Mixing and Pumping by Pairs of Helices in a Viscous Fluid

Amy Buchmann and Lisa J. Fauci, Tulane University
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Eva M. Strawbridge, James Madison University
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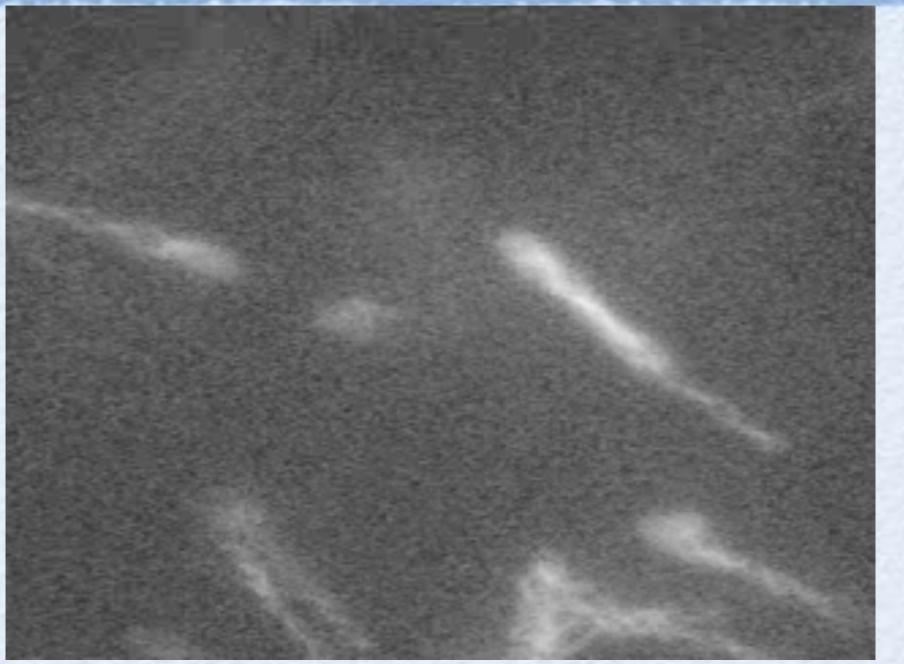
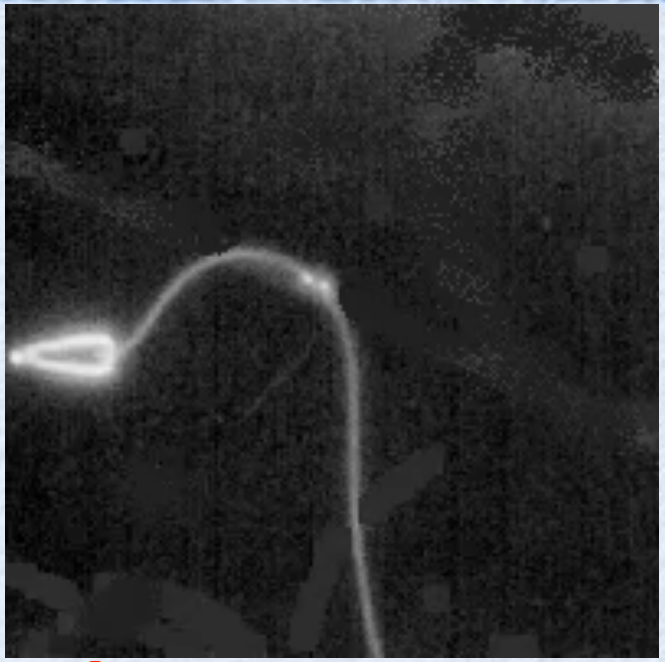
SIAM Conference on the Life Sciences (LS18)
Minneapolis,August 6 - 9, 2018

Examples of Low Re swimmers

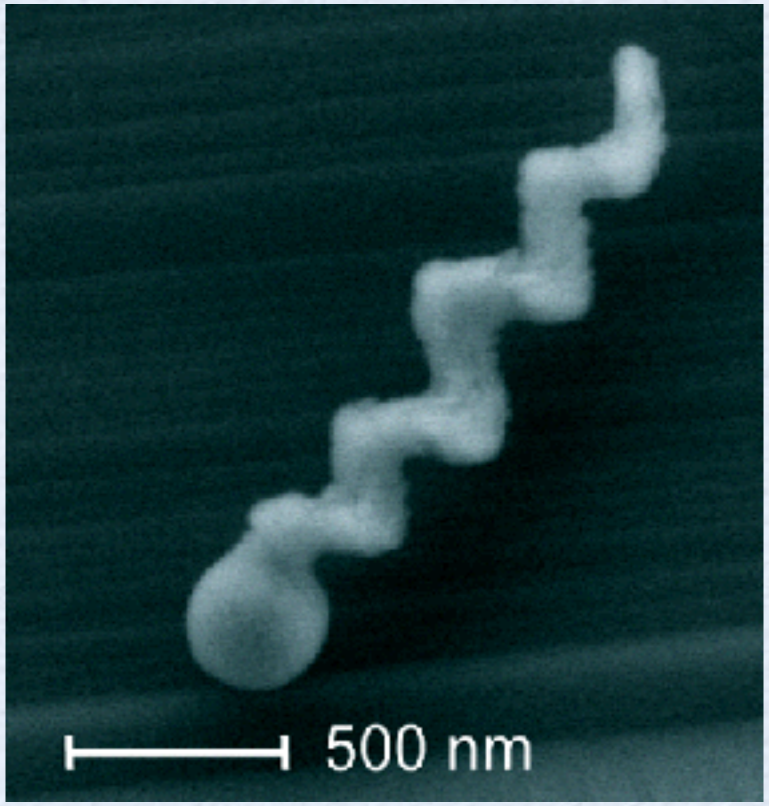


E. Coli bacteria
L. Turner, Harvard

Spirochete
S. Goldstein, UMN



Sea urchin sperm
C. Brokaw, Caltech



Artificial Propellers
Ghosh and Fischer, Nano Letters 2009

Reynolds number

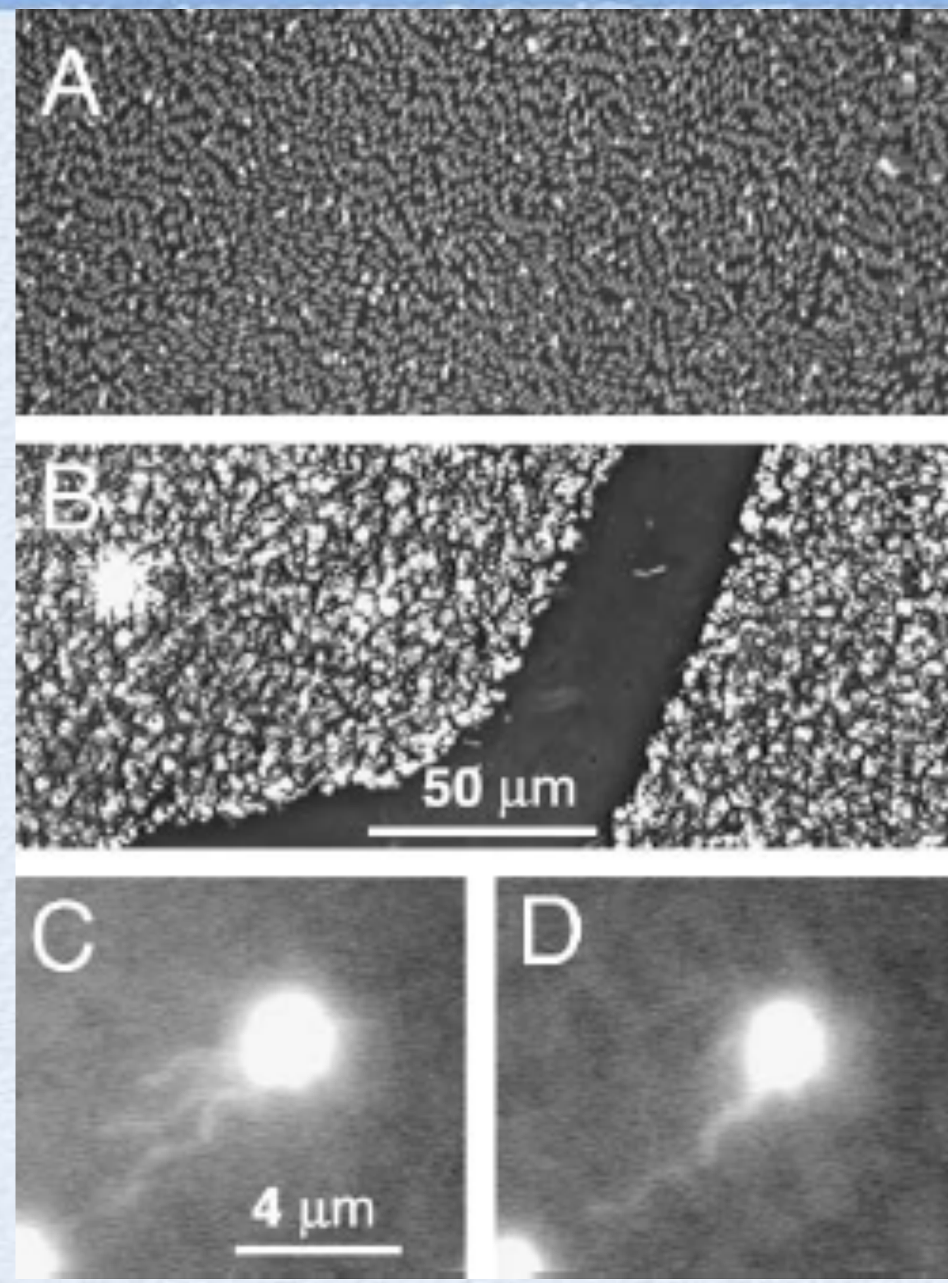
Dimensionless parameter – ratio of inertial forces to viscous forces:

