the model state. This enables the EnKF to guide the state towards the observations.

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Diffusion Coefficient

Most implementations use a specific form of the diffusion coefficient, specifically

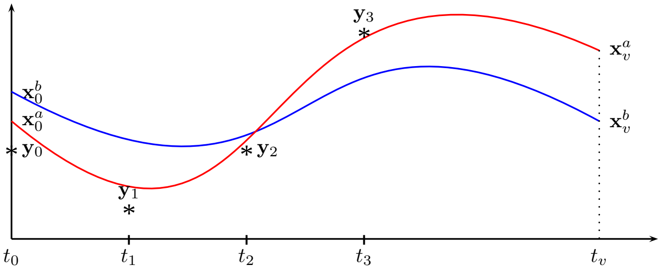
$$D_{LL}\left(Kp, L\right) = 10^{(0.506Kp - 9.325)} L^{10}.$$

We can generalized the form of the diffusion coefficient to

$$D_{LL} = \sum_{i=1}^{N} D_i L^i \tag{4}$$

Schultz and Lanzerotti (1974)explored the use of a variational method to estimate the correct D_i. We propose to use a more sophisticated approach through data assimilation. In particular we will use the 4D-Var.

Four Dimensional Variational Method



The principle of variational methods is to approximate the model simulation trajectory by minimizing a cost functional, that is

$$\mathbf{x}^a = \underset{\mathbf{x}\in\mathbb{R}^n}{\text{argmin}} \mathcal{J}(\mathbf{x})$$

The cost function $\mathcal{J}$ penalizes the distance between the model and observational data over space and time.

4D-Var (Cont.)

For an vector of observations $\mathbf{y}^o \in \mathbb{R}^m$, the 4D-Var cost function is defined as:

$$\mathcal{J}(\mathbf{x}) = \left(\mathbf{x} - \mathbf{x}^f\right)^T \left(\mathbf{P}^f\right)^{-1} \left(\mathbf{x} - \mathbf{x}^f\right) + \sum_{k=1}^{T} \left(\mathbf{y}_k^o - \mathbf{H}_k \mathbf{x}_k\right)^T \mathbf{R}_k^{-1} \left(\mathbf{y}_k^o - \mathbf{H}_k \mathbf{x}_k\right) \qquad (5)$$

In essence we will reformulate our problem within the 4D-Var framework to estimate the coefficients D_i of equation (4).

Tangent and Adjoint are needed for the minimization!

4D-Var (Cont.)

Since the radiation belt model

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(\frac{D_{LL}}{L^2}\frac{\partial f}{\partial L}\right) + F$$

is discretized using Crank Nicholson, the discrete time evolution of the numerical solution is given by

$$\mathbf{A}\mathbf{u}^{t+1} = \mathbf{B}\mathbf{u}^{t}$$

where the matrices $\mathbf{A}$ and $\mathbf{B}$ are functions of the diffusion coefficients. The linear system is typically solved using LU factorization and backward substitution. Hence we need the derivative of the LU factorization and substitution with respect to the diffusion coefficient.
WORK IN PROGRESS

Future Work

- Data assimilation is key to understand and predict radiation belts
- First results with ensemble Kalman filter (EnKF) show great improvement over non-assimilated solutions
- To produce an accurate forecast, future radiation belt models must better account for the various source terms and parameters that would better describe the physical phenomena.
- Need to better specify the diffusion coefficient, variational methods are best equipped for this task
- working on tangent and adjoint of numerical model solution