Re = Density*Length*Velocity / Viscosity

Man swimming: 10,000
Goldfish: 100
Nematode: 1
Sperm cell: .01
Bacteria: .0001

Life at low Reynolds number E.M. Purcell, 1976 American J. Physics

Bacterial Carpets



Biophysical Journal Volume 86 March 2004 1863–1870

Moving Fluid with Bacterial Carpets

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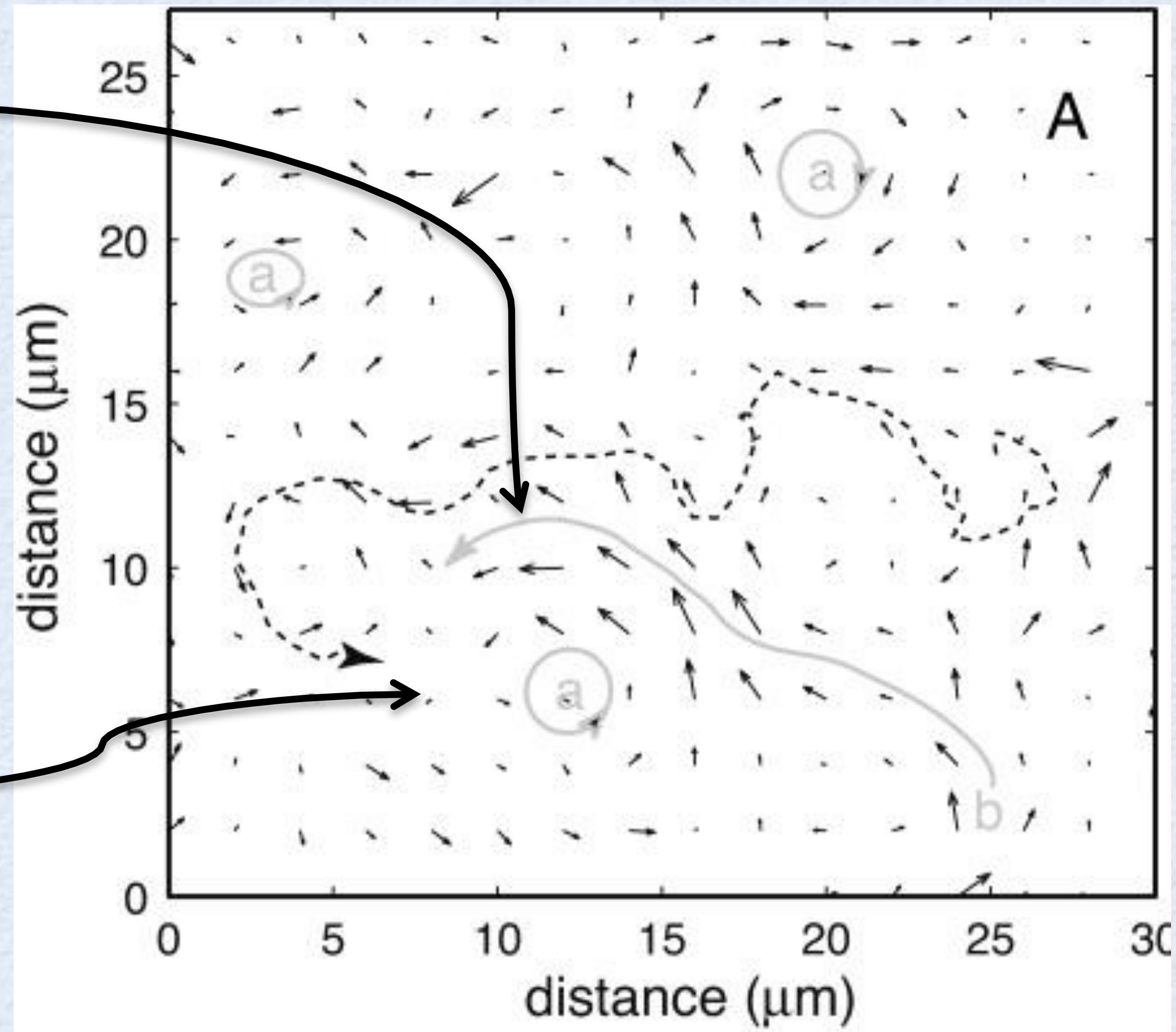
*Rowland Institute at Harvard, Cambridge, Massachusetts 02142; and †Division of Engineering, Brown University, Providence, Rhode Island 02192

Flow structure

"Rivers"

"Whirlpools"



Fundamental questions

These **bacterial carpets** bring up fundamental questions in fluid mechanics regarding the interaction of a collection of helices and finite-volume particles with a Newtonian Stokes fluid.

- How does alignment of helices affect transport?
- How does spatial distribution affect transport?
- How does the presence of the planar wall affect axial thrust and flow features?
- How are finite volume particles transported by bacterial carpet?

Governing equations

Navier-Stokes Equation

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mu\nabla^2\mathbf{u}$$

$$\nabla\cdot\mathbf{u} = 0$$

+ boundary conditions

ρ : fluid density, p : fluid pressure, μ :dynamic viscosity

The condition for Stokes regime to hold

$SrRe = \rho\,\omega\,\ell^2/\mu \ll 1$ $\quad Re = \rho\,\omega\,\ell^2\sin(\kappa)/\mu \ll 1$ (Typically, $10^{-4} - 10^{-3}$)

Fundamental solution: Stokeslet

$$\mu \nabla^2 \mathbf{u} + f_S = \nabla p$$
$$\nabla \cdot \mathbf{u} = 0$$

where $f_S = 8\pi\mu\boldsymbol{\alpha}\delta(\mathbf{x})$

$\delta(\mathbf{x})$: the 3D Dirac delta-function

$\boldsymbol{\alpha}$: the strength of Stokeslet

$$\mathbf{u}_S(\mathbf{x};\boldsymbol{\alpha}) = \frac{\boldsymbol{\alpha}}{|\mathbf{x}|} + \frac{(\boldsymbol{\alpha}\cdot\mathbf{x})\mathbf{x}}{|\mathbf{x}|^3}$$

$$p_S(\mathbf{x};\boldsymbol{\alpha}) = -2\mu\frac{(\boldsymbol{\alpha}\cdot\mathbf{x})}{|\mathbf{x}|^3}$$

More singular solutions can be derived from the Stokeslet by differentiation.

Many important models have been created with the fundamental solutions.

Examples:

Analyses of flagellar motions

Beating motion of cilia

Flows between plates and inside cylinders

Flows in periodic geometries

Slender body theories

Difficulties:

- Shapes
- Interaction between objects
- Instabilities near the singularities

Regularized Stokeslet

$$\mu\nabla^2\mathbf{u} + \mathbf{f} = \nabla\mathbf{p}, \quad \nabla\cdot\mathbf{u} = \mathbf{0}$$

$$\mathbf{f} = \mathbf{f_0}\phi_\epsilon(\mathbf{x}-\mathbf{x_0})$$ the external force

$$\mathbf{u} = \frac{\mathbf{1}}{\mu}\left[(\mathbf{f_0}\cdot\nabla)\nabla\mathbf{B}_\epsilon - \mathbf{f_0}\mathbf{G}_\epsilon\right]$$

$$\nabla^2 G_\epsilon = \phi_\epsilon \text{ and } \nabla^2 B_\epsilon = G_\epsilon$$

Forces are spread over a small ball -- in the case x_0=0

Method of regularized Stokeslets (R. Cortez, SIAM SISC 2001)

Velocity field

For the choice:

$$\phi_\epsilon(\mathbf{x}) = \frac{15\epsilon^4}{8\pi(r^2+\epsilon^2)^{7/2}}.$$

the resulting velocity field is:

$$\mathbf{u}(\mathbf{x}) \quad = \quad \frac{1}{8\pi\mu}\left\{\mathbf{f}_0 \frac{2\epsilon^2+r^2}{(r^2+\epsilon^2)^{3/2}} + \frac{(\mathbf{f}_0\cdot\mathbf{x})\mathbf{x}}{(r^2+\epsilon^2)^{3/2}}\right\}$$

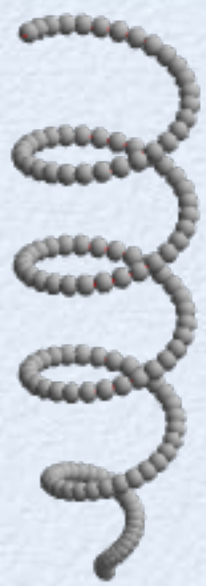
Note:

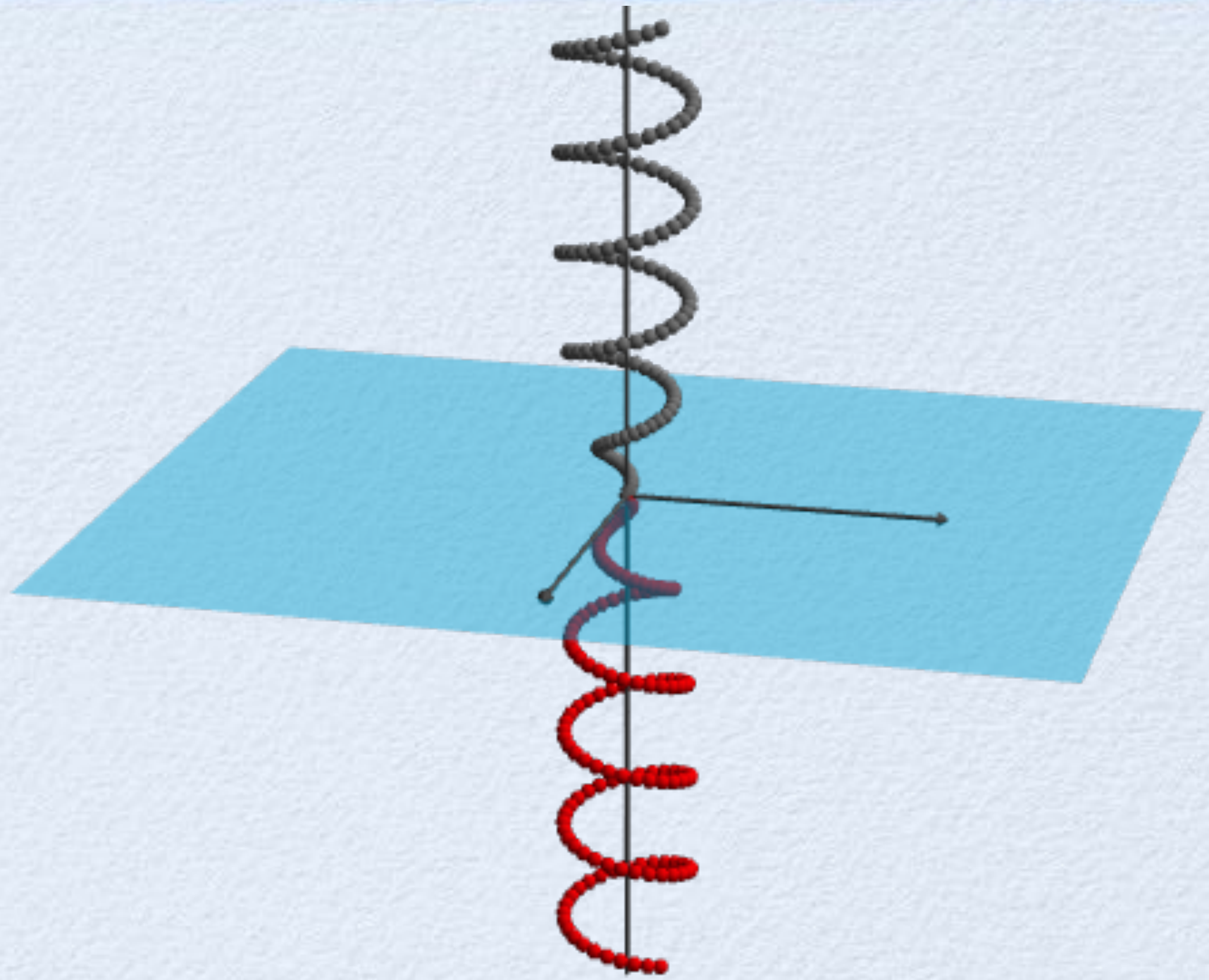
•u(x) is defined everywhere

•u(x) is an exact solution to the Stokes equations, and is incompressible

•Grid–free numerical method

Superposition of singularity and image system



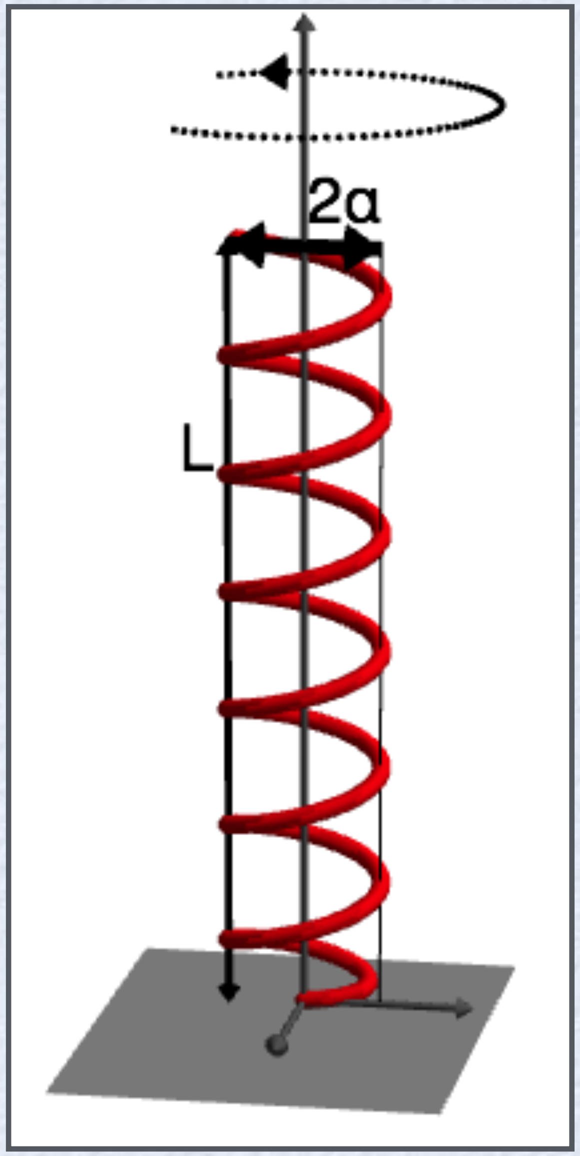


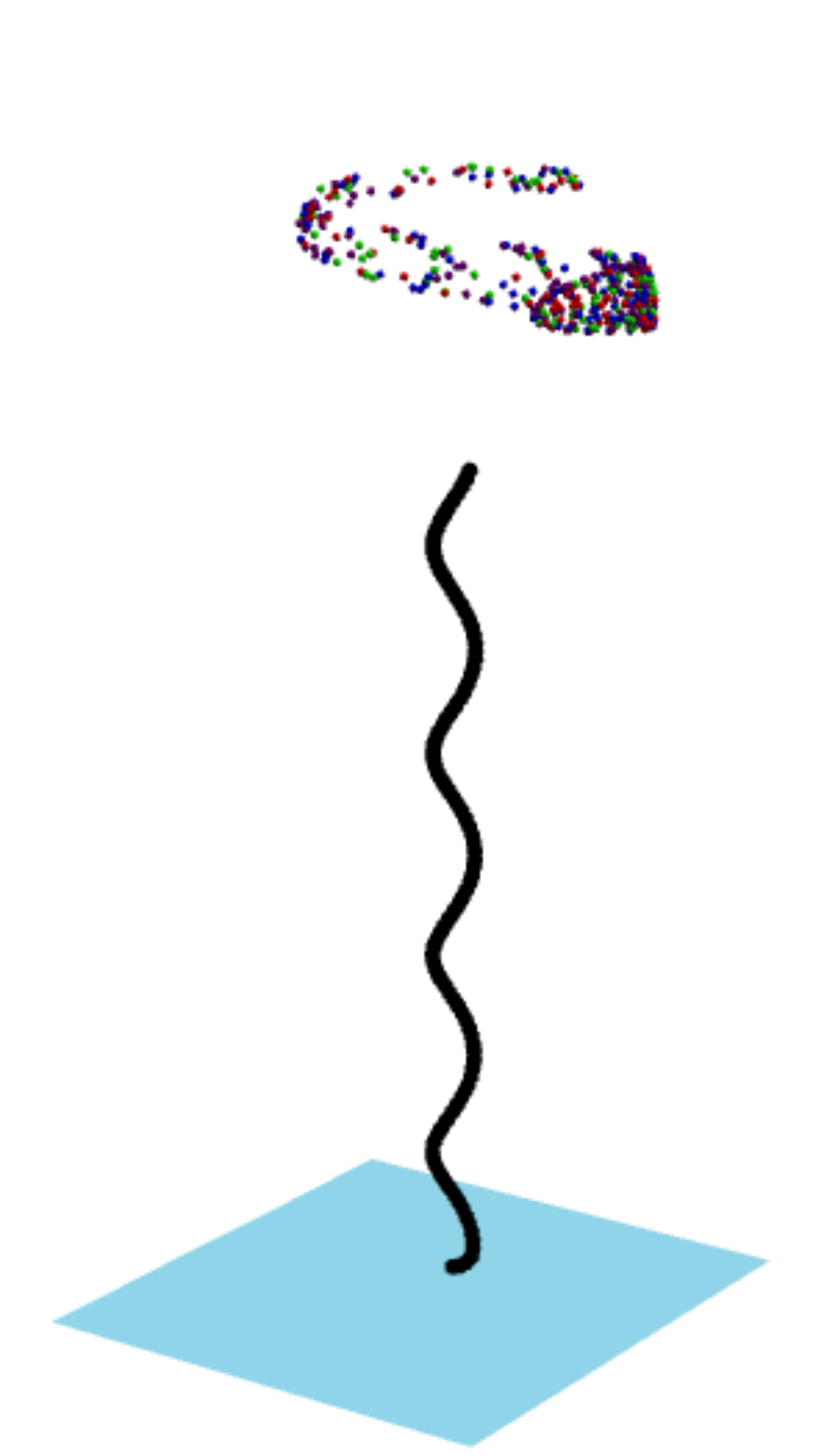
Modeling Helical Flagellum

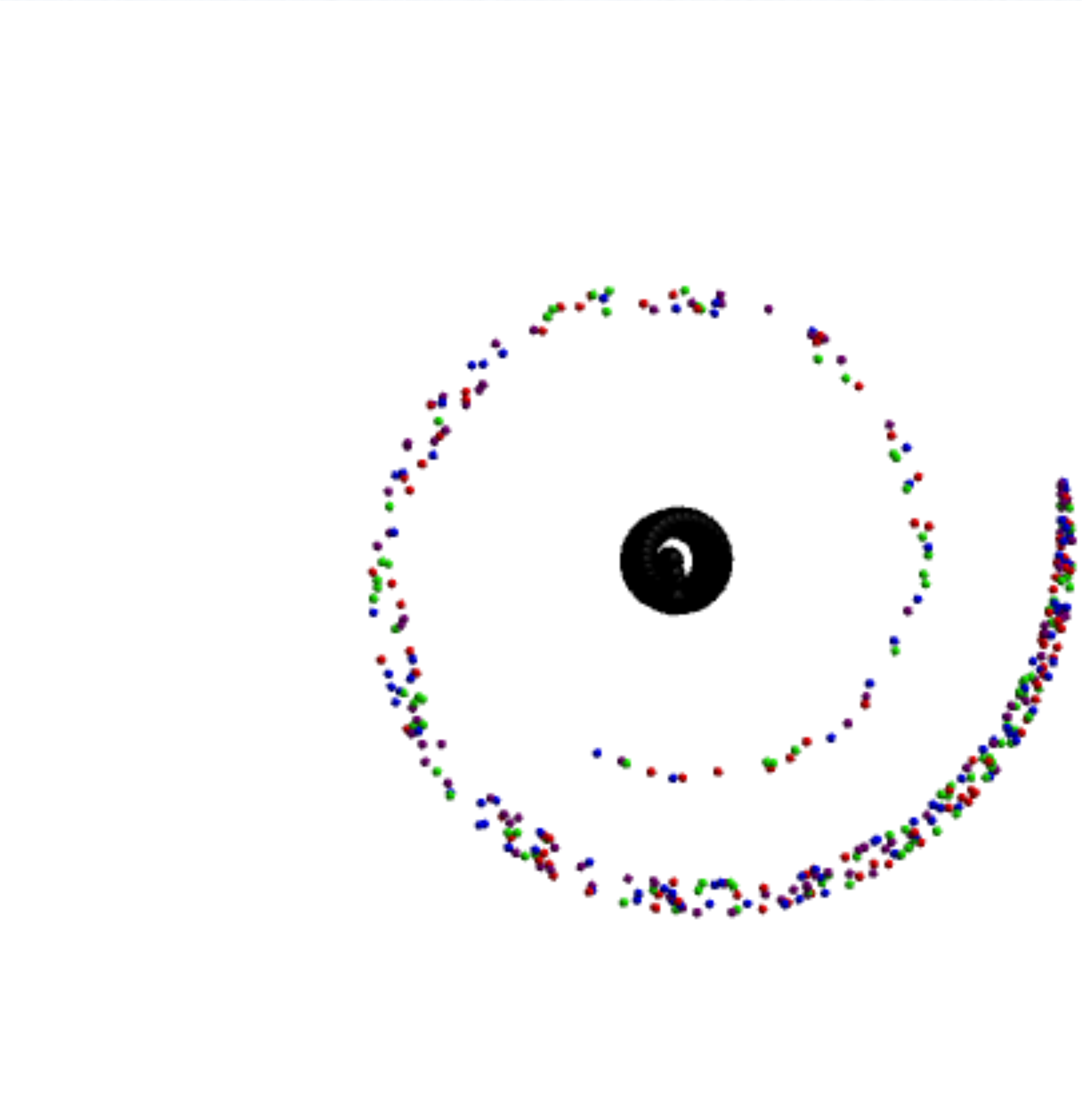
$$U = AF$$

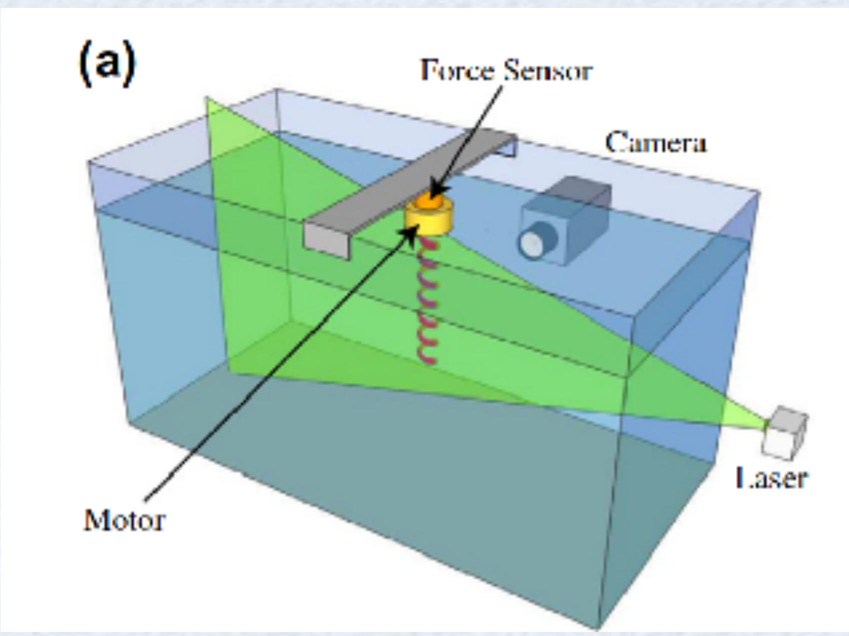
- Solve for the forces on the points of the helix that will give the prescribed velocity.

- Use these forces to find the fluid velocity at any point

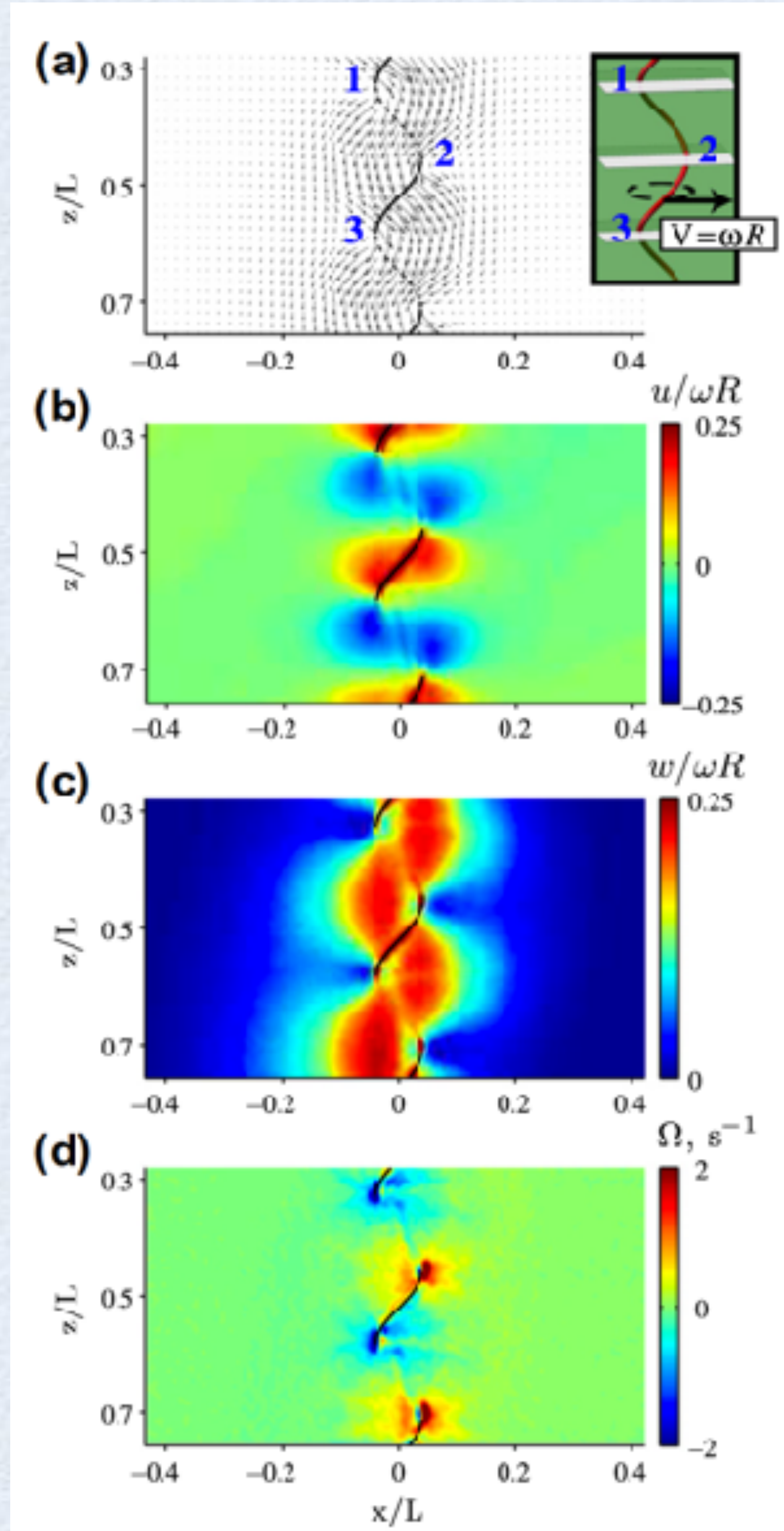




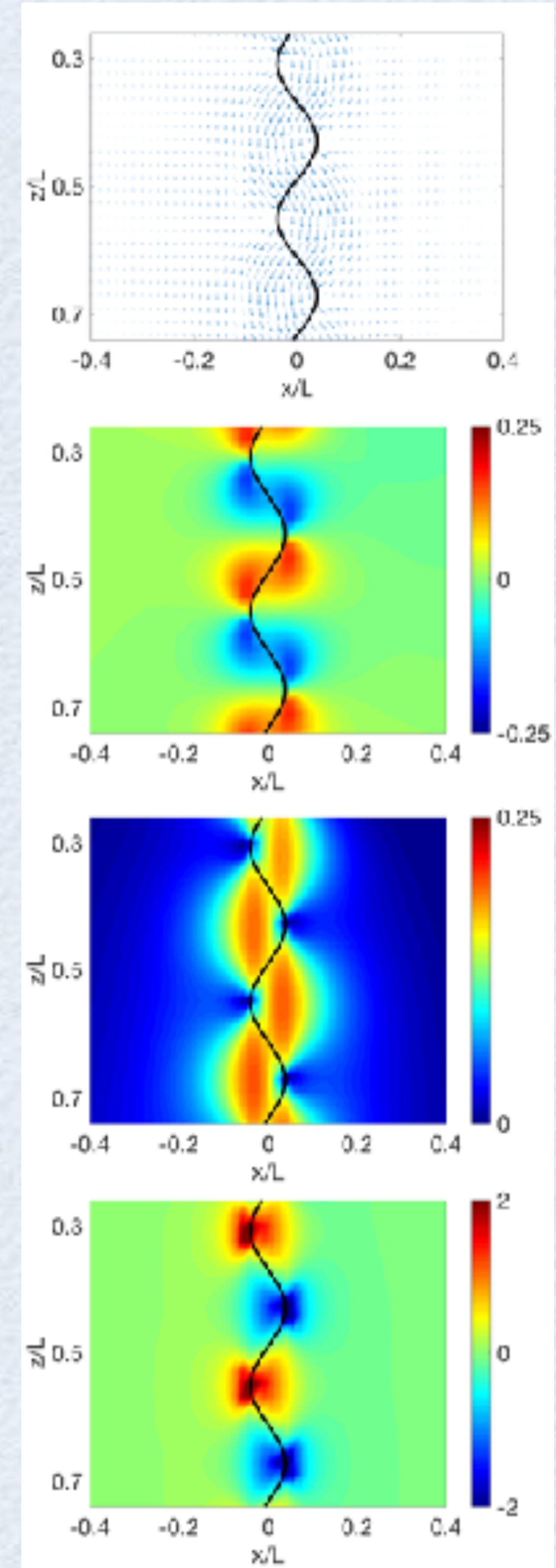




Experiment

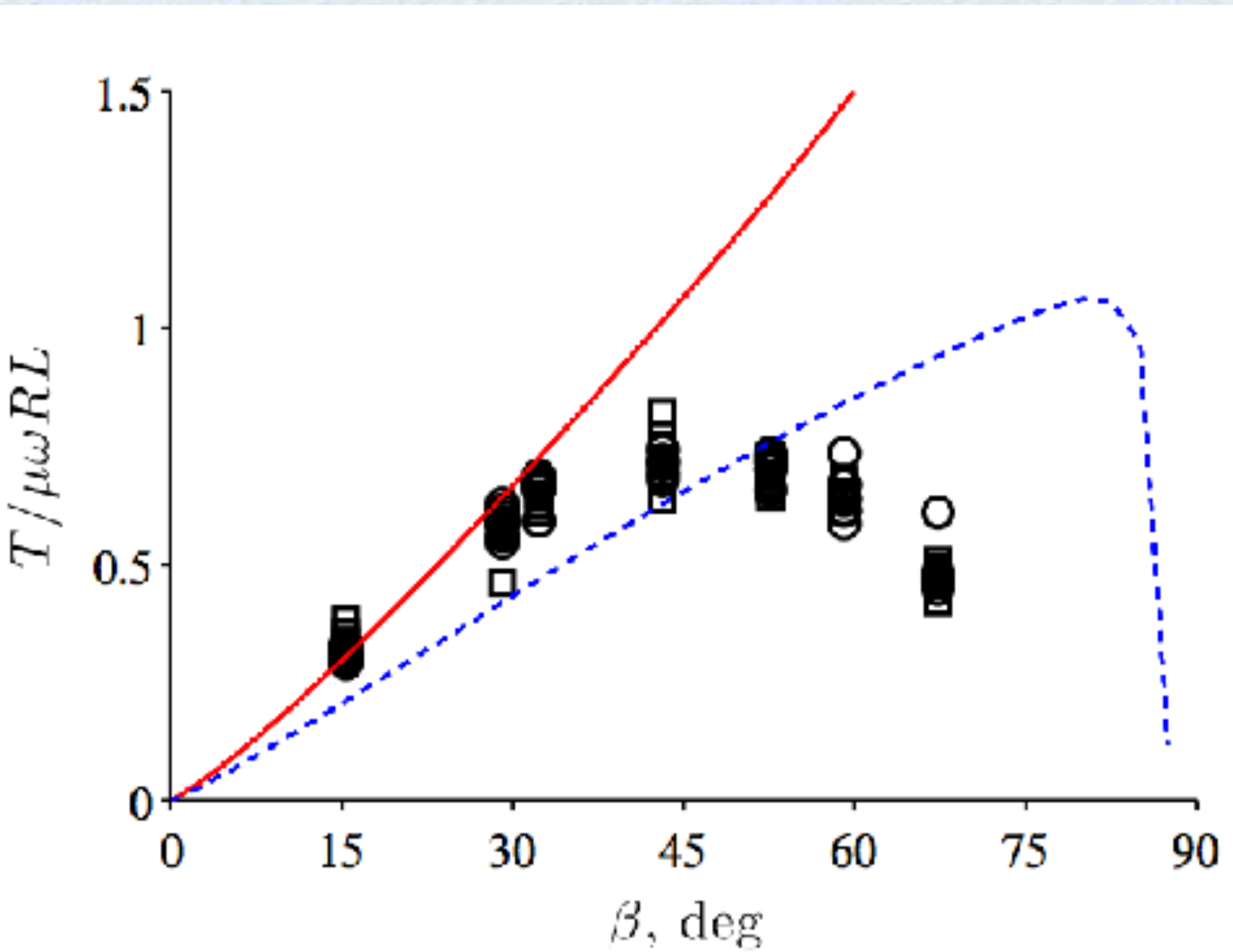


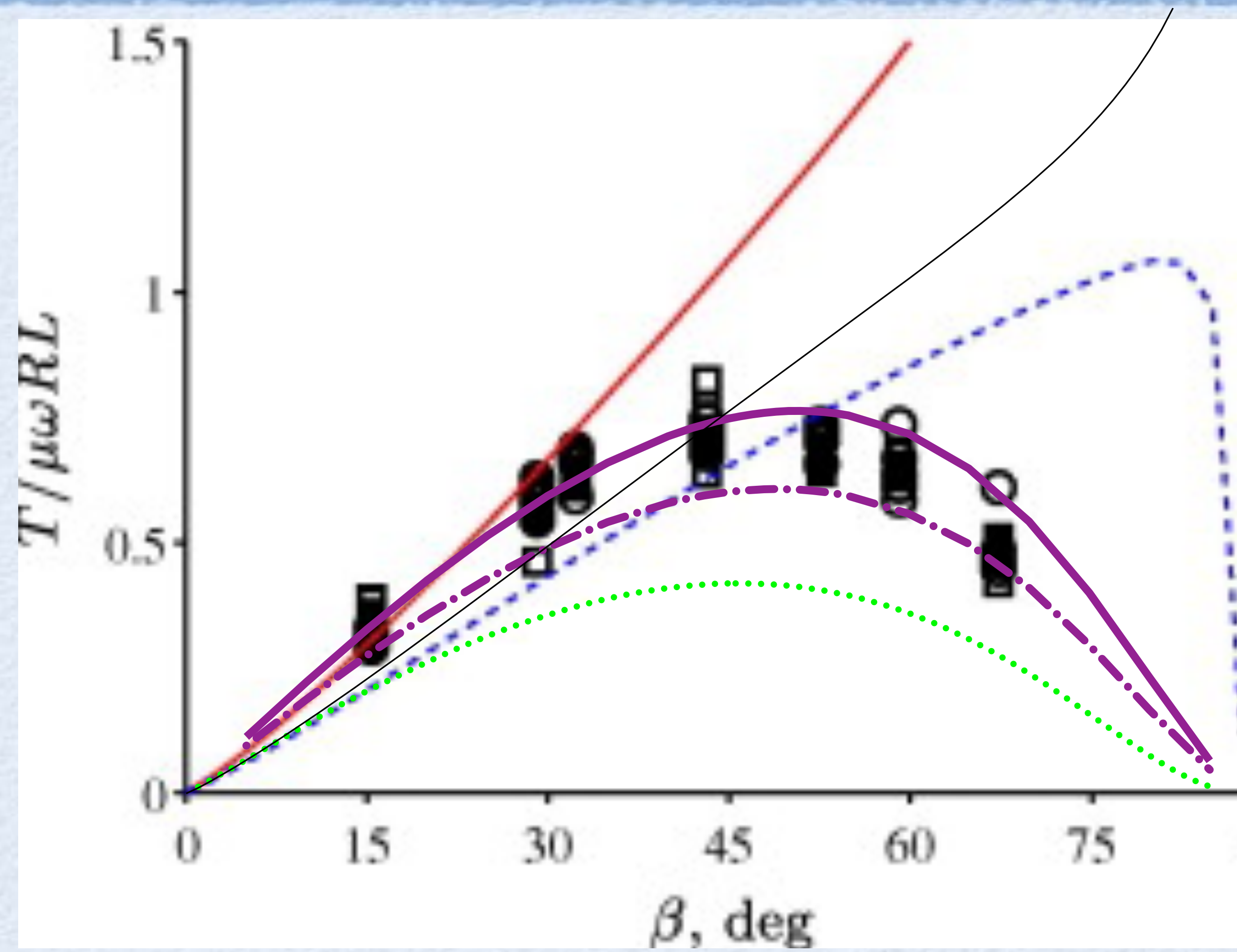
Simulation



S Zhong, KW. Moored, V. Pinedo et al, Exp. Therm. and Fluid Sci., 2013.

Normalized thrust as functions of pitch angle

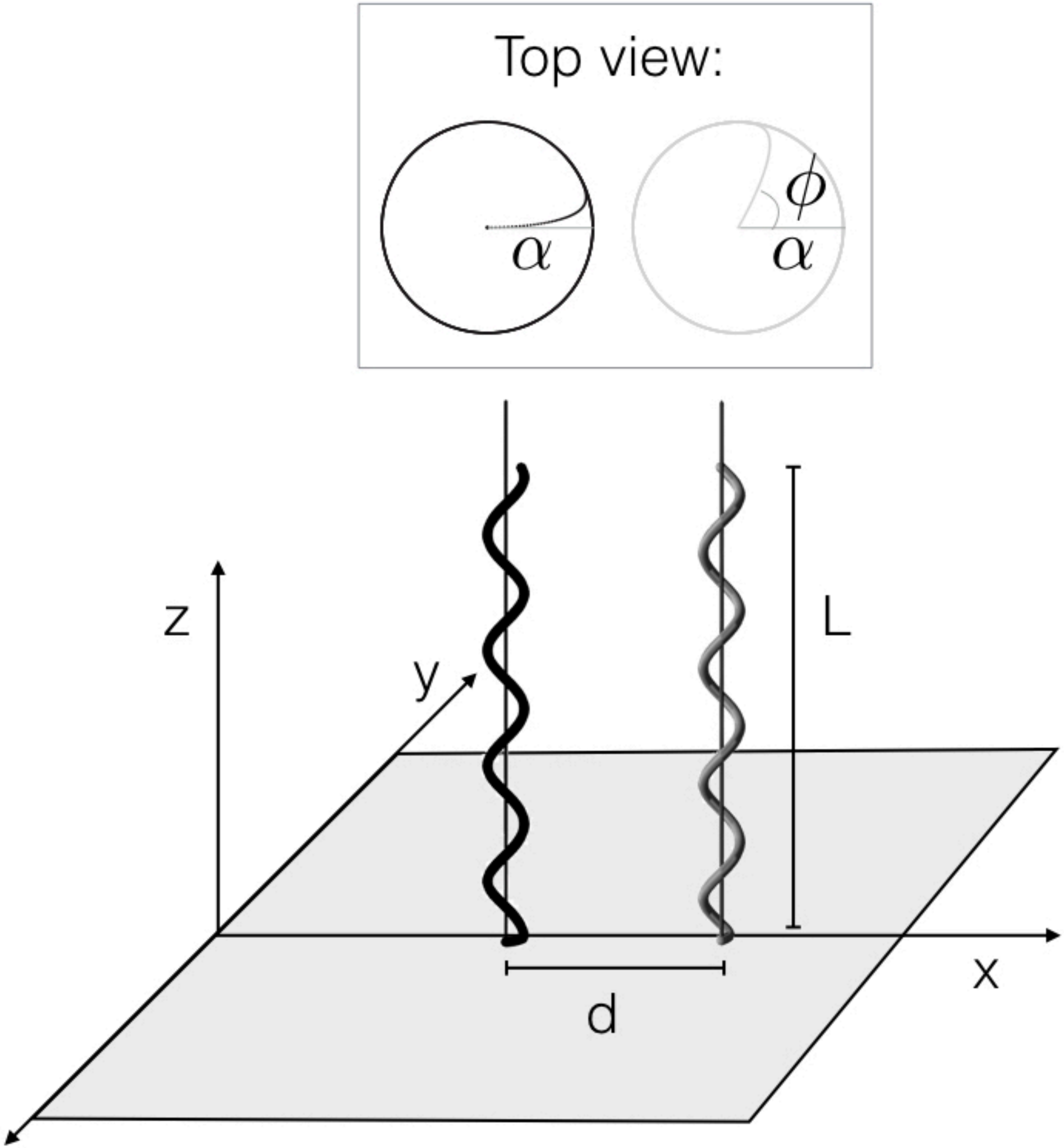




Zhong et al. (2013)

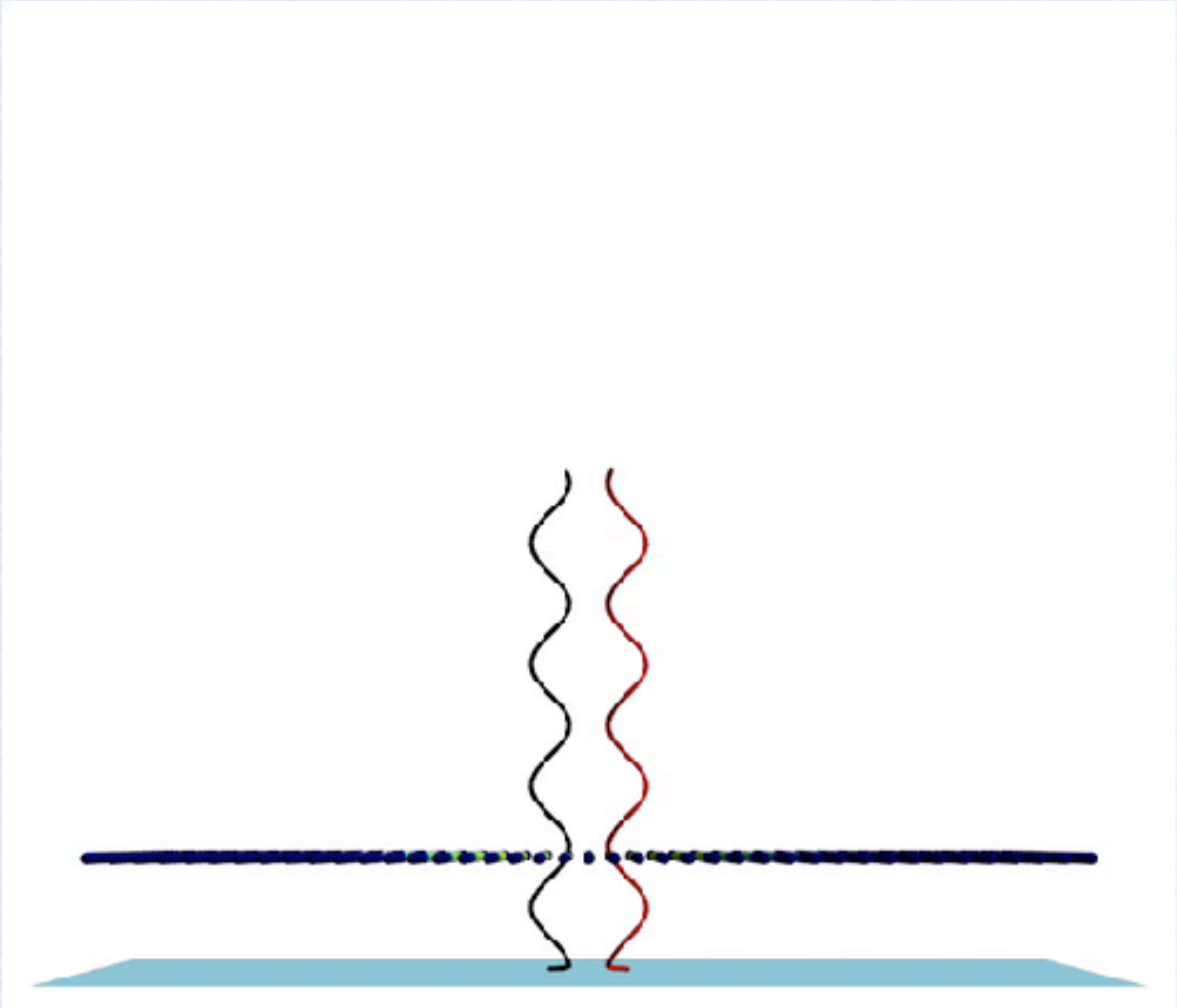
Resistive-force theory
red: Gray and Hancock
blue: Cox, Johnson and Brokaw

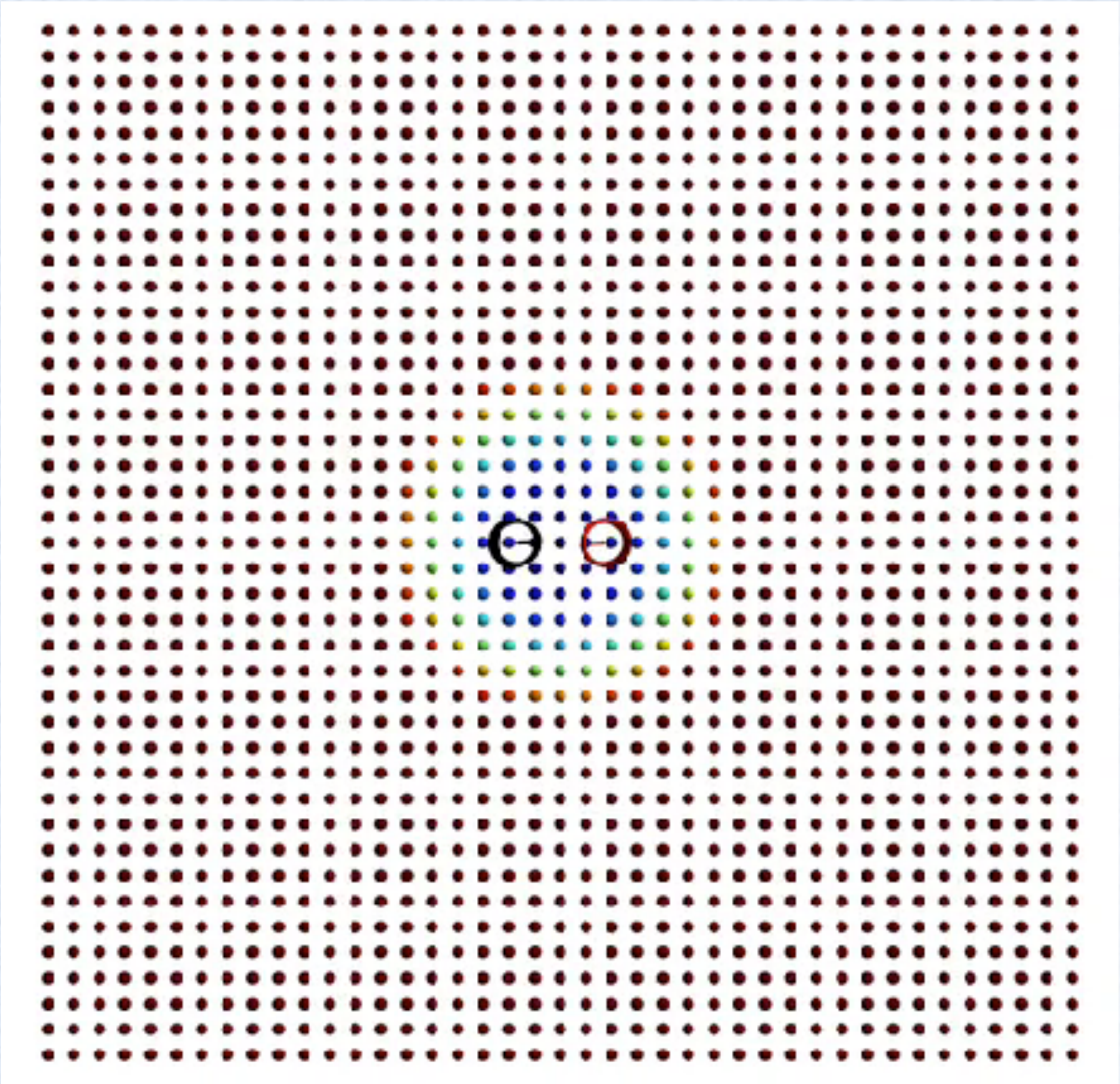
Resistive-force theory
black: Lighthill
Regularized Stokeslet Method:
purple: with wall green: no wall effect



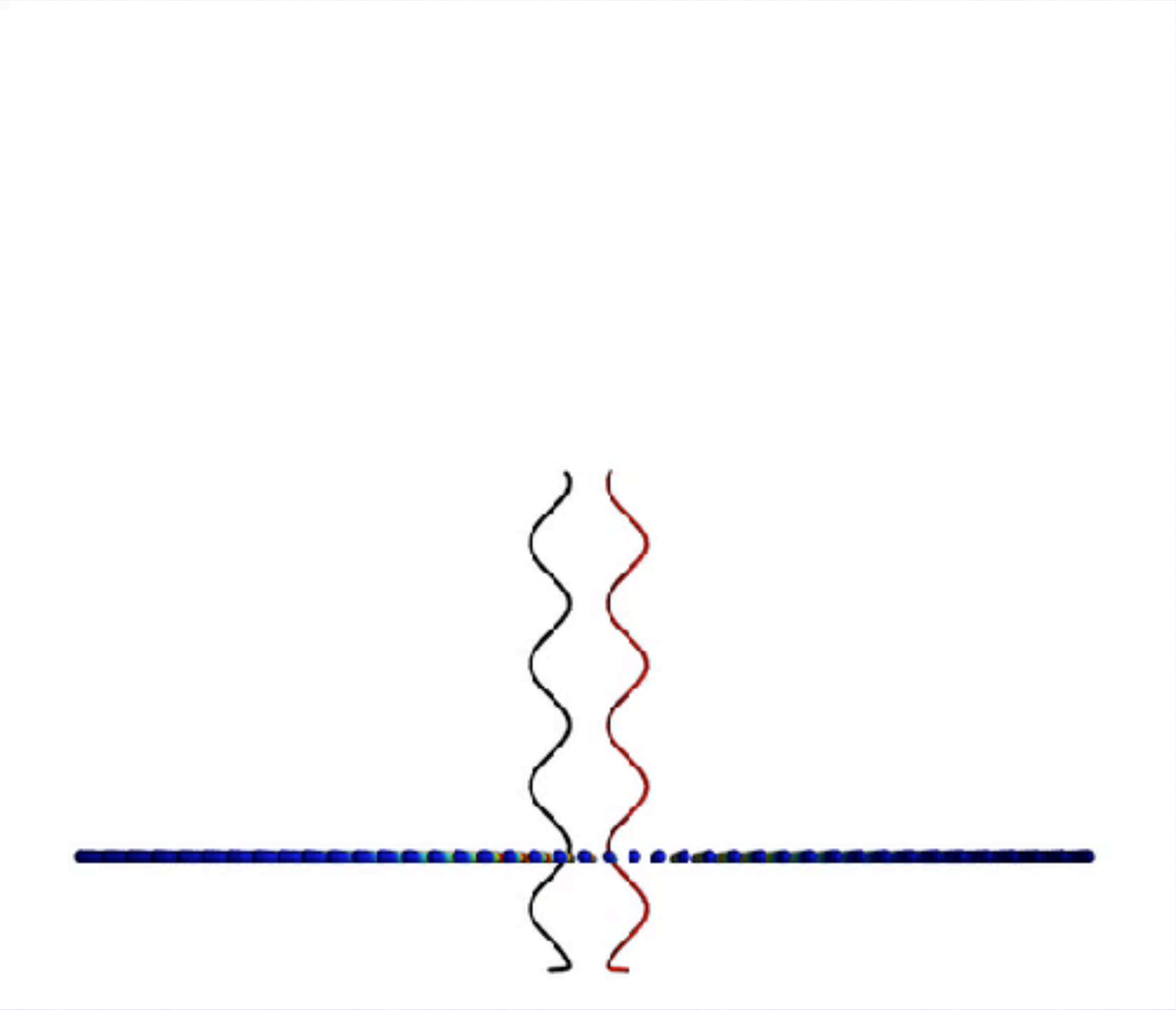
Top view:
α
ϕ
α
z
y
L
x
d

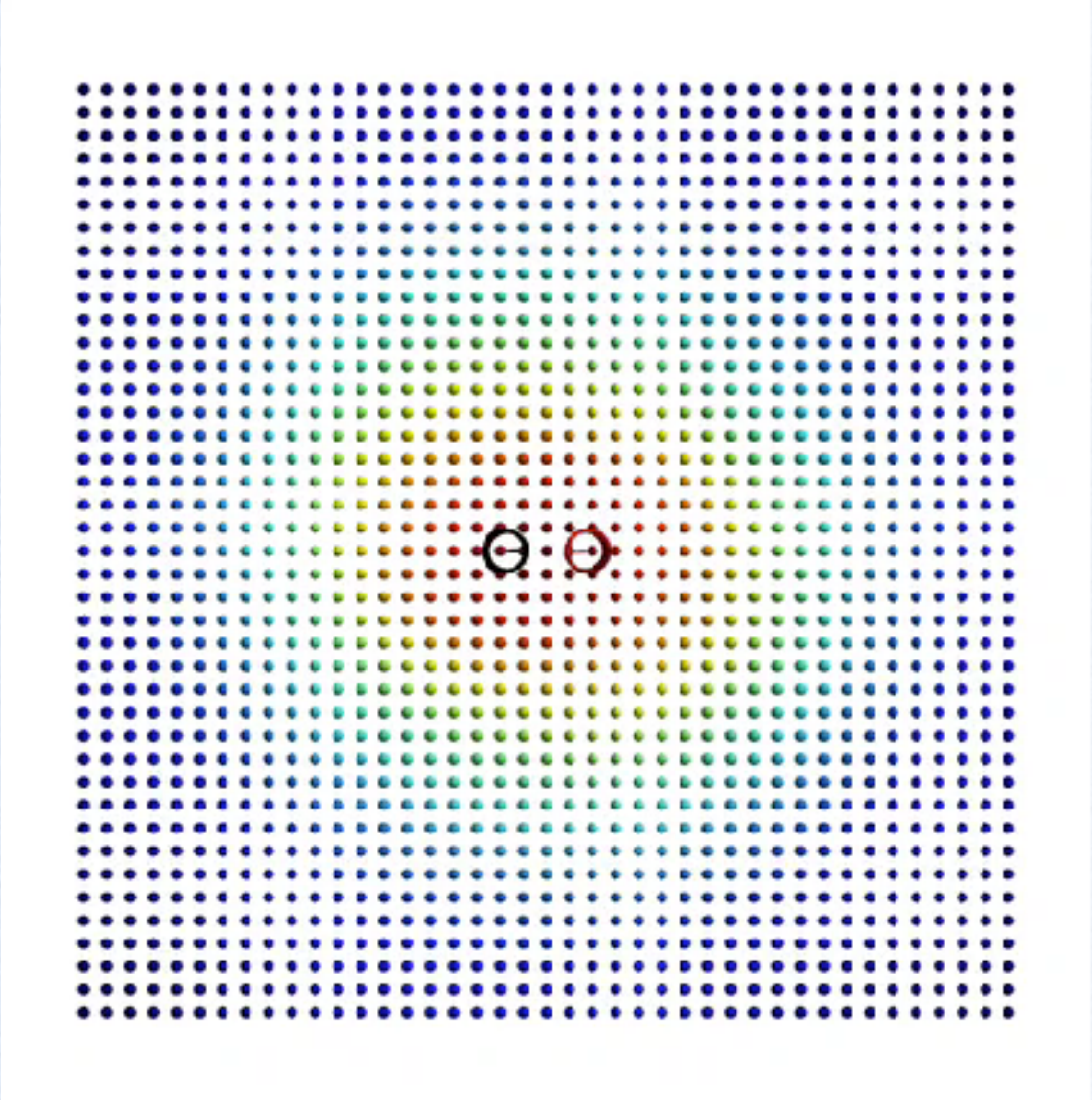
Fluid particle



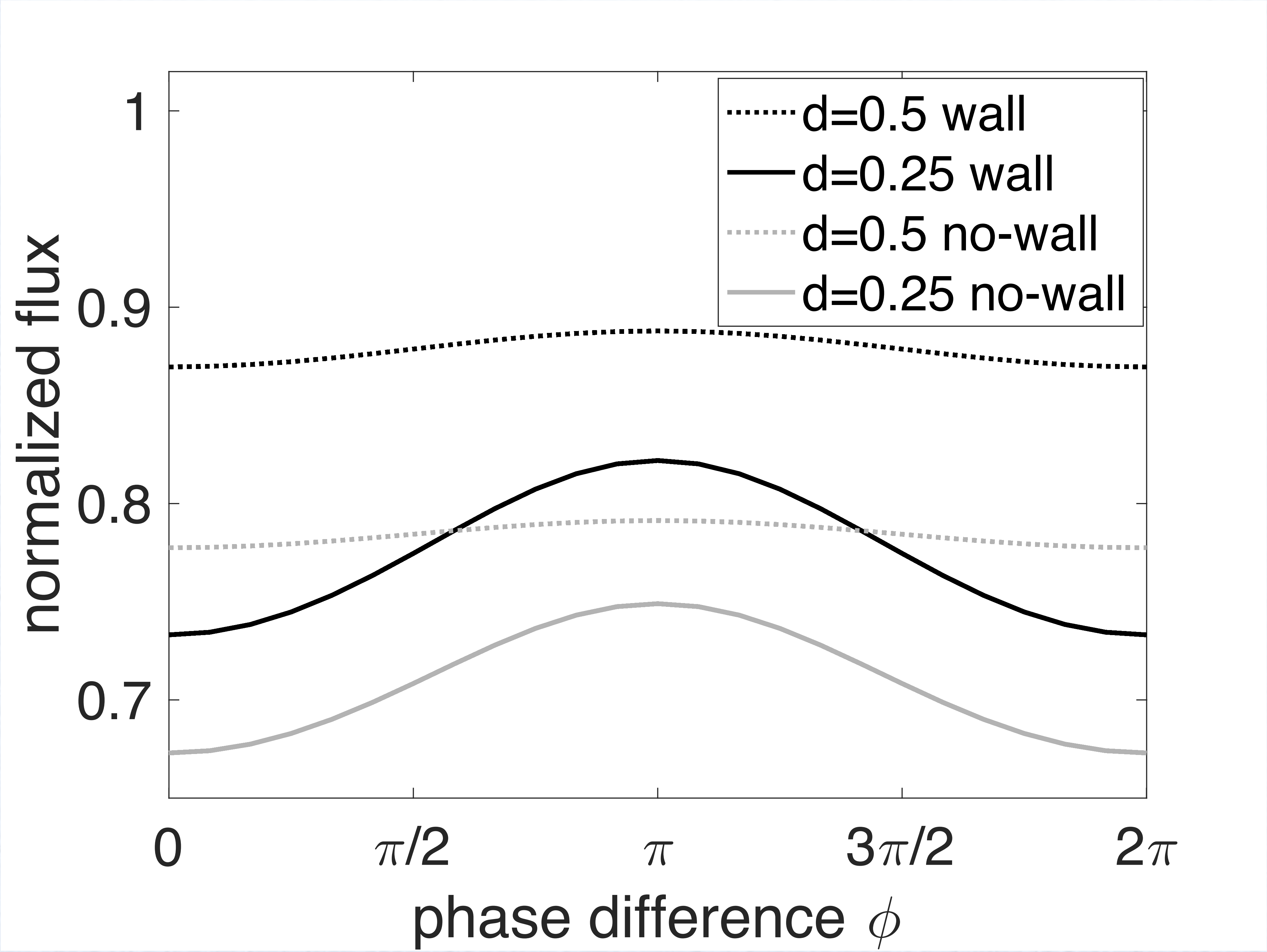


No wall

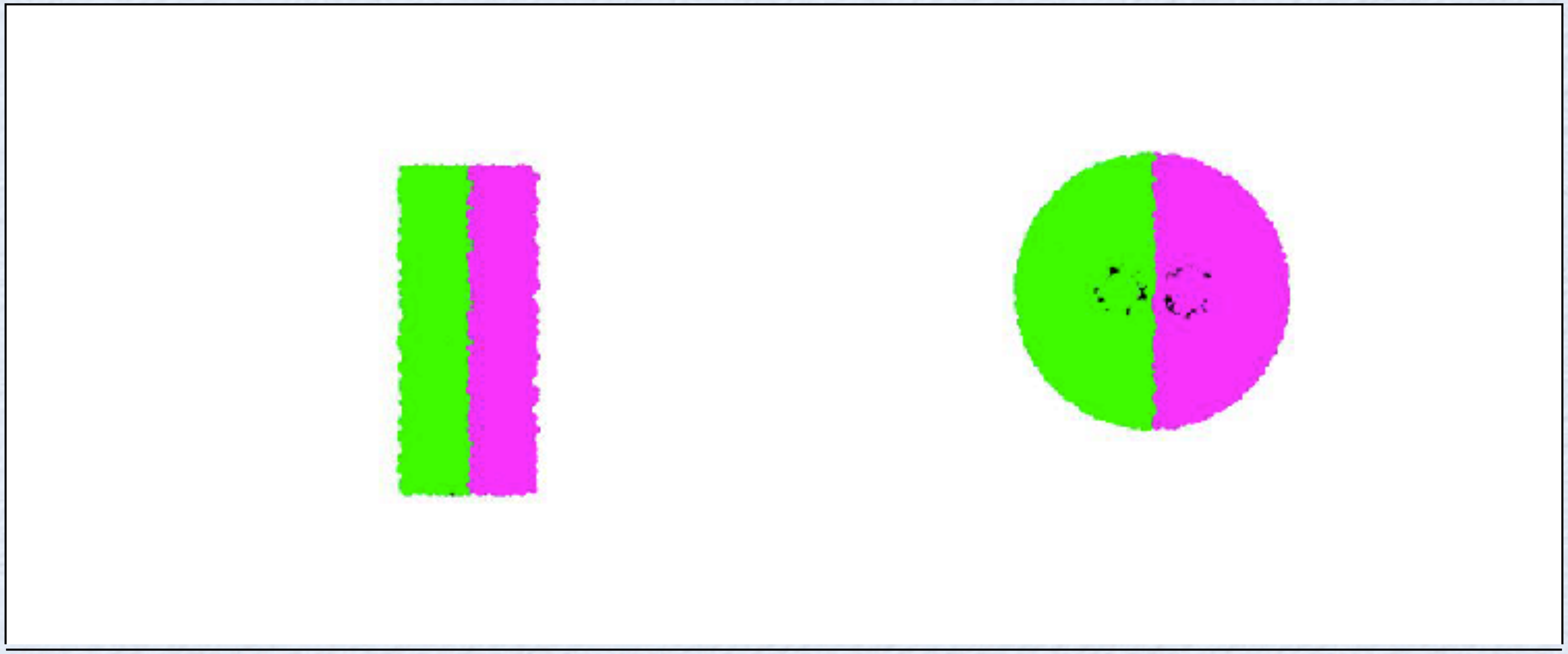




Flux through a filter window



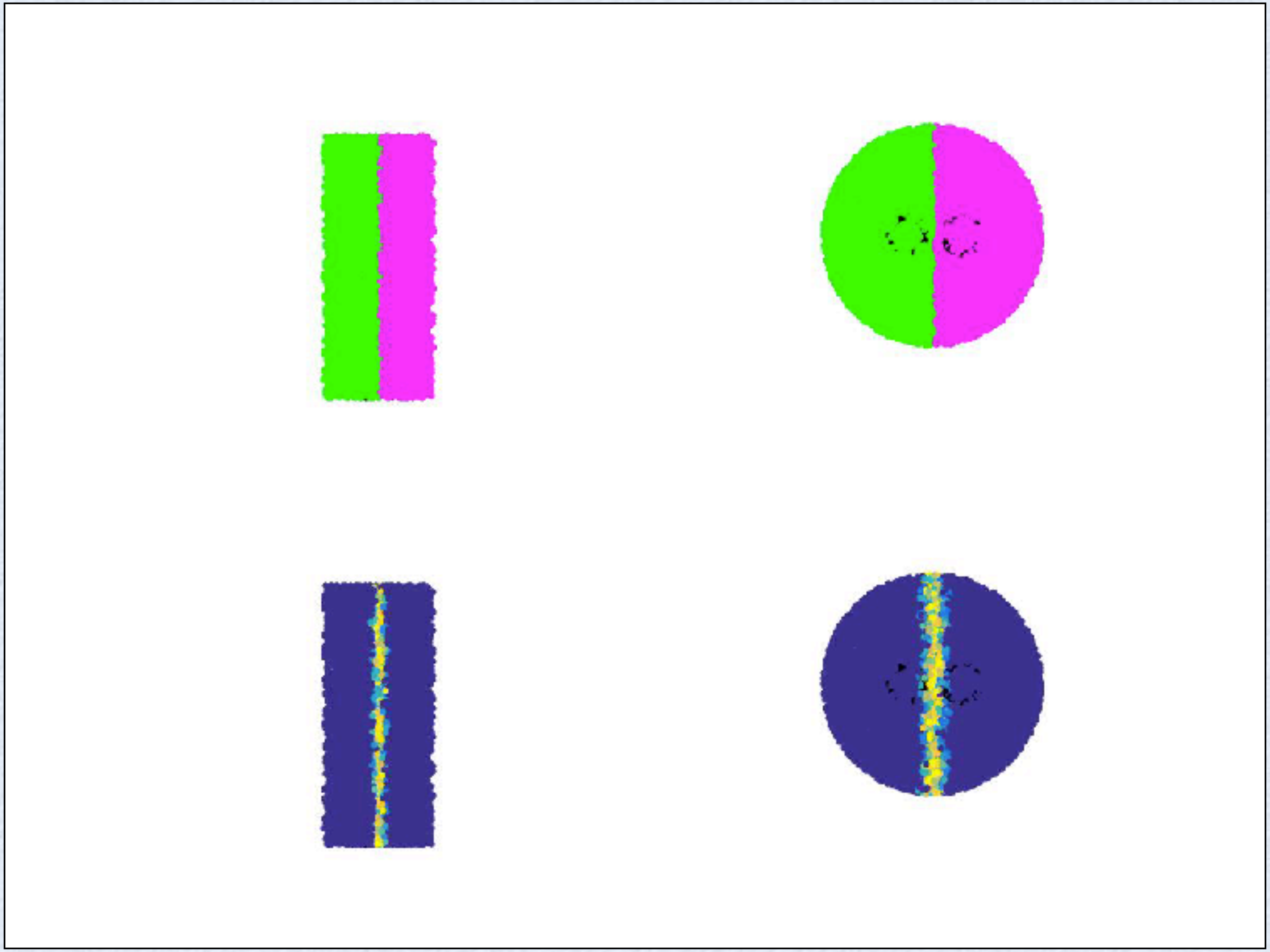
Particle Mixing



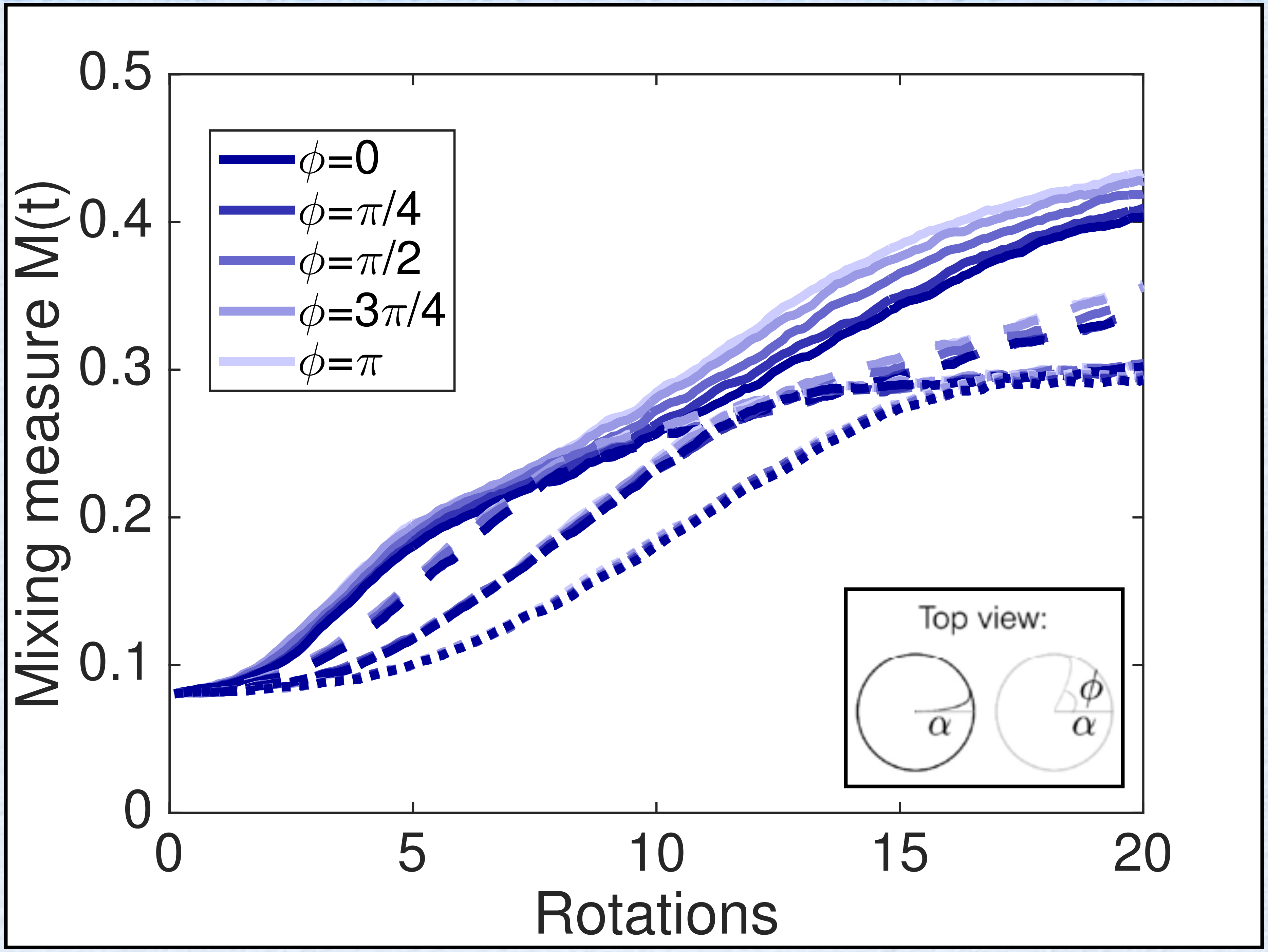
Mixing Measure

M. Robinson, P. Cleary, and J. Monaghan, AIChE journal 54, 1987 (2008)

Particle Mixing



Mixing Measure



Line style denotes helical spacing d = 3α (–), 4α (––), 5α (–·), 6α (· · ·)

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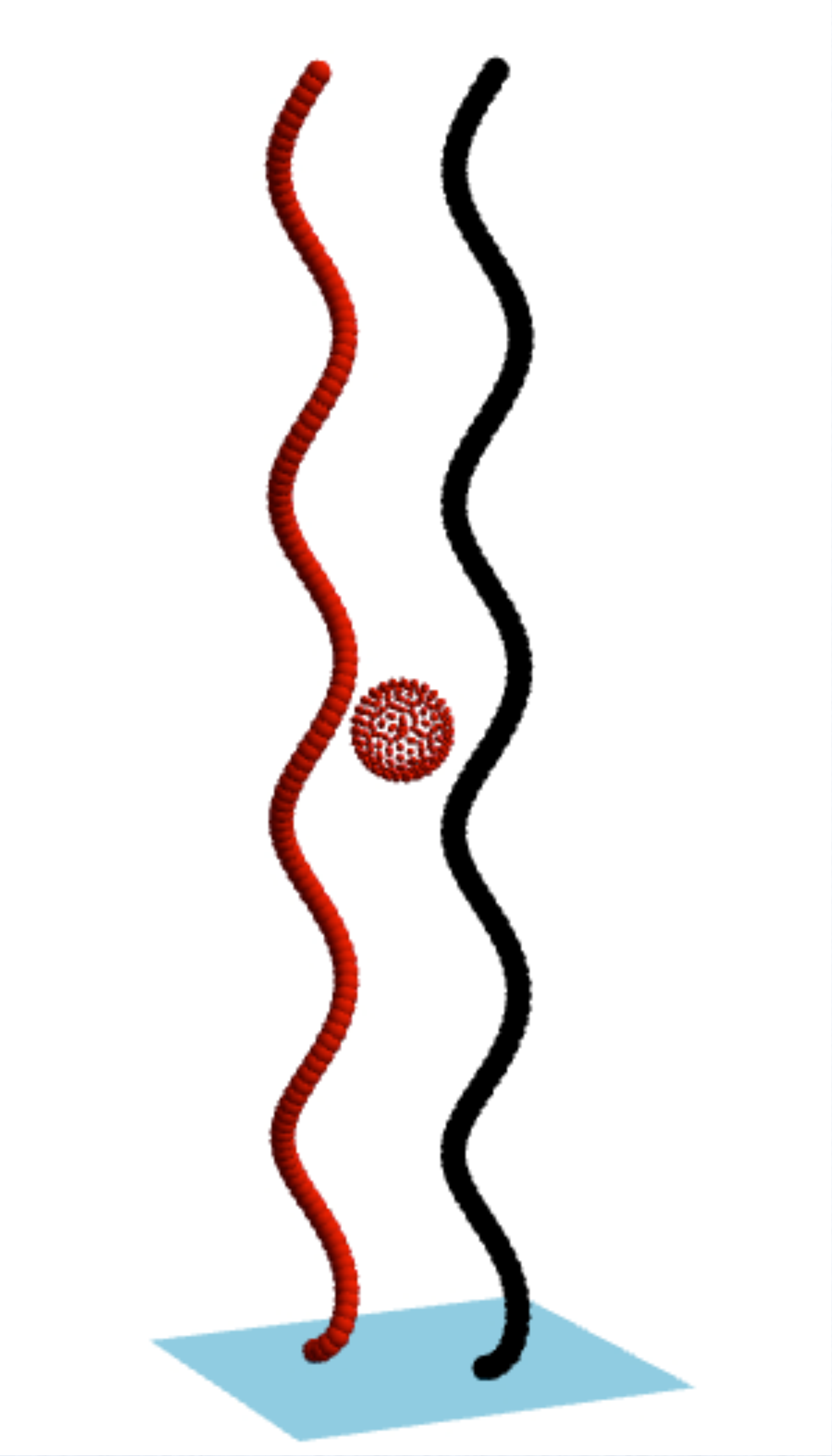

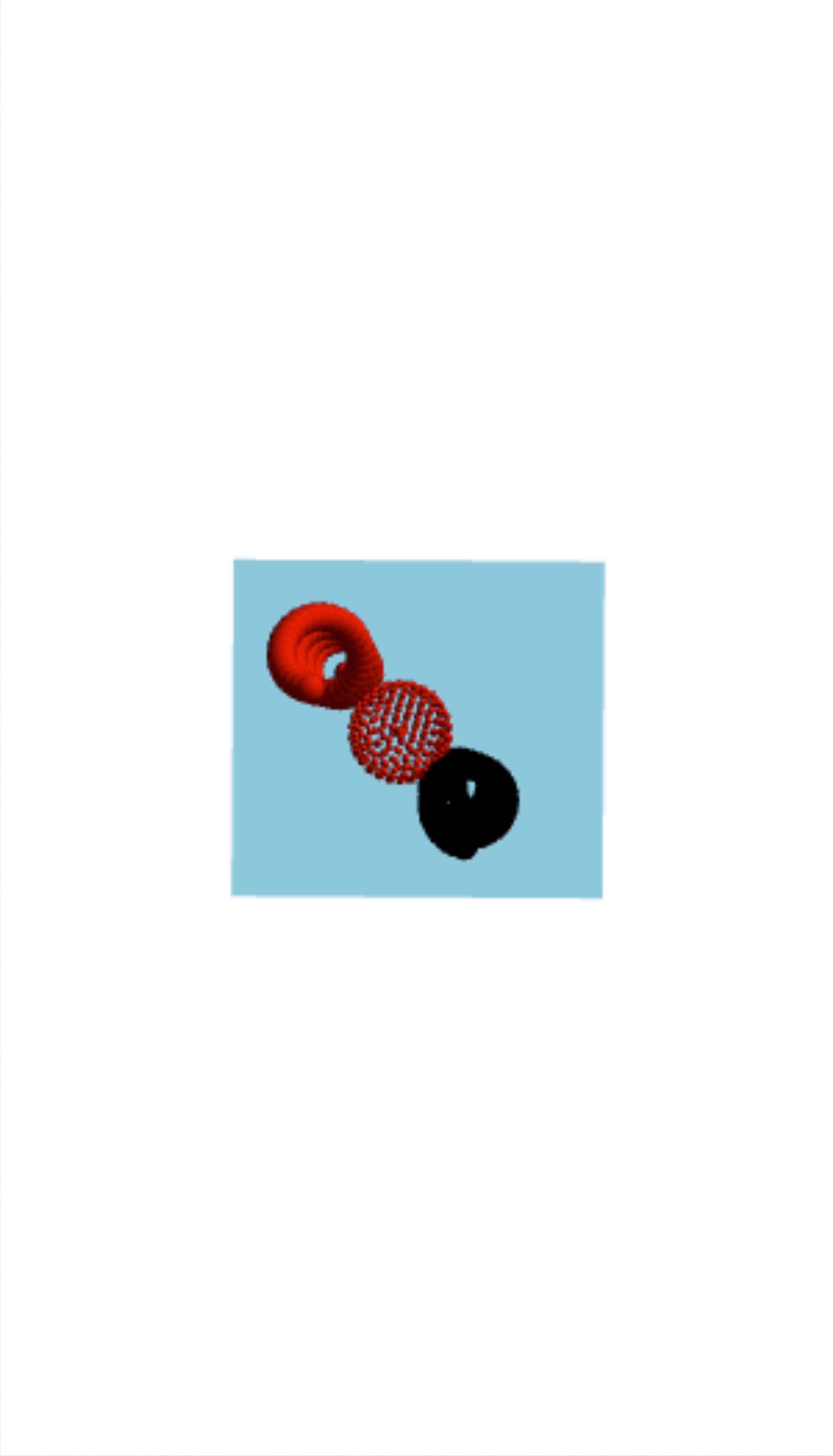

Summary and future work

- Develop a model of a collection of (rotating) helical flagella emanating from a planar wall
- Couple the flagella with elastic particles

We examined:

- Mixing and pumping ability of fluid near flagella
- Flow structure around the rotating flagella
- Interesting dynamics induced by multiple rotating helices

Are two helices twice as effective as one helix?

Future work

- Biologically calibrate parameters
- Flow through channels of bacterial carpets
- Lagrangian coherent structure

Acknowledge

IMA supported program, WhAM!

A Research Collaboration Workshop for Women in Applied Mathematics, Dynamical Systems with Applications to Biology and Medicine,

where discussions of this work first began.

Thank you!